Robust and Gain-Scheduled PID Controller Design for Condensing Boilers by Linear Programming

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Abstract: This paper addresses the water temperature control in condensing domestic boilers. The main challenge of this process under the controller design perspective is the fact that the dynamics of condensing boilers are strongly affected by the demanded water flow rate. Two approaches are presented in this paper. First, a robust PI controller is designed that stabilizes and achieves good performance for closed-loop system for a wide range of the water flow rate. Then, it is shown that if the water flow rate information is used to update the controller gains, a technique known as gain-scheduled control, the performance can be significantly improved. Several models of a boiler in different water flow rate are identified in collaboration with Honeywell, and the effectiveness of the results are illustrated by simulation.

Keywords: Robust control, PID controller, multimodel, gain-scheduled control

1. INTRODUCTION

A condensing boiler is a water heating device designed to recover energy normally discharged to the atmosphere through the flue. It operates through the use of a secondary heat exchanger which most commonly uses residual heat in the flue gas to heat the cooler returning water stream or by having a primary heat exchanger with sufficient surface for condensing to easily take place.

Modern condensing boilers are comprised of microprocessor controlled combustion which adjusts the amount of fuel and air supplied to the burner. This is performed by using an embedded algorithm which considers outdoor temperature, temperature of the supplied water, etc. Continuously controlled units also minimize the on-off cycling for increasing efficiency.

The main characteristic of this process is its nonlinear behaviour, where the dynamics are strongly dependent on the operating conditions defined by the demanded water flow rate. Developing a physical nonlinear model for the plant and computing a nonlinear feedforward control could be considered as a solution to this problem. However, because of great variability of the condensing boilers, this approach needs a considerable effort for modeling. Another approach to model the nonlinear system is to use a finite set of linear models, which locally approximate the original dynamics at different operating points. In such cases, the designed controller must be able to stabilize and guarantee a reasonable performance for all operating conditions.

Many robust controller design methods for multimodel systems are available in the literature. Toscano (2007) proposes a method to design PID and multi-PID to control nonlinear systems using multimodel in the state space representation. The time delay is approximated by a high order transfer function and the controller is tuned using an iterative algorithm with no convergence result. Ge et al. (2002) introduce a robust PID controller design for uncertain systems via LMI approach. The authors use the multimodel paradigm to describe the uncertainties and derived a convex constraint problem to design the controller. In Nyström et al. (1999) a multimodel controller design combined with gain scheduling methods is studied based on a mixed H_2/H_∞ problem. These approaches require the approximation of the time-delay and suffer from the conservatism related to the existence of a common Lyapunov matrix for all closed loop systems. Gain scheduled controllers have been widely and successfully applied to divers fields and many approaches are available to design such controllers. Blanchett et al. (2000) and Zhao et al. (1993) developed a fuzzy gain scheduling strategies for PID controllers for process control where reasonable performance has been reported with simple control schemes. For a detailed review of gain-scheduling analysis and design refer to Leith and Leithead (2000).

In this paper the method given in Karimi et al. (2007) is applied to design a robust PID controller for the condensing boiler problem. The main feature of the method is that the stability, robustness margins and some performance specifications are guaranteed by linear constraints in the Nyquist diagram. Therefore, a PID controller can be designed using linear programming. An extension to this method to design gain-scheduled controllers was presented by Kunze et al. (2007) and is applied to design a gain-scheduled PI controller for the condensing boiler where the gains are function of the requested water flow rate.

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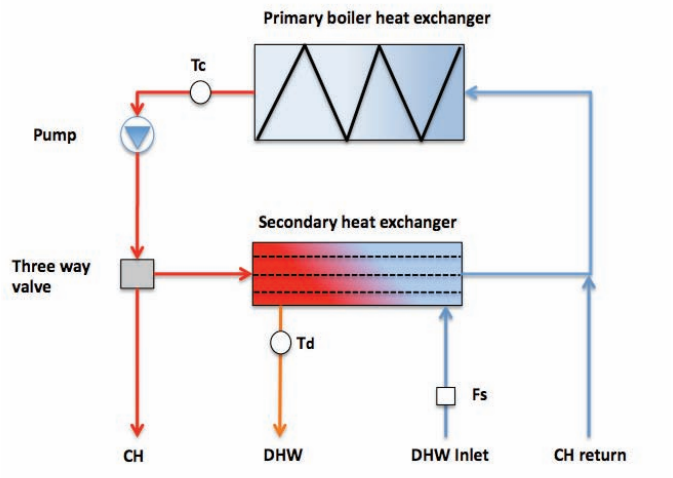

