On input design for direct data-driven controller tuning ⋆

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Abstract: In recent years, noniterative Correlation-based Tuning (CbT) and Virtual Reference Feedback Tuning (VRFT) have been proposed as an alternative to the standard model-based approach for model-reference control design. In this work, the problem of input design for direct data-driven controller tuning methods is investigated. For bounded input energy, the excitation signal is designed such that the bias on the expectation of the control criterion is reduced. The above strategy is numerically tested on a benchmark example.

Keywords: VRFT; correlation-based tuning; input design; direct data-driven control

1. INTRODUCTION

In system identification theory, optimal experiment design is about finding the operating conditions that provide the most informative data for modeling the plant. However, depending on the intended model application, the optimal experiments appear to be very different. In control applications, the model is used to design a suitable controller, and therefore the final aim for identification and input design is not to accurately describe the mathematical structure of the system, but to obtain a closed-loop behaviour with some desired properties. Recently, the term "identification for control" has been introduced to refer to identification from a control-oriented perspective (see H. Hjalmarsson [2005] for a survey). In this research area, assessing the model quality by experiment design is of primary importance, as is witnessed by a large set of contributions, see *e.g.* M. Gevers [1996] and M Gevers et al. [1986]. The problem is generally difficult due to many reasons, one of which is the fact that control-relevant criteria typically depend on the ideal controller, designed with respect to the true system. Moreover, to the authors' knowledge, only the case of full-parameterization is treated, *i.e.* the case where the real system belongs to the model set, except for X. Bombois et al. [2006], where upper bounds on modeling error are considered to deal with undermodeling.

As far as the authors are aware, input design for direct data-driven controller tuning, *i.e.* the case where a fixed-order controller is directly identified from data without modeling the plant, has not been considered yet. These methods can be very useful when a mathematical description of the plant is a costly and time-consuming undertaking. However, as in standard system-identification, a deep understanding of the asymptotic variability of the estimate is needed, as this setting corresponds to all real situations. This paper attempts to obtain some insight into statistical properties of noniterative data-driven techniques, *i.e.* noniterative Correlation-based Tuning (CbT) and Virtual Reference Feedback Tuning (VRFT), whereof the interesting feature is that they provide a global solution to a model-reference control issue via simple least squares techniques. The above methodologies have been only recently introduced, respectively in A. Karimi et al. [2007] and in M.C. Campi et al. [2002]. Iterative data-driven methods are instead not subjects of the present work.

The main goal of the paper is to carry out an input design methodology to reduce the bias effect related to the presence of noise that affects the expected value of the final control criterion. This will be shown straightforward as, unlike the standard "model-based" approach, the "direct" philosophy will allow one to establish an explicit relationship between the aforementioned expected value and the input spectrum.

The outline of the paper is as follows. In Section 2, preliminaries on noniterative CbT and VRFT are briefly recalled. The main analysis results and the input design procedure are discussed in Section 3, while Section 4 demonstrates the effectiveness of the above method on the benchmark simulation example proposed in I. D. Landau et al. [1995]. Some concluding remarks end the paper.

2. BACKGROUNDS

Consider the unknown LTI SISO stable plant $G(q^{-1})$, where q^{-1} denotes the backward shift operator. The objective of the model-reference control problem is to design a linear, fixed-order controller $K(q^{-1}, \rho)$, parameterized through ρ, for which the closed-loop system matches the given stable strictly proper reference model $M(q^{-1})$ (see

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Figure 1). More specifically, let the controller parameterization be $K(q^{-1}, \rho) = \beta^T(q^{-1})\rho$, where $\beta(q^{-1})$ is a vector of n linear discrete-time transfer operators.
Formally, the aim is to find the vector of parameters that minimizes the (filtered) $\mathcal{L}_2$-norm of the difference between the reference model and the achieved closed-loop system:

$$J(\rho) = \left\| \left(\frac{GK(\rho)}{1+GK(\rho)} - M \right) W \right\|_2^2, \qquad (1)$$

where $W(q^{-1})$ is a user-defined frequency-weighting filter. Consider that an open-loop collection of input-output

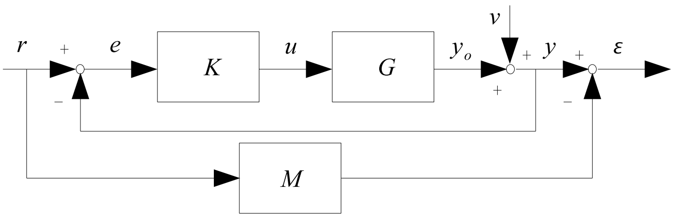

Fig. 1. Model reference control problem.

