Comment on “Explicit equations for infiltration” by Prabhata K. Swamee, Pushpa N. Rathie and Luan Carlos de S.M. Ozelim, Journal of Hydrology 426–427 (2012) 151–153

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Swamee et al. (2012) presented an explicit approximation for the (implicit) three-parameter infiltration equation of Parlange et al. (1982). Our main purpose is to suggest how they might improve their formula based on an existing approximation for the three-parameter infiltration equation, given by Parlange et al. (2002).

The infiltration formulas of Green and Ampt (1911), Talsma and Parlange (1972) and Parlange et al. (1982), given here as Eqs. (1)-(3), respectively, are:

$$t_* = F_* - \ln(1 + F_*), \tag{1}$$

$$t_* = F_* + \exp(-F_*) - 1, \tag{2}$$

$$t_* = F_* - (1 - \alpha)^{-1}\ln[\alpha^{-1} - (\alpha^{-1} - 1)\exp(-\alpha F_*)], \tag{3}$$

where we have used the notation of Swamee et al. (2012), i.e., t_* is dimensionless time, F_* is dimensionless infiltration and α is the interpolation parameter introduced by Parlange et al. (1982). Equations (1) and (2) are, respectively, the $\alpha = 0$ and $\alpha = 1$ limits of Eq. (3), i.e., Eq. (3) interpolates between Eqs. (1) and (2).

To find infiltration as a function of time, $F_*(t_*)$, each of Eqs. (1)-(3) must be inverted. For the inversion of Eq. (3), Swamee et al. (2012) presented the approximation:

$$F_* = \left[(1.94 + 0.72\alpha^{1.76}){t_*}^{0.74+0.384\alpha^{1.25}} + {t_*}^{\left(0.7-0.24\alpha^{0.844}\right)^{-1}}\right]^{0.7-0.24\alpha^{0.844}}, \tag{4}$$

taking $\alpha = 0$ and 1 to invert Eqs. (1) and (2), respectively. Swamee et al. (2012) developed Eq. (4) as an alternative to using the analytical approximations of the Lambert W function (Barry et al., 1993, 1995, 2000, 2002, 2005). Parlange et al. (2002) provided the inversion:

$$F_* = t_* + (1 - \alpha)^{-1}\ln\left[1 + (\alpha^{-1} - 1)\sqrt{1 - f}\right], \tag{5}$$

where

$$f = \exp\left[-2\alpha^2 t_* \left(\frac{1 + A\sqrt{2t_*} + 2Bt_*}{1 + C\sqrt{2t_*} + 2Bt_*\sqrt{2\alpha}}\right)\right], \tag{6}$$

$$A = \frac{1}{2} + \frac{\lambda - 2\alpha}{3}, \tag{7}$$

$$B = \frac{1 + \sqrt{2\alpha}}{12}\left(\frac{4\lambda - 11\alpha}{3} + 1\right), \tag{8}$$

$$C = \frac{1}{6} + \frac{\lambda}{3} \tag{9}$$

and

$$\lambda = \frac{35}{17}\alpha - \frac{3}{2}\alpha^{\frac{1}{4}}\exp\left(-\frac{15}{4}\sqrt{\alpha}\right). \tag{10}$$

Equation (5) is valid for all t_* and has a maximum relative error of 0.048% (Parlange et al., 2002).

Certainly, Eq. (5) is more complex than Eq. (4), but, by construction, it satisfies three important conditions, none of which are obeyed by Eq. (4). First, for $\alpha = ½$, Eq. (5) becomes:

$$F_* = t_* + 2\ln\left[1 + \sqrt{1 - \exp\left(-\frac{t_*}{2}\right)}\right], \tag{11}$$

which is the exact solution to Eq. (3) for this case (Parlange et al., 2002). Second, the exact short-time limit of Eq. (3) is:

$$\lim_{t_* \to 0} F_* = \sqrt{2t_*}, \tag{12}$$

which is also given by Eq. (5). Third, the long-time limit of Eq. (3) is, for $\alpha \neq 0$:

$$\lim_{t_* \to \infty} F_* = t_* - (1 - \alpha)\ln(\alpha), \tag{13}$$

and, for $\alpha = 0$:

$$\lim_{t_* \to \infty} F_* = t_* + \ln(t_*). \tag{14}$$

A systematic comparison was made of the relative and absolute errors of Eqs. (4) and (5) over the range of applicability of Eq. (4), with results given for both relative and absolute errors in Fig. 1. For the relative error, the ordinate indicates the number of leading digits that are correct. Because Eq. (4) was based on an optimization procedure over the range, $10^{-3} \le t_* \le 10^3$, it is not surprising that it loses accuracy outside this range (not plotted). It would be, we suggest, useful to amend Eq. (4) to satisfy the conditions given by Eqs. (11)-(14). This would result in a simple, useful but more accurate expression.