Fig. 1. Simplified boiler scheme

The article is organized as follows: the process and control problem are described in Section 2. Section 3 introduces the robust control design method together with the simulation results. Section 4 shows how the system performance can be improved by a gain-scheduled controller. Some concluding remarks are given in Section 5.

2. SYSTEM DESCRIPTION

Figure 1 shows a simplified scheme of a domestic boiler. Most boilers work on two different modes: central heating (CH) and domestic hot water (DHW) that normally are mutually exclusive. The returning water from central heating (CH return) will be heated up in the primary tube-in-tube heat exchanger which is located close to the combustion chamber. A sensor monitors the water temperature leaving the primary exchanger (T_c). If CH mode is activated, then the control system will define the necessary power to keep T_c in the desired set-point. The heated water will thus be driven around the building by a pump and will come back to the boiler in a closed loop.

When the water flow-rate sensor (F_s) detects a demand for domestic water, the DHW mode will be used. Here, a three way valve will switch and the CH loop will be closed through the secondary heat exchanger, i.e., during DHW operation there is no water circulation around the building, but it returns to the primary heat exchanger right after leaving the secondary. In this mode the cold water for domestic use (DHW inlet) coming from the supply system will be heated up in the secondary heat exchanger (typically plate heat exchanger) by the hot water of the central heating. The controller will define the necessary power to maintain the temperature T_d in the set point. It is worth to point out that when T_d is being controlled the temperature T_c can vary freely.

There are two main control loops: the first one will control the temperature of the CH water leaving the primary heat exchanger and the second controller will maintain constant the temperature of the DHW at the output of the secondary heat exchanger. As it has been said, generally it is not possible to control both temperature at the same time. It means that there will be a switching between the two control loops. The supervisor will switch automatically

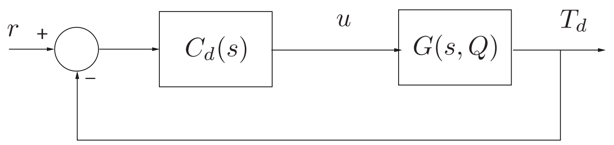

Fig. 2. *Controller in domestic hot water mode*

from the CH to the DHW controller whenever the flow-rate detects a demand for domestic hot water.

The control problem is different for each mode. In the CH controller the dynamics of the system are slow and the main disturbance is the ambient temperature which has a very large time scale. Furthermore, the water is driven around a closed loop with constant flow rate.

The goal of the DHW controller is to maintain the domestic hot water T_d at the set point and reject the disturbances as fast as possible without oscillation since the performance of the controller affects directly the comfort of the user. The control of the domestic hot water temperature is much more critical than the CH controller. The conditions may change very quickly and the controller must respond fast. Here, the disturbances are mainly the water flow rate Q which is demanded and the temperature of water coming from the supply system.

The flow rate can vary drastically in a short period of time, which creates strong disturbances. In fact, one of the main challenges controlling this process is the fact that the dynamics of the system depend strongly on the demanded water flow rate, as we shall see in the following. For this reason, the focus of this paper is the robust design of the DHW controller.

Figure 2 shows the block diagram of the DHW control system. The controlled variable is the temperature T_d of the water at outlet of the secondary heat exchanger; disturbance Q is the demanded water flow rate and u is the output of the controller used to keep the temperature at the set point. This value will then be converted to the real actuator input which depends on the boiler.

The results in this work are based on a domestic condensing boiler comprised by a variable speed fan. The fan speed is proportional to a pressure difference which drives fuel into the burner. i.e, higher speed indicates higher power. The working range of the fan speed in this case is from 1000 rpm to 5350 rpm.