(I/O) data $\{u(t), y(t)\}_{t=1,\ldots,N}$ is available. Let the output $y(t)$ be affected by additive noise $v(t) = H(q^{-1})e(t)$, where $H(q^{-1})$ is an unknown stable LTI system and $e(t)$ is a zero mean white Gaussian noise with variance of σ^2.
In standard "indirect" data-driven approaches, the above objective can be achieved by identifying from data a model $\hat{G}$ of the plant and designing a model-based controller $K(\hat{G})$. Unfortunately, the controller that makes $J(\rho) = 0$ is given by $K_o(q^{-1}) = M(q^{-1})/(G(q^{-1})(1 - M(q^{-1})))$, that might be of very high order since it depends on the unknown and possibly high-order plant $G(q^{-1})$. Furthermore, $K_o(q^{-1})$ might be non-causal. It follows that model-based design of fixed-order controller is not a trivial task and the use of "direct" techniques that do not need the plant model may sometimes be preferable.

2.1 Noniterative correlation-based Tuning

By using $u(t)$ as reference signal and approximating the actual sensitivity function with the ideal one, the model-matching error $\varepsilon(t, \rho)$ can be directly computed from data, as in the noiseless setting it is expressed by

$$\begin{aligned}\varepsilon(t,\rho) &= M(q^{-1})r(t) - (1-M(q^{-1}))K(q^{-1},\rho)G(q^{-1})r(t)\\ &= M(q^{-1})u(t) - (1-M(q^{-1}))K(q^{-1},\rho)y(t).\end{aligned}$$

In case of noiseless environment and full parameterization of $K(q^{-1}, \rho)$, *i.e.* when $K_o(q^{-1})$ belongs to the controller set, it is straightforward that the minimizer of the two-norm of $\varepsilon(t, \rho)$ is exactly $K_o(q^{-1})$. When data are collected in a noisy environment, the method resorts to the correlation approach to identify the controller. Specifically, an extended instrumental variable $\zeta(t)$ correlated with $u(t)$ and uncorrelated with $v(t)$ is introduced to decorrelate the error signal $\varepsilon(t)$ and the reference signal $r(t)$. $\zeta(t)$ is defined as

$$\zeta(t) = [u(t+l), \ldots, u(t), \ldots, u(t-l)]^T, \qquad (2)$$

where l is a sufficiently large integer. The correlation function is defined as

$$f_{N,l}(\rho) = \frac{1}{N}\sum_{t=1}^{N} \zeta(t)\varepsilon(t,\rho) \qquad (3)$$

and the correlation criterion as

$$J_{N,l}(\rho) = f_{N,l}^T(\rho) f_{N,l}(\rho). \qquad (4)$$

The global minimizer of (4) tends to K_o as N and l tend to infinity and l/N to zero (see K. van Heusden, A. Karimi and D. Bonvin [2010] for technical details).
When the optimal controller does not belong to the controller set, the criteria (4) and (1) no longer share the same minimum point. However, it has been proven in K. van Heusden, A. Karimi and D. Bonvin [2010] that the optimizer of (4) tends w.p.1 to the optimizer of (1) as $N, l \to \infty$ and $l/N \to 0$, if data in $\varepsilon(t, \rho)$ are prefiltered by $L_C(q^{-1})$, defined as

$$L_C(e^{-j\omega}) = \frac{\left(1 - M(e^{-j\omega})\right) W(e^{-j\omega})}{\Phi_u(\omega)}, \qquad (5)$$

where $\Phi_u(\omega)$ denotes the spectral density of $u(t)$. Notice that such prefilter may be non-causal but it can be implemented off-line.

