Robust joint reconstruction of misaligned images using semi-parametric dictionaries

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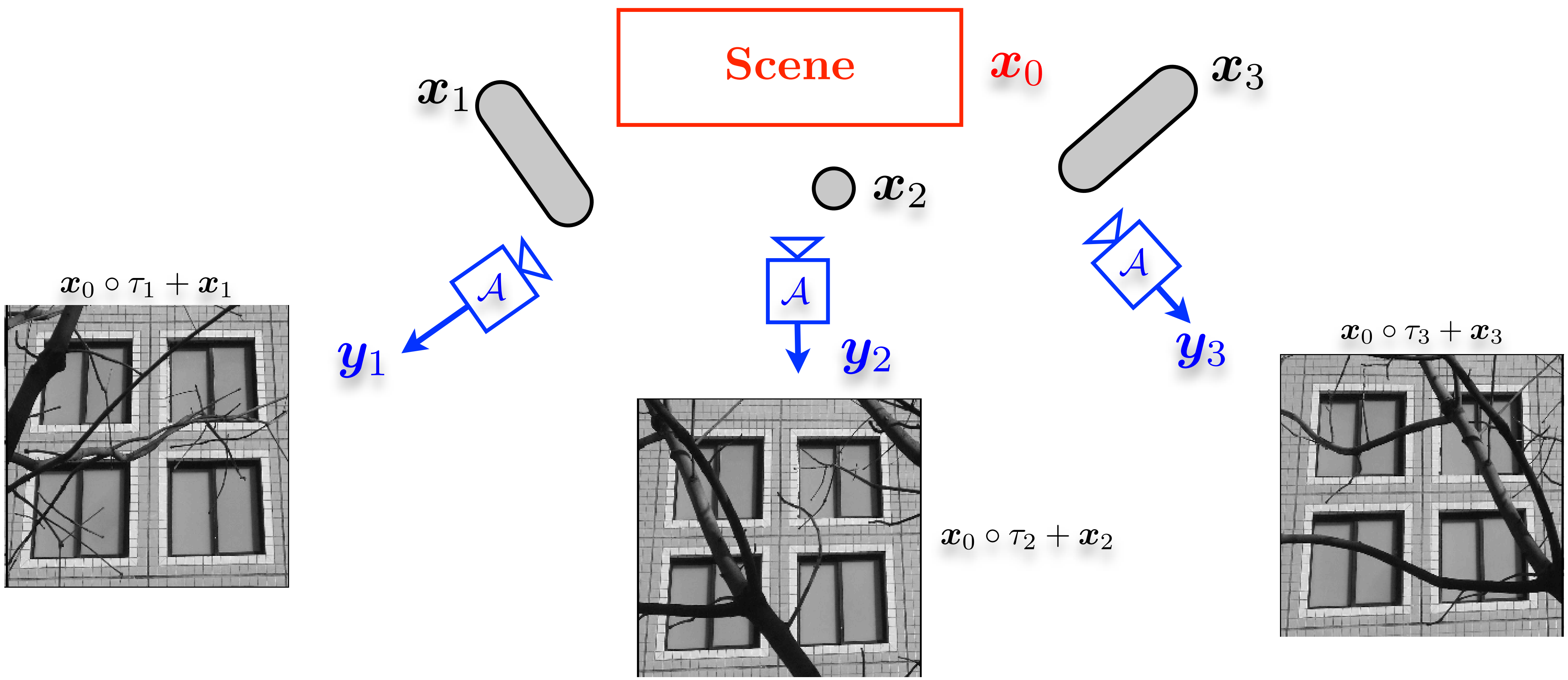
- I -

Motivation

Motivation

- A scene x_0 is observed from different point of views, providing l noisy measurement vectors $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_l \in \mathbb{R}^m$.
- The observation system is modeled by a linear operator $\mathcal{A}$.
- This scene undergoes geometric transformations $\tau_1, \ldots, \tau_l$ that depend on the position of the observer.
- The scene is partly occluded by some objects $x_1, \ldots, x_l$.