Our goal is to design the controller C_d which offers stability, fast response and small overshoot for a whole range of water flow rate. The first step in the controller design is to obtain the system's model. Therefore, a series of open-loop experiments were carried out in order to identify models at different values of water flow rate Q. In this case, we define 6 operating points: $Q = [3, 4, 5, 6, 7, 8]$ l/min.

The identified systems are basically first order models in the form

$$G_i(s) = \frac{K_i}{\tau_i s + 1} e^{-\theta_i s} \tag{1}$$

where the parameters are given on Table 1. As it can be seen, the static gain K_i of the system ranges from 0.0093 up to 0.0216 while the time constant τ_i goes from 31.8 s

Table 1. Models parameters in different operating points

Model:	$K_i[^\circ C/rpm]$	$\tau_i[s]$	$\theta_i[s]$
G_1 (8 l/min)	0.0093	31.8	6.2
G_2 (7 l/min)	0.0103	34.1	6.0
G_3 (6 l/min)	0.0117	34.8	6.7
G_4 (5 l/min)	0.0139	37.6	9.8
G_5 (4 l/min)	0.0170	57.4	12
G_6 (3 l/min)	0.0216	62.1	15.2

up to 62.1 s. As expected using energy balance, with low flow rates the static gain of the system is higher. When the water flow rate is higher the heat transfer coefficient is also higher. This explains the smaller time constant.

3. CONTROL DESIGN METHOD

While the design of a controller that matches the specifications for a single operating point seems to be trivial, finding a fixed controller which works properly in all operating points might not be a simple task. To solve this problem a method is presented to design robust controllers for multi-model uncertainty using linear programming. The main feature of this method is that the stability and some robustness margins are guaranteed by linear constraints in the Nyquist diagram and the method is applicable to multiple models as well (see Karimi et al. (2007)).

3.1 Design of robust controllers

Consider the class of linear time-invariant continuous-time SISO systems with no pole in the right-half plane (RHP). In order to design the controller, it is assumed that either a parametric transfer function with time delay or a non-parametric spectral model is available. Suppose that the frequency domain is discretized in a sufficiently large finite number of points N. The set of models is given in the following by

$$\mathbb{M} := \{G_i(j\omega_k) \quad i = 1, \dots, m; \quad k = 1, \dots, N\} \tag{2}$$

where m is the number of models in the set.

A PID controller can be defined by $C(s) = \rho^T \phi(s)$ with

$$\rho^T = [K_P, K_I, K_D] \tag{3}$$

$$\phi^T(s) = [1, \frac{1}{s}, \frac{s}{1+T_f s}] \tag{4}$$

where the derivative filter time constant T_f is supposed to be known.

Let the open loop transfer function be defined as

$$L_i(s) = C(s)G_i(s)$$

With this parametrization, every point on the Nyquist diagram of $L_i(j\omega) = C(j\omega)G_i(j\omega)$ is a linear function of the controller parameters ρ.

$$L_i(j\omega) = \rho^T\phi(j\omega)G_i(j\omega) = \rho^T[\mathcal{R}_i(\omega) + j\mathcal{I}_i(\omega)] \tag{5}$$

with

$$\mathcal{R}_i(\omega) = \mathcal{R}_e\{\phi(j\omega)G_i(j\omega)\} \tag{6}$$

$$\mathcal{I}_i(\omega) = \mathcal{I}_m\{\phi(j\omega)G_i(j\omega)\} \tag{7}$$

where $\mathcal{R}_e\{\cdot\}$ and $\mathcal{I}_m\{\cdot\}$ stand for real and imaginary part of a complex value.

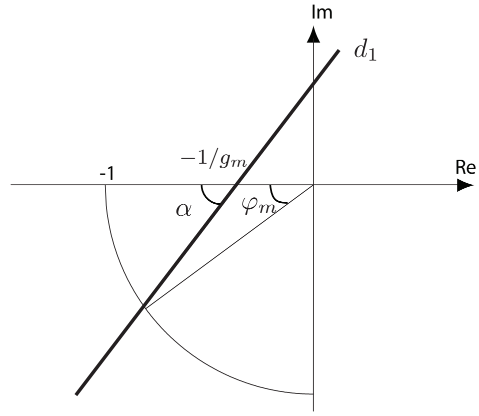

Fig. 3. Lower bounds on the gain and phase margins and linear constraints in the Nyquist diagram