2.2 Virtual Reference Feedback Tuning

The idea of Virtual Reference Feedback tuning was first proposed in G.O.Guardabassi et al. [1997] with the name of Virtual Reference Direct Design (VRD^2) and subsequently fixed and extended in M.C. Campi et al. [2002], S. Formentin et al. [2011] and M.C. Campi et al. [2006], respectively for LTI, LPV and nonlinear systems.
The main idea to minimize (1) without identifying $G(q^{-1})$ is to build a "virtual" closed-loop system, where the input and output signals are equal to $u(t)$ and $y(t)$ and the closed-loop transfer function is assumed to correspond to $M(q^{-1})$. From such loop, the so-called "virtual reference" $r_v(t)$ and "virtual error" $e_v(t)$ signals can be computed as $r_v(t) = M^{-1}(q^{-1})y(t)$ and $e_v(t) = r_v(t) - y(t)$. The control design issue is then reduced to an identification problem, where the optimal controller is the one that generates $u(t)$ when fed by $e_v(t)$. The criterion to be minimized is then

$$J_{VR}^N(\rho) = \frac{1}{N}\sum_{t=1}^{N} \left(u_L(t) - K(q^{-1},\rho)e_L(t)\right)^2, \qquad (6)$$

where $u_L(t) = L(q^{-1})u(t)$, $e_L(t) = L(q^{-1})e_v(t)$ and $L(q^{-1})$ is a suitable prefilter such that (6) is equal to the second-order Taylor expansion of (1) in the neighborhood of the minimum point (see M.C. Campi et al. [2002]). More specifically, the frequency response of $L(q^{-1})$ must be such that

$$L(e^{-j\omega}) = \frac{M(e^{-j\omega})\left(1 - M(e^{-j\omega})\right) W(e^{-j\omega})}{\Phi_u^{1/2}(\omega)}, \qquad (7)$$

where $\Phi_u^{1/2}(\omega)$ denotes a spectral factor of $\Phi_u(\omega)$.
For the final estimate not to be biased, instrumental variables are used to counteract the effect of noise (see L. Ljung [1999]).

3. OPTIMAL INPUT DESIGN

In this section, the expectation of the control criterion is computed for noniterative CbT and VRFT in case of large N and full-order controller parameterization and input design is proposed to cope with bias reduction. Once the optimal spectrum is known, the input signal can be generated by means of well-known spectral factorization techniques (see *e.g.* A.H. Sayed et al. [2001]).

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3.1 Noniterative correlation-based Tuning

A preliminary asymptotic analysis of the variability of the controller parameter estimate for noniterative CbT has been already published in K. van Heusden, A. Karimi and D. Bonvin [2010]. The main result states that the expected value of the correlation criterion (4) can be approximately expressed by

$$\mathbb{E}\left[J_{N,l}(\rho)\right] \approx \widetilde{J}_{N,l}(\rho) + \\ + \frac{\sigma^2(2l+1)}{2\pi N}\int_{-\pi}^{\pi}\frac{|1-M|^4\,|K(\rho)|^2\,|H|^2\,|W|^2}{\Phi_u(\omega)}\,d\omega \tag{8}$$

where $\widetilde{J}_{N,l}(\rho)$ is a windowed estimate of $J(\rho)$ over N in the absence of noise.

Equation (8) clearly shows that the frequency shaping of the input signal used in the experiment is strictly related to the bias effect on the identification criterion (by inverse relationship). Furthermore, the choice of the parameter l represents a trade-off as, to find a good estimate of $J(\rho)$, l must be large as possible (see K. van Heusden, A. Karimi and D. Bonvin [2010]), whereas (8) clearly shows that the bias increases as l increases.
The following theorem provides a way to optimally shape $\Phi_u(\omega)$ in order to minimize the bias effect on $\mathbb{E}\left[J_{N,l}(\rho)\right]$ (and subsequently on $\mathbb{E}\left[J(\rho)\right]$) for sufficiently large l.