[1] Peng et al., CVPR, pp. 763-770, 2010

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- II -

Problem formulation

Problem formulation

- We discretize the images on a square grid of $\sqrt{n} \times \sqrt{n}$ pixels, $n \geqslant m$: $\boldsymbol{x}_0, \ldots, \boldsymbol{x}_l \in \mathbb{R}^n$.
- The linear operator $\mathcal{A}$ is represented by $\mathsf{A} \in \mathbb{R}^{m \times n}$.
- The transformations $\tau_1, \ldots, \tau_l$ belong to a transformation group represented by p parameters $\theta_j \in \mathbb{R}^p, \forall j \in \{1, \ldots, l\}$.
- The transformed image $x_0 \circ \tau_j$ on the discrete grid is obtained by applying $\mathsf{S}(\theta_j)$ to $\boldsymbol{x}_0$ (e.g., bilinear or bicubic spline interpolation).
- The observation model reads as

$$\underbrace{\begin{bmatrix} \boldsymbol{y}_1 \\ \vdots \\ \boldsymbol{y}_l \end{bmatrix}}_{\boldsymbol{y}} = \begin{bmatrix} \mathsf{AS}(\theta_1) & \mathsf{A} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{AS}(\theta_l) & 0 & \ldots & \mathsf{A} \end{bmatrix} \underbrace{\begin{bmatrix} \boldsymbol{x}_0 \\ \vdots \\ \boldsymbol{x}_l \end{bmatrix}}_{\boldsymbol{x}} + \begin{bmatrix} \boldsymbol{n}_1 \\ \vdots \\ \boldsymbol{n}_l \end{bmatrix}.$$

Problem formulation

- The inverse ill-posed problem is regularized by assuming that the scene $\boldsymbol{x}_0$ and the occluding objects $\boldsymbol{x}_1, \ldots \boldsymbol{x}_l$ are sparse in a wavelet basis $\mathsf{W} \in \mathbb{R}^{n\times n}$.
- The decomposition of $\boldsymbol{x}_j$ in W is denoted $\boldsymbol{\alpha}_j \in \mathbb{R}^{n\times n}$, $1 \leqslant j \leqslant l$.
- We want to solve the following non-convex problem:

$$\min_{\boldsymbol{\alpha},\theta} \|\boldsymbol{\alpha}\|_1 + \underset{?}{\kappa} \|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{y}\|_2^2 \ \text{ s.t. } \ \theta \in \mathcal{T},$$

$$\text{with } \ \boldsymbol{\alpha} = [\boldsymbol{\alpha}_0, \ldots, \boldsymbol{\alpha}_l]^{\mathsf{T}} \ \text{ and } \ \mathsf{A}(\theta) = \begin{bmatrix} \mathsf{AS}(\theta_1)\mathsf{W} & \mathsf{AW} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{AS}(\theta_l)\mathsf{W} & 0 & \ldots & \mathsf{AW} \end{bmatrix}.$$

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- We want to solve the following non-convex problem:

$$\min_{\boldsymbol{\alpha}, \boldsymbol{z}, \theta} \|\boldsymbol{\alpha}\|_1 + \underbrace{\kappa}_{\text{Large}} \|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{z}\|_2^2 \quad \text{s.t.} \quad \begin{cases} \|\boldsymbol{y}_1 - \boldsymbol{z}_1\|_2 \leqslant \epsilon_1 \\ \vdots \\ \|\boldsymbol{y}_l - \boldsymbol{z}_l\|_2 \leqslant \epsilon_l \end{cases} \quad \text{and } \theta \in \mathcal{T},$$

$$\text{with } \boldsymbol{\alpha} = [\boldsymbol{\alpha}_0, \ldots, \boldsymbol{\alpha}_l]^{\mathsf{T}}, \qquad \mathsf{A}(\theta) = \begin{bmatrix} \mathsf{AS}(\theta_1)\mathsf{W} & \mathsf{AW} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathsf{AS}(\theta_l)\mathsf{W} & 0 & \ldots & \mathsf{AW} \end{bmatrix},$$

and $\boldsymbol{z} = [\boldsymbol{z}_0, \ldots, \boldsymbol{z}_l]^{\mathsf{T}}$.