The classical robustness indicators like gain and phase margins as well as the performance indicator, the crossover frequency ω_c, are nonlinear functions of the controller parameters. The optimization methods with constraints on these values leads to non-convex optimization problems and cannot be solved efficiently. The basic idea is to present some linear constraints on the controller parameters that guarantee a lower bound on the robustness margins and the crossover frequency (Karimi et al. (2007)). To proceed, consider that a lower bound, g_m, for the gain margin and a lower bound, φ_m, for the phase margin are given. Then, in the Nyquist diagram, the straight line d_1 that passes through $(-1/g_m, 0)$ and $(-\cos\varphi_m, -\sin\varphi_m)$ is completely known and can be represented by (see Fig 3):

$$y = \frac{\sin\varphi_m}{g_m\cos\varphi_m - 1}(g_m x + 1) \tag{8}$$

If the Nyquist plot of the open loop transfer function lies to the right of d_1, lower bounds on the gain and phase margins are ensured. This can be presented by a set of linear constraints on the controller parameters ρ:

$$\rho^T\mathcal{I}_i(\omega_k) < \frac{\sin\varphi_m}{g_m\cos\varphi_m - 1}\big(g_m\rho^T\mathcal{R}_i(\omega_k) + 1\big)$$
$$\text{for } i = 1, \dots, m \quad , \quad \text{for } k = 1, \dots, N \tag{9}$$

These constraints can be simplified using α, the slope of d_1, as follows:

$$\rho^T(\cot\alpha\mathcal{I}_i(\omega_k) - \mathcal{R}_i(\omega_k)) - 1/g_m < 0$$
$$\text{for } i = 1, \dots, m \quad , \quad \text{for } k = 1, \dots, N \tag{10}$$

The closed-loop performance can be given by a lower bound, ω_x, on the crossover frequency. This can also be presented as a set of linear constraints in the Nyquist diagram. Figure 4 shows a line d_2 in the complex plane, which is tangent to the unit circle centered at the origin. The part of d_2 between d_1 and the imaginary axis is a linear approximation of the unit circle in this region. The open loop Nyquist curve intersects d_2 at a frequency, ω_x, which is always less than or equal to the crossover frequency ω_c. Hence, the frequency ω_x can be used as a lower approximation of the crossover frequency. The constraints can be summarized in the following way: the Nyquist curve must lie below d_1 and above d_2 for frequencies greater than ω_x. It must also lie below d_2 for frequencies less than or equal to ω_x. The angle β should be chosen small