Theorem 1. Let the input energy of $u(t)$ be bounded by the application-dependant parameter γ_u, *i.e.*

$$\int_{-\pi}^{\pi}\Phi_u(\omega)\;d\omega < \gamma_u. \tag{9}$$

The optimal expression of the input spectrum $\Phi_u^o(\omega)$ for bias minimization of (8) is given by

$$\Phi_u^o(\omega) = \mu\,|1-M|^2\,|K(\rho)|\,|H|\,|W| \tag{10}$$

where

$$\mu = \frac{\gamma_u}{\int_{-\pi}^{\pi}|1-M|^2\,|K(\rho)|\,|H|\,|W|d\omega}. \tag{11}$$

Proof. The proof follows the line of L. Ljung [1999] (Chapter 13.6) for high-order black box models. In this case, the result is not asymptotic in the controller order as the derivation of (8) does not require this assumption.

□

Notice that the optimal signal with spectrum $\Phi_u^o(\omega)$ cannot be directly implemented as it depends on the identified controller $K(q^{-1},\rho)$ and on the noise model $H(q^{-1})$. The first problem is typical of any input design procedure (see *e.g.* M. Gevers [1996]) and, in system-identification theory, is addressed with a sequential approach, *i.e.* a preliminary model is first estimated from a persistently exciting set of data and such model is used to derive the optimal input. The final result is certainly suboptimal, however, it can be improved via iterative procedures (see H. Hjalmarsson [2005]). Analogously, also the second task is typical of any input design problem, as illustrated in M Gevers et al. [1986], and can be managed via preliminary knowledge or identification.

3.2 Virtual Reference Feedback Tuning

As already mentioned in Section 2.2, instrumental variables are used in VRFT to deal with measurement noise. Specifically, for a given N, the parameter vector are computed as

$$\hat{\rho}_N = R_N^{-1} r_N, \tag{12}$$

$$R_N = \frac{1}{N}\sum_{t=1}^{N}\phi_2(t)\phi_1^T(t),\; r_N = \frac{1}{N}\sum_{t=1}^{N}\phi_2(t)u_L(t).$$

The regressor is defined as

$$\phi_1(t) = \beta(M^{-1}-1)Ly(t), \tag{13}$$

that is

$$\begin{aligned}\phi_1(t) &= \beta(M^{-1}-1)Ly_o(t) + \beta(M^{-1}-1)Lv_1(t)\\ &= \phi_o(t) + \tilde{\phi}_1(t),\end{aligned}$$

where $y_o(t)$ is the noiseless output, *i.e.* $y_o(t) = Gu(t)$, and $v_1(t)$ is the realization of noise $v(t)$ in the identification experiment.
The basic instrumental variable is instead defined as

$$\phi_2(t) = \beta(M^{-1}-1)Ly_2(t), \tag{14}$$

where $y_2(t)$ is a second set of output data. Specifically, $y_2(t)$ might be selected as the noiseless output, *i.e.* $y_2(t) = y_o(t)$, that can be obtained by feeding a full-order model of the system, if available, with the same input u used in the experiment. Otherwise, $y_2(t)$ can be derived by feeding again the system with u; in this case, the output of this second experiment would be $y_2(t) = y_o(t) + v_2(t)$, where $v_2(t)$ is a second realization of the noise $v(t)$. In the latter case, analogously to $\phi_1(t)$, the instrumental variable can be rewritten as

$$\phi_2(t) = \phi_o(t) + \tilde{\phi}_2(t), \tag{15}$$

where

$$\tilde{\phi}_2(t) = \beta(M^{-1}-1)Lv_2(t). \tag{16}$$

Basic instrumental variables asymptotically guarantee consistent results but increase the variance of the estimate (see T. Soderstrom et al. [2005]). In controller identification, this fact may critically jeopardize the final performance.
As a matter of fact, consider the expected value (with respect to the noise) of the second order Taylor expansion of $J(\hat{\rho}_N)$ around the global minimum ρ_o (recall that, in the minimum, the first order derivative is zero)

$$\mathbb{E}\left[J(\hat{\rho}_N)\right] \approx J(\rho_o) + \frac{1}{2}\,\mathbb{E}\left[(\hat{\rho}_N-\rho_o)^T\frac{\partial^2 J}{\partial\rho^2}|_{\rho_o}(\hat{\rho}_N-\rho_o)\right].$$