- III -

Method

Method

- Objective function: $\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta) = \|\boldsymbol{\alpha}\|_1 + \kappa \,\|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{z}\|_2^2 + i_{\mathcal{B}(\boldsymbol{y},\epsilon)}(\boldsymbol{z}) + i_{\mathcal{T}}(\theta)\,.$

  Indicator functions of the sets $\mathcal{B}(\boldsymbol{y}, \epsilon) = \{\boldsymbol{z} = \{\boldsymbol{z}_j\}_{1 \leqslant j \leqslant l} : \|\boldsymbol{y}_j - \boldsymbol{z}_j\|_2 \leqslant \epsilon_j\}$ and $\mathcal{T}$.

- Solve this non-convex problem using a proximal method (Attouch et al.):
  - Initializations: set $k = 0$, $\boldsymbol{\alpha}^0 = \mathbf{0} \in \mathbb{R}^{(l+1)n}$, $\boldsymbol{z}^0 = \boldsymbol{y}$, $\theta^0 \in \mathcal{T}$, choose $0 < \lambda_{\min} \leqslant \lambda_{\boldsymbol{z}}, \lambda_\theta, \{\lambda_{\boldsymbol{\alpha}}^k\}_{k \in \mathbb{N}} \leqslant \lambda_{\max}$ and $b > 0$.
  - Repeat:

    1) $(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}) \in \underset{\boldsymbol{\alpha}, \boldsymbol{z}}{\operatorname{argmin}}\, \mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta^k) + \lambda_z \|\boldsymbol{z} - \boldsymbol{z}^k\|_2^2 + \lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha} - \boldsymbol{\alpha}^k\|_2^2.$

    2) Find $\theta^{k+1} \in \mathcal{T}$ such that:

    $$\kappa \,\|\mathsf{A}(\theta^{k+1})\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2 + \lambda_\theta \|\theta^{k+1} - \theta^k\|_2^2 \leqslant \kappa \,\|\mathsf{A}(\theta^k)\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2,$$

    $$\|\nabla_\theta \mathcal{L}(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}, \theta^{k+1})\|_2^2 \leqslant b \|\theta^{k+1} - \theta^k\|_2^2.$$

    3) $k \leftarrow k + 1$.

- Under some mild conditions, the algorithm converges to a critical point of $\mathcal{L}$.

[2] Attouch et al., Mathematics of Operations Research, vol. 35(2), pp. 438-457, 2010.

[3] Attouch et al., J. Mathematical programming, 2011.

Method

- Objective function: $\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta) = \|\boldsymbol{\alpha}\|_1 + \kappa \|\mathsf{A}(\theta)\,\boldsymbol{\alpha} - \boldsymbol{z}\|_2^2 + i_{\mathcal{B}(\boldsymbol{y},\epsilon)}(\boldsymbol{z}) + i_{\mathcal{T}}(\theta)$.

  Indicator functions of the sets $\mathcal{B}(\boldsymbol{y}, \epsilon) = \{\boldsymbol{z} = \{\boldsymbol{z}_j\}_{1 \leqslant j \leqslant l} : \|\boldsymbol{y}_j - \boldsymbol{z}_j\|_2 \leqslant \epsilon_j\}$ and $\mathcal{T}$.

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  - Repeat:

    1) $(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}) \in \underset{\boldsymbol{\alpha}, \boldsymbol{z}}{\operatorname{argmin}} \, \mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{z}, \theta^k) + \lambda_z \|\boldsymbol{z} - \boldsymbol{z}^k\|_2^2 + \lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha} - \boldsymbol{\alpha}^k\|_1$.

    2) Find $\theta^{k+1} \in \mathcal{T}$ such that:

    $$\kappa \|\mathsf{A}(\theta^{k+1})\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2 + \lambda_\theta \|\theta^{k+1} - \theta^k\|_2^2 \leqslant \kappa \|\mathsf{A}(\theta^k)\,\boldsymbol{\alpha}^{k+1} - \boldsymbol{z}^{k+1}\|_2^2,$$

    $$\|\nabla_\theta \mathcal{L}(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}, \theta^{k+1})\|_2^2 \leqslant b \|\theta^{k+1} - \theta^k\|_2^2.$$

    3) $k \leftarrow k + 1$.

  until *convergence*.

[4] Attouch et al., Mathematics of Operations Research, vol. 35(2), pp. 438-457, 2010.
[5] Attouch et al., J. Mathematical programming, 2011.