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Figure

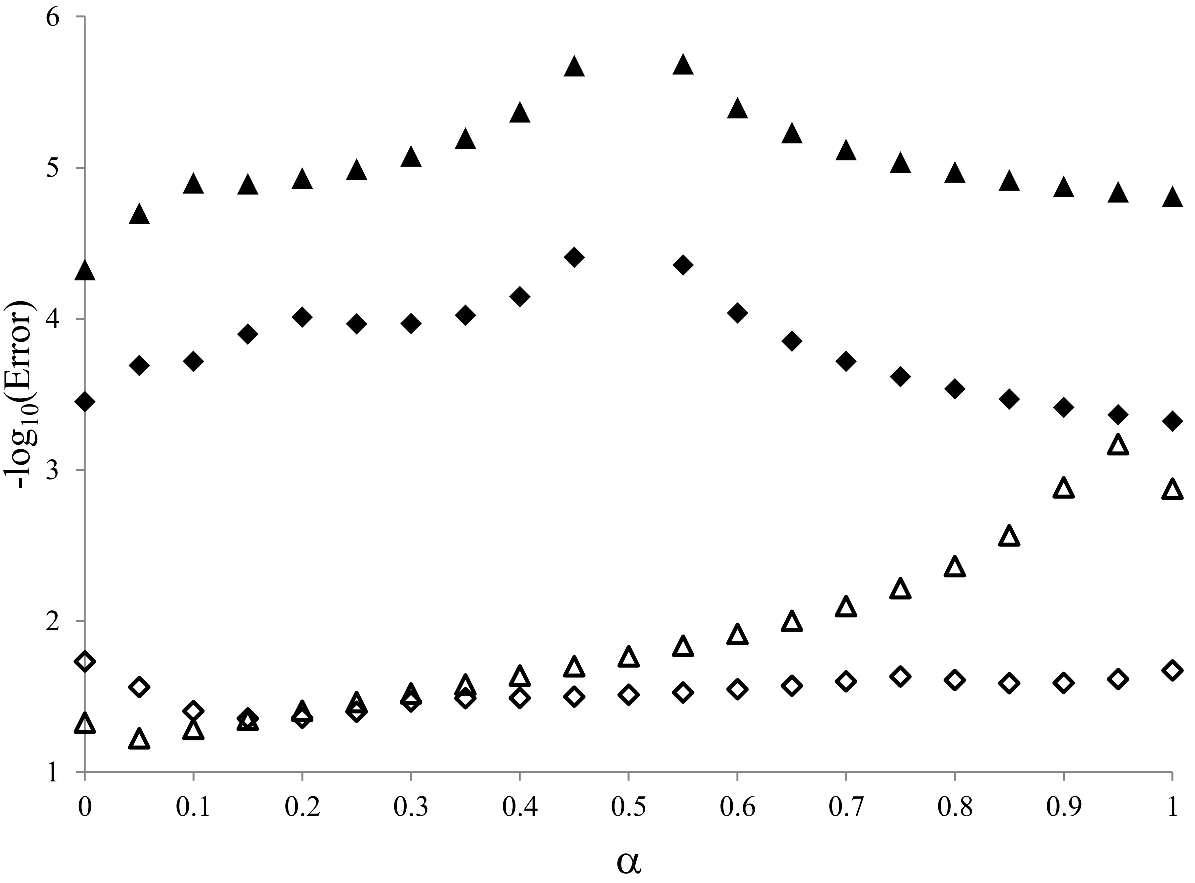

Figure 1. Maximum errors of Eqs. (4) and (5), given by line-drawn and filled symbols, respectively, calculated for $10^{-3} \leq t_* \leq 10^3$. Diamonds show the relative error and triangles the absolute error. In the case of the relative error (diamonds), plotting in this form indicates the number of digits of accuracy (Barry et al., 1995). Equation (5) is exact for $\alpha = ½$, thus the number of correct digits is, in principle (i.e., subject to machine computation/round-off errors only), infinite.