The previous expression can be rewritten, thanks to the cyclic property of the trace operator, as

$$\mathbb{E}\left[J(\hat{\rho}_N)\right] \approx J(\rho_o) + \frac{1}{2}\,tr\left\{\mathbb{E}\left[(\hat{\rho}_N-\rho_o)(\hat{\rho}_N-\rho_o)^T\right]\Lambda\right\},$$

where Λ is the Hessian computed in ρ_o, that has been taken out of the $\mathbb{E}\left[\cdot\right]$ argument as it is deterministic. An approximate expression of $\mathbb{E}\left[J(\hat{\rho}_N)\right]$ is

$$\mathbb{E}\left[J(\hat{\rho}_N)\right] \approx J(\rho_o) + \frac{1}{2N}tr\left\{P_{IV}\Lambda\right\}, \tag{17}$$

where P_{IV} is the variance of the asymptotic distribution.

Analogously to the CbT case, the objective of reducing the effect of noise would be to minimize the bias error on $\mathbb{E}\left[J(\hat{\rho}_N)\right]$. Note that $tr\{P_{IV}\Lambda\} \leq tr\{P_{IV}\}tr\{\Lambda\}$. Therefore by minimizing the trace of P_{IV} an upper bound on

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the bias error will be minimized. This can be achieved by making P_{IV} as close as possible to the covariance of the parameter estimates using optimal instrumental variables, P_{IV}^o. The following proposition provides a fast way to suitably build the input signal according to the aforementioned rationale.

Proposition 1. If ϕ_2 is built according to (14), where $y_2(t) = y_o(t)$, P_{IV}^o is achieved if

$$\Phi_u^o(\omega) = |1 - M|^4 |K_o|^2 |H|^2 |W|^2 . \tag{18}$$

Proof. Since the controller is fully parameterized, $u_L = \phi_o^T \rho_o$, that is $u_L = \phi_1^T \rho_o - \tilde{\phi}_1^T \rho_o$. Then, r_N in (12) becomes

$$r_N = \frac{1}{N}\sum_{t=1}^{N} \phi_2(t)\phi_1^T(t)\rho_o - \frac{1}{N}\sum_{t=1}^{N} \phi_2(t)\tilde{\phi}_1^T(t)\rho_o$$

and $\sqrt{N}\,(\hat{\rho}_N - \rho_o)$ can be rewritten as

$$\sqrt{N}\,(\hat{\rho}_N - \rho_o) = R_N^{-1}\frac{1}{\sqrt{N}}\sum_{t=1}^{N} \phi_2(t)\tilde{\phi}_1^T(t)\rho_o. \tag{19}$$

According to L. Ljung [1999], as $N \to \infty$,

$$\frac{1}{\sqrt{N}}\sum_{t=1}^{N} \phi_2(t)\tilde{\phi}_1^T(t)\rho_o \to \mathcal{N}(0, P_o) \tag{20}$$

where

$$P_o = \lim_{N\to\infty} \frac{1}{N}\,\mathbb{E}\left[\sum_{t=1,s=1}^{N} \phi_2(t)\tilde{\phi}_1^T(t)\rho_o\rho_o^T\tilde{\phi}_1(s)\phi_2^T(s)\right].$$

Following the same procedure in T. Soderstrom et al. [1983] (Appendix A8.1), since $\dim(\phi_2) = \dim(\rho) = n$, the variance expression can be rewritten as

$$P_o = \sigma^2\mathbb{E}\left[F(q^{-1})\phi_2(t)\right]\left[F(q^{-1})\phi_2(t)\right]^T , \tag{21}$$

where

$$F = K_o(1 - M)^2 HW\Phi_u^{-1/2}. \tag{22}$$

Subsequently, $P_{IV} = R_o^{-1}P_oR_o^{-1}$, where

$$R_o = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \phi_o(t)\phi_o^T(t), \tag{23}$$

as $R_N \to R_o$ for $N \to \infty$ (see T. Soderstrom et al. [1983]). Since the optimal covariance is $P_{IV}^o = \sigma^2 R_o^{-1}$, the input spectrum has to be selected such that $P_o = \sigma^2 R_o$. From (21) it follows that, under the hypothesis of $\phi_2(t) = \phi_o(t)$, such optimal input is the one that makes $|F|^2 = 1$, that is

$$\Phi_u^o(\omega) = |1 - M|^4 |K_o|^2 |H|^2 |W|^2 . \tag{24}$$

As a matter of fact, when $|F|^2 = 1$ and $\phi_2(t) = \phi_o(t)$,

$$P_o = \sigma^2\mathbb{E}\left[\phi_2(t)\phi_2^T(t)\right] = \sigma^2 R_o, \tag{25}$$

and $P_{IV} = P_{IV}^o$, that is the thesis.