Method

- By construction $\mathcal{L}$ is not increasing. Indeed, $k \geqslant 0$,

$$\mathcal{L}(\boldsymbol{\alpha}^{k+1}, \boldsymbol{z}^{k+1}, \theta^{k+1}) + \lambda_{\boldsymbol{z}} \|\boldsymbol{z}^{k+1} - \boldsymbol{z}^k\|_2^2 + \lambda_\theta \|\theta^{k+1} - \theta^k\|_2^2 + \lambda_{\boldsymbol{\alpha}}^k \|\boldsymbol{\alpha}^{k+1} - \boldsymbol{\alpha}^k\|_1 \leqslant \mathcal{L}(\boldsymbol{\alpha}^k, \boldsymbol{z}^k, \theta^k).$$

- We also have:

$$\sum_{k=0}^{+\infty} \|\boldsymbol{z}^{k+1} - \boldsymbol{z}^k\|_2^2 + \|\theta^{k+1} - \theta^k\|_2^2 + \|\boldsymbol{\alpha}^{k+1} - \boldsymbol{\alpha}^k\|_1 < +\infty.$$

- Convergence to a critical point of $\mathcal{L}$?

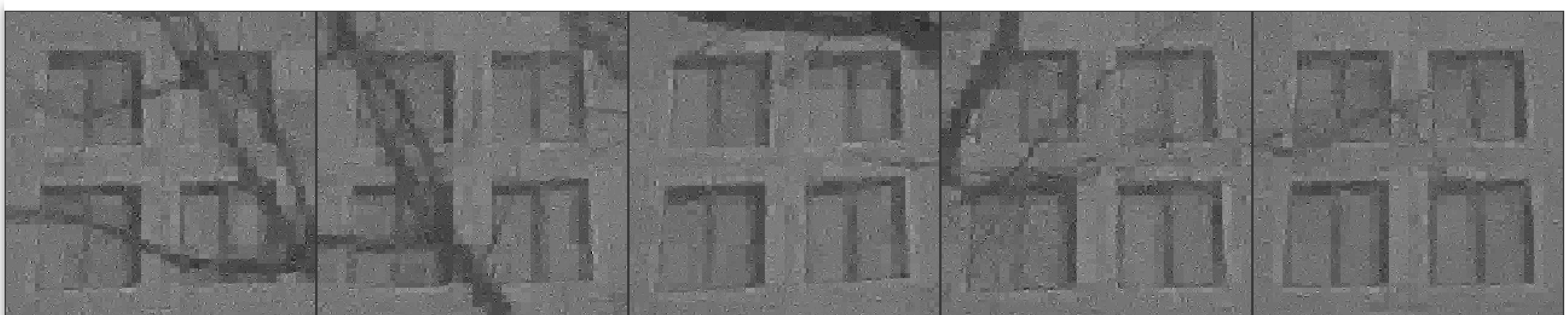
- IV -

Simulation results

Simulations results

- $m = 0.1n$ measurements per image obtained with the spread spectrum technique [6].
- The transformations are assumed to be homographies modeled by 8 unknown parameters.

Original images



Solving the Basis Pursuit independently for each image

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

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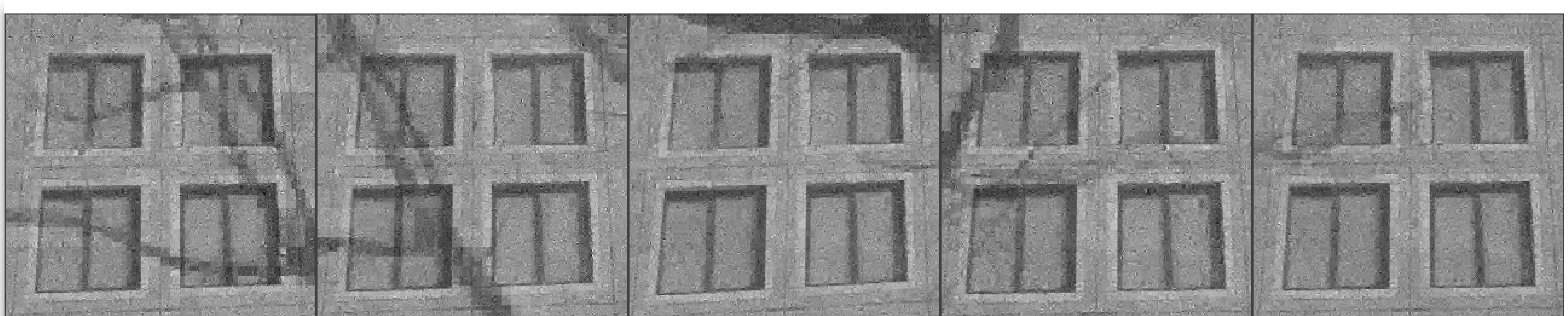