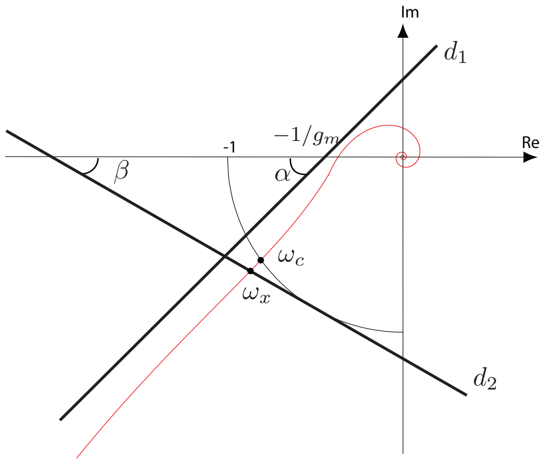

Fig. 4. *Linear constraints for robustness and performance*

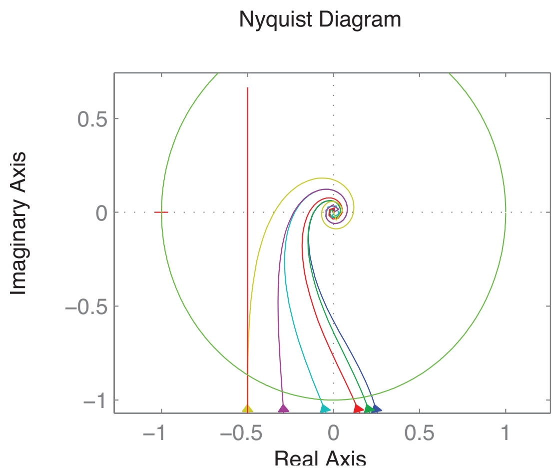

Fig. 5. Nyquist diagram of L_i

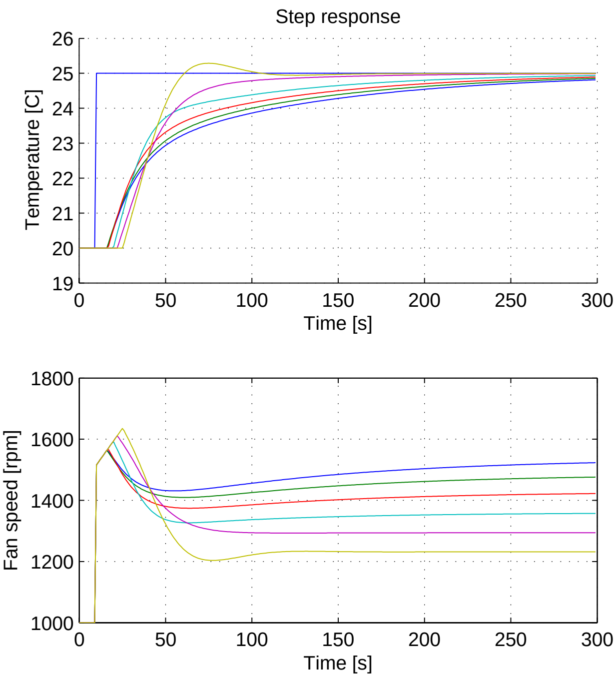

Fig. 6. Step response of the closed-loop system for all models

enough such that the Nyquist curve cannot approach the critical point -1 from the left side. In other words d_2 should intersect the negative real axis at $x < -2 + 1/g_m$ (or $\sin\beta < (2 - 1/g_m)^{-1}$).

The closed-loop performance can be optimized by increasing iteratively ω_x or by maximizing K_I which leads to minimizing the integrated error (IE) (Åström and Hägglund (2005)). This can be described by the following linear optimization problem

$$\max K_I$$

subject to

$$\begin{aligned} &\rho^T(\cot\alpha\mathcal{I}_i(\omega_k) - \mathcal{R}_i(\omega_k)) - 1/g_m < 0 \quad \omega_k > \omega_x, \\ &\rho^T(\cos\beta\mathcal{I}_i(\omega_k) + \sin\beta\mathcal{R}_i(\omega_k)) > -1 \quad \omega_k > \omega_x, \\ &\rho^T(\cos\beta\mathcal{I}_i(\omega_k) + \sin\beta\mathcal{R}_i(\omega_k)) \le -1 \quad \omega_k \le \omega_x, \\ &k = 1, \ldots, N \quad i = 1, \ldots, m \end{aligned} \tag{11}$$

The constraints given by (11) are valid only if $L_i(s)$ has one or two integrators. For more general cases like unstable systems or systems with complex poles on the imaginary axis as well as other type of closed-loop performance (H_∞, loop-shaping) see Karimi and Galdos (2010).

3.2 Simulation results

In this section a PI controller is designed by the proposed method for the boiler temperature control problem. The simulation results are also presented.

Consider a PI controller which should guarantee robust stability and performance for all operating points presented by models G_1 to G_6 in Table 1. The design specifications are $\varphi_m = 60°$, $g_m = 2$. These margins lead to $\alpha = 90°$. Since the open-loop bandwidth varies significantly from one model to another, no constraint for crossover frequency is considered. The phase margin was chosen in order to avoid a large overshoot and to guarantee the robustness with respect to some uncertainties, for instance, the temperature of the inlet water, which have not been described by this set of models.