□

It should be said that, when the instrumental variable is selected using a second experiment, the previous result no longer holds. However, such instrumental variable is an unbiased estimate of the previous one, as $\mathbb{E}[\phi_2(t)] = \mathbb{E}[\phi_o(t)]$. Moreover, when (18) is employed in this setting, (21) becomes

$$P_o = \sigma^2 R_o + \sigma^2\mathbb{E}\left[\tilde{\phi}_2(t)\tilde{\phi}_2^T(t)\right]. \tag{26}$$

Then, if SNR is high, the additional term is one order smaller than the $\sigma^2 R_o$ and the input (24) can be reasonably used also in this case. In Section 4, the effectiveness of this input choice with instrumental variable built from a second experiment will be shown.

For energy-constrained signals with a given bound γ_u, the optimal input can be, *e.g.*, suitably scaled so as to exploit all the available energy without changing the input frequency weighting. This means that the optimal input for VRFT becomes

$$\Phi_u^o(\omega) = \nu\,|1 - M|^4 |K_o|^2 |H|^2 |W|^2 , \tag{27}$$

where

$$\nu = \frac{\gamma_u}{\int_{-\pi}^{\pi} |1 - M|^4 |K_o|^2 |H|^2 |W|^2 d\omega}. \tag{28}$$

Remarks

- (27) is different from (10) in the power of frequency weighting and in the expression of the controller ($K(\rho)$ is substituted by K_o). However, the latter is only a formal distinction, since the desired value of ρ is exactly ρ_o. Moreover, as in practice neither $K(\hat{\rho}_N)$ and K_o are known "a-priori", the optimal input expression can only employ a preliminary estimate of the controller.
- Notice that, in the VRFT case, the time-realization of a suitably shaped input signal can be straightforwardly found without using complex spectral factorization techniques. As a matter of fact, in order to get such an input, it is sufficient to feed the filter $L_u = \sqrt{\nu}(1 - M)^2 K_o HW$ by means of a white noise with unit variance.
- Also in identification for control, quantitative measurements of the expected value of $J(\rho)$ can be derived from variability of G and K (see *e.g.* H. Hjalmarsson [2005], M. Gevers [1996] and M Gevers et al. [1986]) but computations become complicated.

4. A SIMULATION EXAMPLE: THE FLEXIBLE TRANSMISSION SYSTEM

The example proposed herein for testing the above input design strategy is the flexible transmission system introduced as a benchmark for digital control design in I. D. Landau et al. [1995].

The plant is described by the discrete-time model

$$G(q^{-1}) = \frac{0.28261q^{-3} + 0.50666q^{-4}}{A(q^{-1})} \tag{29}$$

where $A(q^{-1}) = 1{-}1.41833q^{-1}{+}1.58939q^{-2}{-}1.31608q^{-3}{+}0.88642q^{-4}$ and the sampling time is $T_s = 0.05s$. The measurement noise is supposed to be white and such that the signal-to-noise ratio is the parameter SNR such that $H(q^{-1}) = var[y_o(t)]/SNR$, where $var[y_o(t)]$ is the variance of $y_o(t)$. Moreover, the frequency-weighting function $W(q^{-1}) = 1$ and the set of available controllers is

$$K(q^{-1}, \rho) = \frac{\rho_0 + \rho_1 q^{-1} + \rho_2 q^{-2} + \rho_3 q^{-3} + \rho_4 q^{-4} + \rho_5 q^{-5}}{1 - q^{-1}}.$$

The control objective is defined by a reference model that allows the perfect matching to be achieved, *i.e.*

$$M(q^{-1}) = \frac{G(q^{-1})K(q^{-1})}{1 + G(q^{-1})K(q^{-1})}, \tag{30}$$

where the optimal controller is in the controller set and its parameters are

$$\rho_o = [0.2045, -0.2715, 0.2931, -0.2396, 0.1643, 0.0084].$$

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Table 1. Mean values (100 iterations) of the achieved performance J for different SNRs and sizes N of the dataset: the CbT case.