Reconstruction with the proposed method

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

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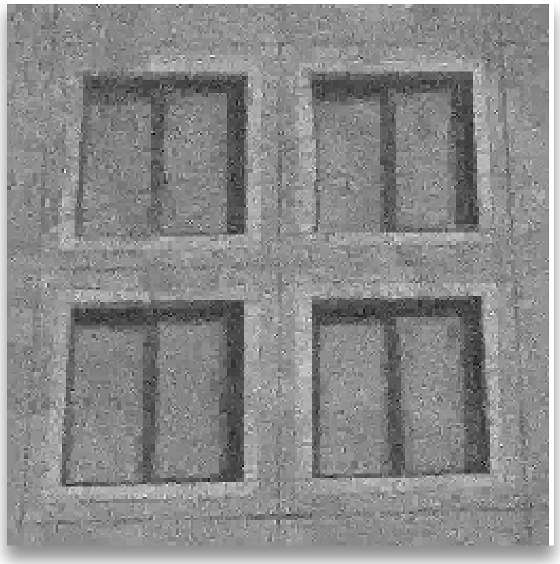

Background image

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

Simulations results

- $m = 0.1n$ measurements per image obtained with the spread spectrum technique [6].
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Original images

Foreground images

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

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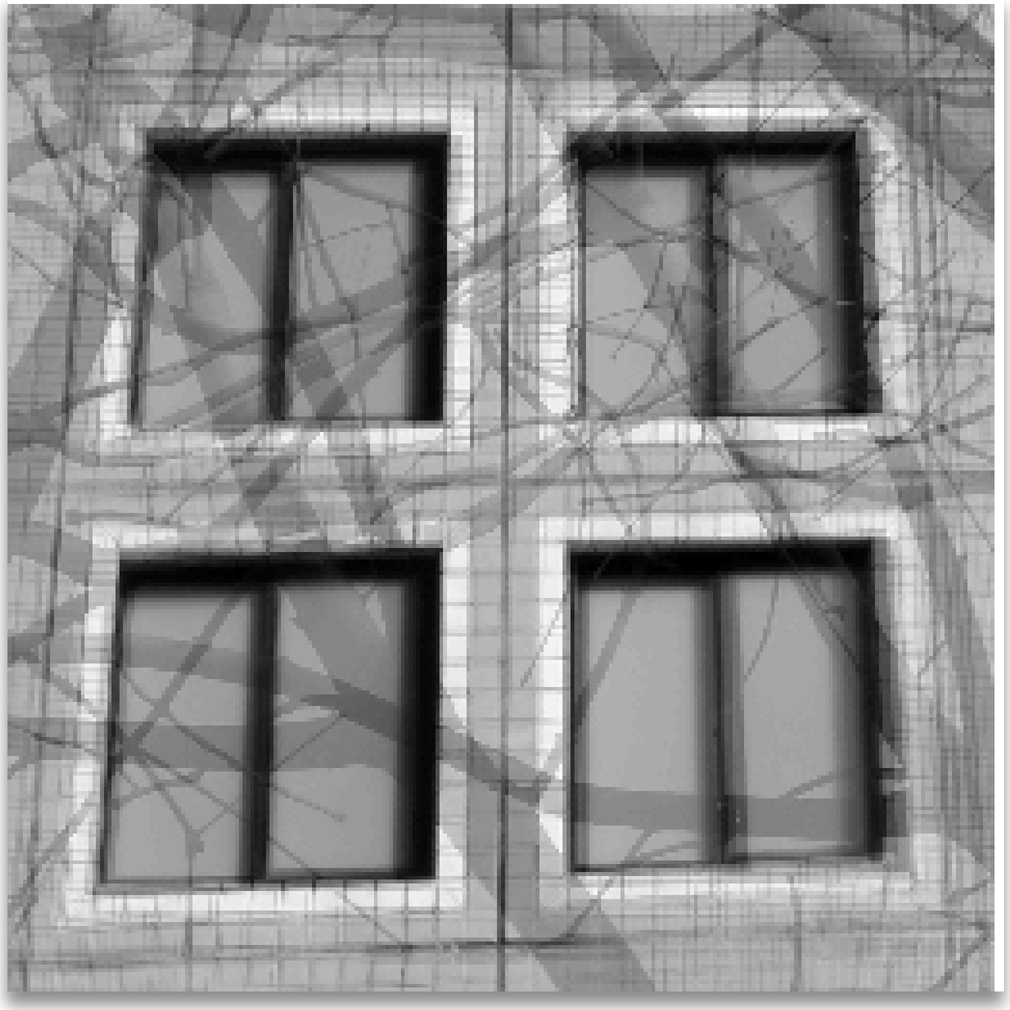