SNR ↓ N →	100	500	1000
5	WN: 0.1344 OI: 0.0547	WN: 0.0690 OI: 0.0273	WN: 0.0527 OI: 0.0183
10	WN: 0.0931 OI: 0.0400	WN: 0.0478 OI: 0.0187	WN: 0.0379 OI: 0.0135
20	WN: 0.0661 OI: 0.0285	WN: 0.0356 OI: 0.0131	WN: 0.0267 OI: 0.0092

With simulation parameters above and $\gamma_u = 1$, the optimal input spectra for CbT and VRFT are shaped as illustrated in Figure 2, where the magnitude plot of $G(q^{-1})$ is also showed. Notice that in both the cases, the input energy is low in the frequencies corresponding to the resonances, whereas it is higher around the desired bandwidth and at high frequencies, where the desired sensitivity function $S(q^{-1}) = 1 - M(q^{-1})$ is high.

To verify the effectiveness of the proposed strategy, a

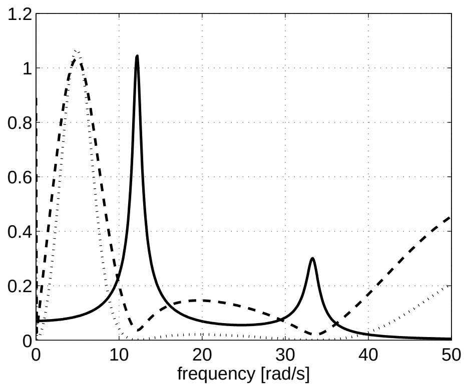


Fig. 2. Normalized magnitude plot of G (solid), optimal input spectrum for CbT (dashed) and for VRFT (dotted) on the linear scale.

Monte-Carlo simulation with 100 running experiments is performed, using a different noise realization for each experiment, and an estimate of $\mathbb{E}\,[J(\hat{\rho})]$ in the minimum $\hat{\rho}$ is computed by sample mean. The case of PRBS and the case of optimal input signal are then compared for different values of signal-to-noise ratios SNR and number of data N. Results for CbT and VRFT are illustrated respectively in Table 1 and in Table 2, while Figures 3 and 4 show the case of $SNR = 5$ and $N = 1000$ as an example. In each figure, the ideal closed-loop behaviour $M(q^{-1})$ is also shown. For CbT, the length of the instrumental variable vector is $l = 35$, which corresponds to the length of the impulse response of M. Notice that since VRFT requires two sets of data for using the instrumental variable, each experiment is characterized by half the samples for a fair comparison between the methods. From numerical results, it is clear that the use of the optimal input not only improves the closed-loop performance but also makes CbT and VRFT comparable in terms of statistical behaviour. Obviously, this fact is not completely costless as a first experimental session is required to estimate $K(\rho)$ and H required by (10) and (27). However, the proposed approach seems to the authors a very good trade-off for all applications where experiments are not too costly and high accuracy is required.

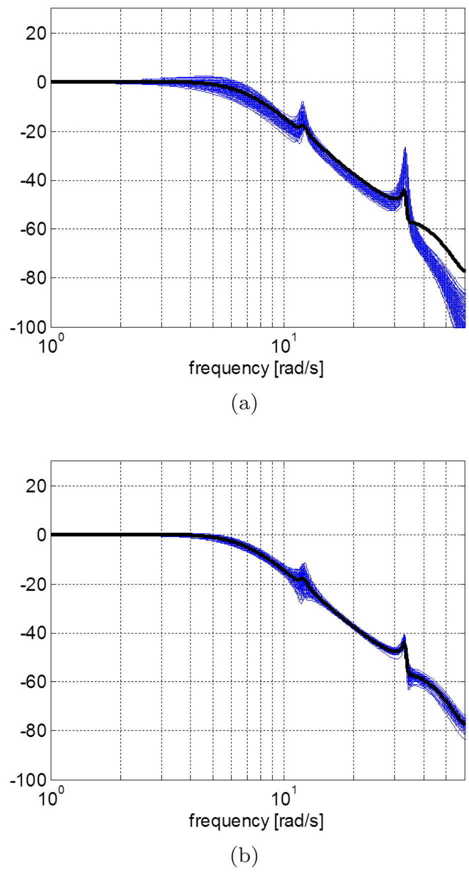


Fig. 3. Magnitude Bode plots of M (black line), achieved closed-loop performance with PRBS input for the 100/100 stabilizing CbT controllers (blue lines, above), with optimal input signal for 100/100 stabilizing controllers (blue lines, below). In the example, N = 1000, SNR = 5.

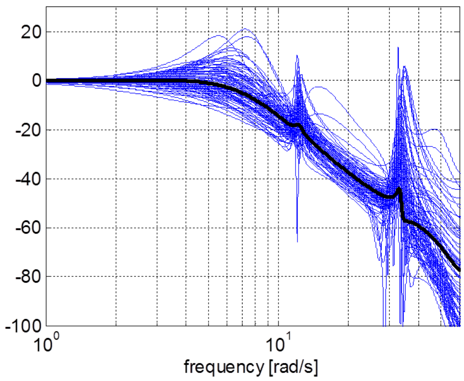
Table 2. Mean values (100 iterations) of the achieved performance J for different SNRs and sizes N of the dataset: the VRFT case.

SNR ↓ N →	100	500	1000
5	WN: 0.3028 OI: 0.2664	WN: 0.1841 OI: 0.1296	WN: 0.1769 OI: 0.0378
10	WN: 0.2392 OI: 0.0907	WN: 0.1179 OI: 0.0363	WN: 0.1333 OI: 0.0333
20	WN: 0.1993 OI: 0.0857	WN: 0.1048 OI: 0.0313	WN: 0.0560 OI: 0.0172

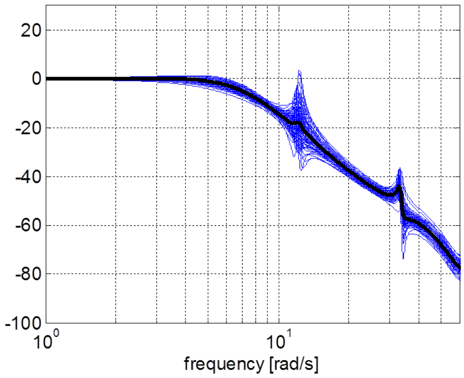
Finally notice that when the uncertainty is high, some VRFT controllers may destabilize the system (see again example in Figure 4). Bias reduction via optimal input design also allows one to reduce the probability of obtaining such controllers without adding any additional (and conservative) constraint (see *e.g.* K. van Heusden, A. Karimi and D. Bonvin [2010]) to the design procedure.

5. CONCLUSIONS AND FUTURE WORKS

In this work, statistical properties of direct data-driven controller tuning have been analyzed and optimal input design have been proposed to increase closed-loop per-



(a)



(b)

Fig. 4. Magnitude Bode plots of M (black line), achieved closed-loop performance with PRBS input for the 95/100 stabilizing VRFT controllers (blue lines, above), with optimal input signal for 100/100 stabilizing controllers (blue lines, below). In the example, N = 1000, SNR = 5.

formance. Specifically, the innovative contributions of the paper can be summarized as follows:

- CbT and VRFT have been demonstrated to have the same problem for large N; specifically, the expected value of the cost function is biased when data are noisy;
- the analytical expression of the control-relevant input signal has been provided; an open-loop experiment with such input allows the bias effect to be reduced;
- the proposed methodology directly connects the final cost function and the input spectrum expression in a straightforward way. The latter can then be directly exploited to improve the closed-loop performance;
- the benchmark numerical example has shown to behave much better when the control-relevant input is used; moreover, unstabilizing controllers are no longer obtained.

Future work will focus on the analysis of controller underparameterization and iterative approaches for input design.

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