Superposed unregistered images.

Superposed registered images.

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

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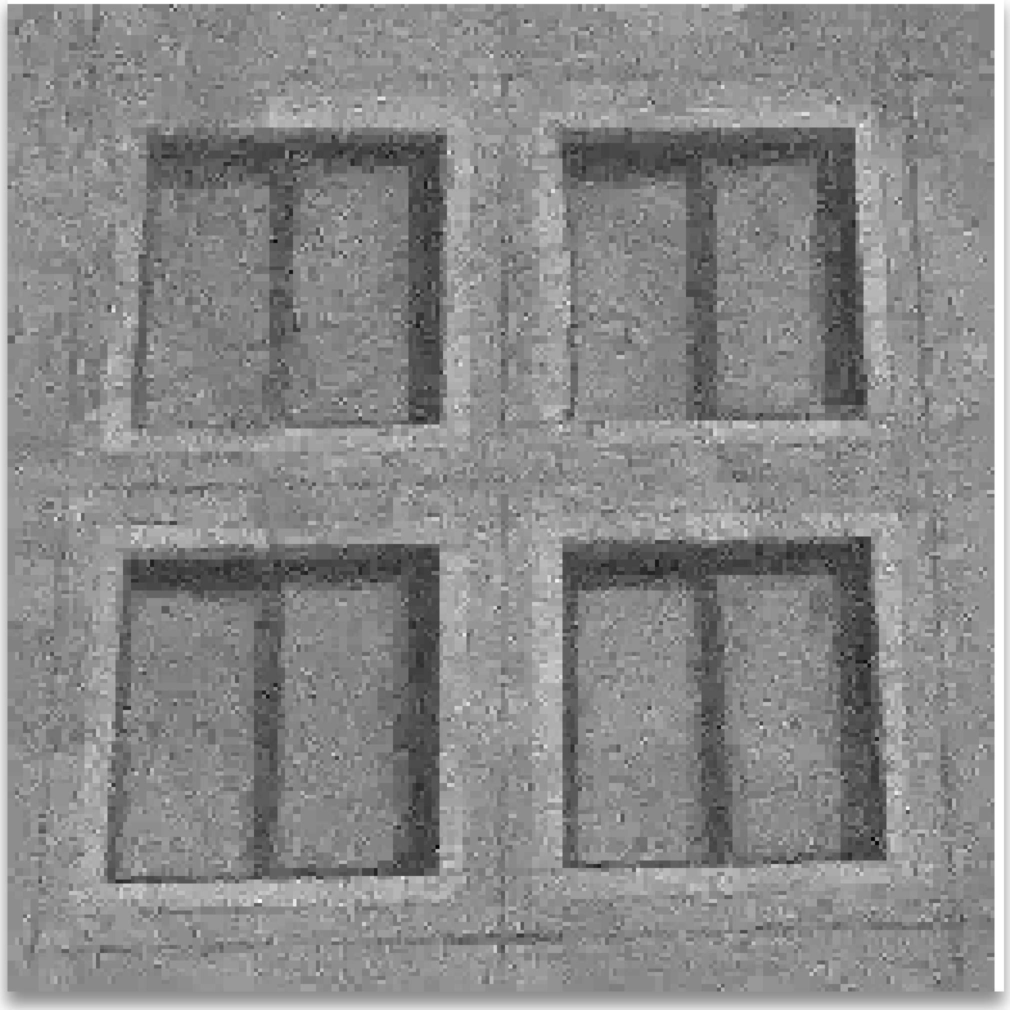
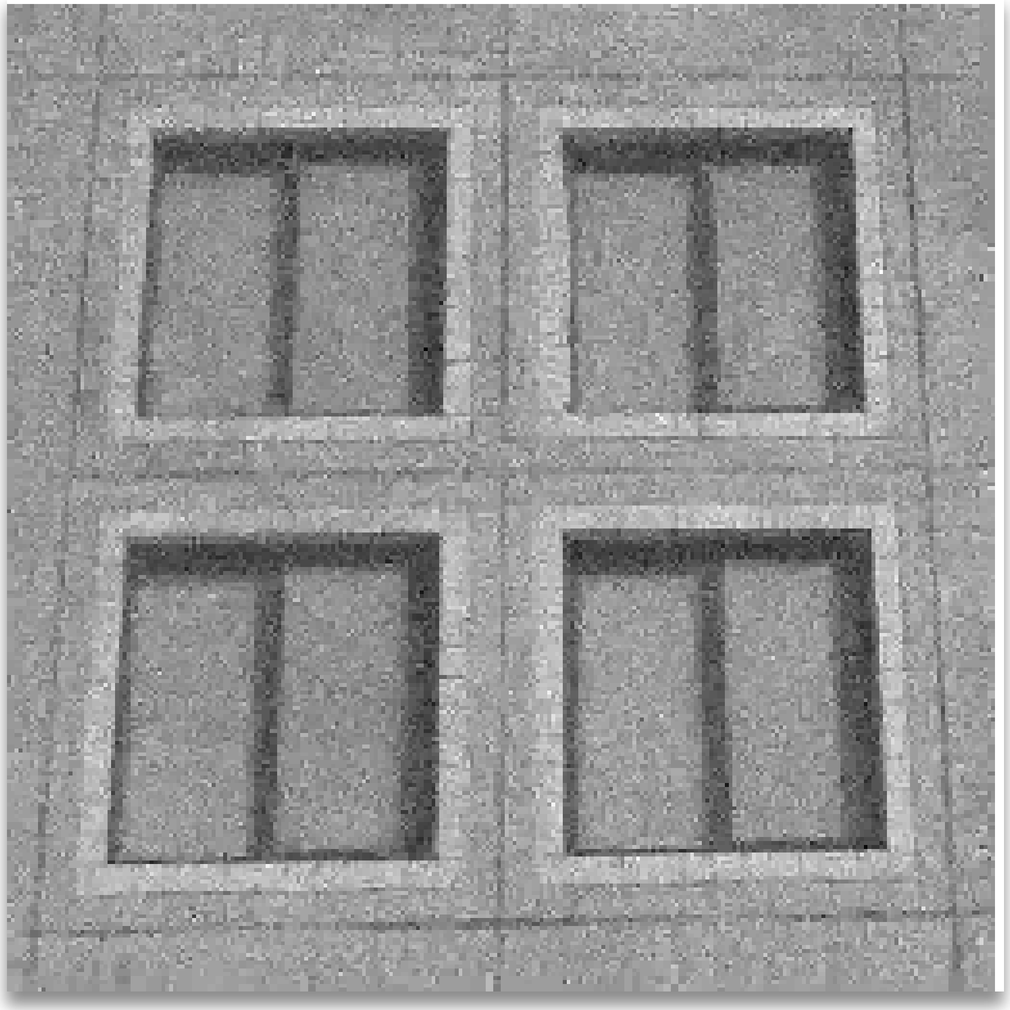


Estimated background image with
5 measurements vectors



Estimated background image with
10 measurements vectors

[6] Puy et al., EURASIP Journal on Advances in Signal Processing, vol. 2012(6), 2012.

Simulations results

- Repeat the experiments for different number of measurements and noise levels on the following 5 images:

Castle-R20 dataset available at cvlab.epfl.ch (Strecha et al., CVPR, 2008).

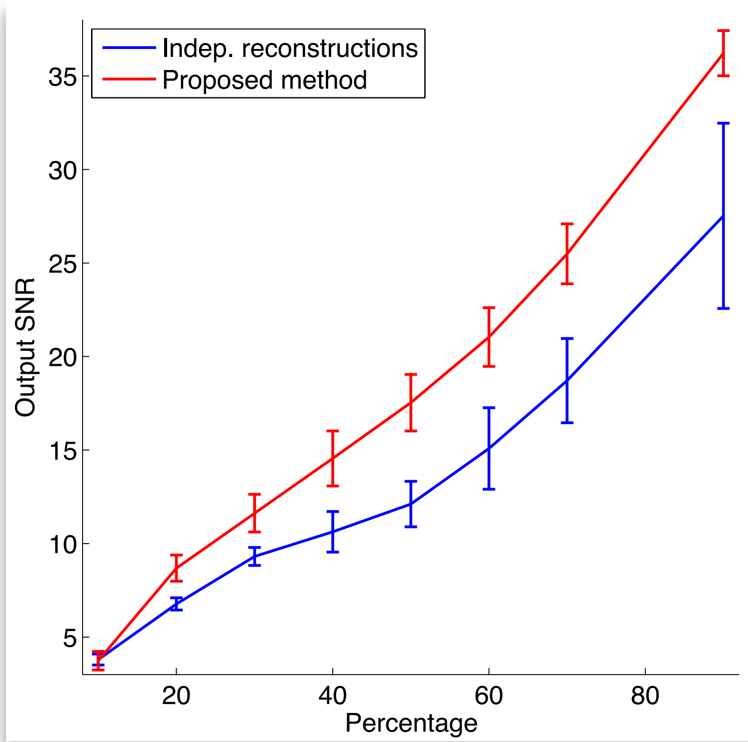


Image quality vs. Nb. measurements

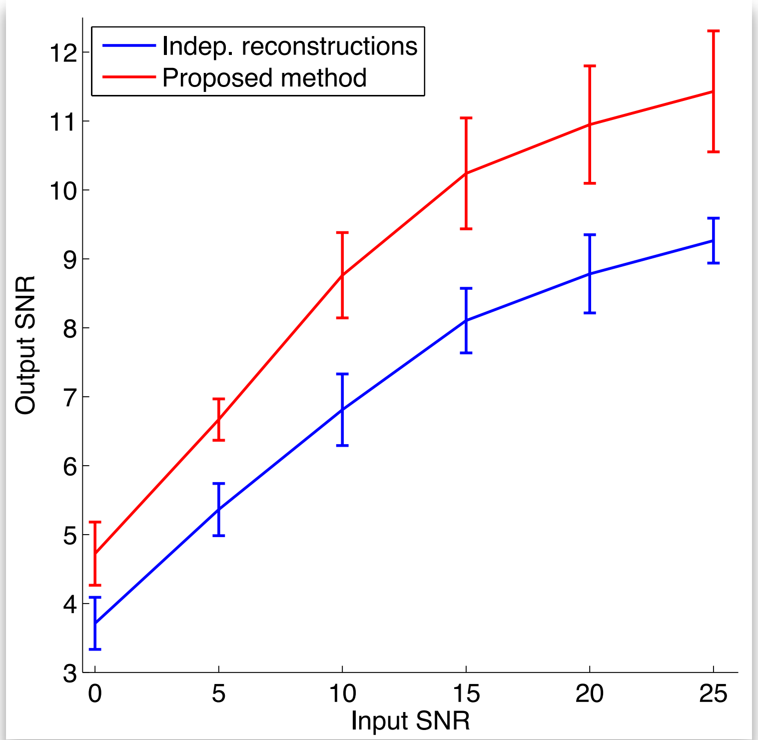


Image quality vs. Noise level

Simulations results

- Repeat the experiments for different number of measurements and noise levels on the following 5 images:

From 30% of measurements

Superposed unregistered images.

Superposed registered images.

- V -

Conclusion & Perspectives

Conclusion & Perspectives

- We proposed a method for compressed multi-view imaging that:
  - unifies the reconstruction and the registration in the same setting.
  - is robust to occlusions and noise measurements.
  - separates automatically the background image from the foreground images (occlusions)

- Application to free breathing coronary MRI or magnetic resonance spectroscopic imaging.