Using the specifications aforementioned, the optimization problem stated on Eq. (11) is solved by `linprog` of MATLAB optimization toolbox using $N = 150$ frequency points logarithmically spaced between 0.005 and 2 rad/s and $m = 6$ models. The following controller is found:

$$C_1(s) = \frac{103.2s + 1.584}{s} \tag{12}$$

Figure 5 shows the Nyquist plot of $L_i = C_1 G_i$. As it can be seen, the controller C_1 respects the constraints for all the models, i.e., in each case the Nyquist diagram of the open loop transfer function L_i is at the right hand side of the line d_1 which cross the real axis at the point -0.5 with an angle $\alpha = 90°$. Figure 6 shows the step response for a change in the temperature set point for all the six models. It is clear that a disturbance in T_d (because of a sudden change of water flow rate) will be rejected with the same dynamic. The settling time varies from 90s to about 250s in different operating points.

4. GAIN-SCHEDULED CONTROLLER

So far, we have presented a method to design robust fixed order controllers for a set of linear time-invariant SISO systems. This method guarantees robustness and some performance requirements for all models in the set. However, it can lead to high conservative solutions in case there is a big difference between the models. The performance can be improved if the parameters of the controller are not fixed, but function of a given variable (the scheduling parameter) of the system which defines the operating point of the plant. This strategy is known as gain-scheduled control.

The gain-scheduled controller can be computed from a set of models of the system in different operating points or directly from a Linear Parameter Varying (LPV) model. In this paper, we will tune a gain-scheduled controller by linear programming for which the scheduling parameter is the domestic water flow rate Q. In other words, we will try to improve the performance of the closed loop system by using the water flow rate information inside the controller. The mathematical formulation of the control design procedure was proposed in Kunze et al. (2007) is recalled briefly for designing PID gain-scheduled controllers.

Consider the gain-scheduled controller $C(s,Q)$ linearly parametrized by:

$$C(s,Q) = \rho^T(Q)\phi(s) \tag{13}$$

where the basis function vector $\phi(s)$ is defined in Eq. (4) and $\rho^T(Q)$ is given by

$$\rho^T(Q) = [K_P(Q),\, K_I(Q),\, K_D(Q)] \tag{14}$$

Every gain is a polynomial function of order δ of the scheduling parameter and is defined as

$$K_P(Q) = K_{P_\delta} Q^\delta + \cdots + K_{P_1} Q + K_{P_0}$$
$$K_I(Q) = K_{I_\delta} Q^\delta + \cdots + K_{I_1} Q + K_{I_0}$$
$$K_P(Q) = K_{D_\delta} Q^\delta + \cdots + K_{D_1} Q + K_{D_0}$$

With this parametrization, we now can state an optimization problem in a similar way as presented before with the difference that :

$$\rho(Q) = \begin{bmatrix} K_{P_\delta} & \cdots & K_{P_1} & K_{P_0} \\ K_{I_\delta} & \cdots & K_{I_1} & K_{I_0} \\ K_{D_\delta} & \cdots & K_{D_1} & K_{D_0} \end{bmatrix} \begin{bmatrix} Q^\delta \\ \vdots \\ Q \\ 1 \end{bmatrix}$$

When a set of models is available, the optimization methods presented above can be directly applied to compute a gain-scheduled controller as the number of models and the number of frequency points are finite. On the other hand, If an LPV model is available, there will be an infinity number of models corresponding to different values of the scheduling parameter. This problem can be solved by gridding the scheduling parameter.

Consider the case that the robustness margins are given and control objective is to optimize the load disturbance rejection performance of the closed-loop. This corresponds to maximizing $K_I(Q)$ for all values of the scheduling parameter Q. In practice Q is gridded to obtain a set of m models for different operating points. The linear optimization problem can thus be stated and is given in the following.

$$\max \sum_{i=1}^{m} K_I(Q_i)$$

subject to

$$\begin{aligned} &\rho(Q_i)^T(\cot\alpha \mathcal{I}_i(\omega_k) - \mathcal{R}_i(\omega_k)) - 1/g_m < 0 \quad \omega_k > \omega_x, \\ &\rho(Q_i)^T(\cos\beta \mathcal{I}_i(\omega_k) + \sin\beta \mathcal{R}_i(\omega_k)) > -1 \quad \omega_k > \omega_x, \\ &\rho(Q_i)^T(\cos\beta \mathcal{I}_i(\omega_k) + \sin\beta \mathcal{R}_i(\omega_k)) \le -1 \quad \omega_k \le \omega_x, \\ &k = 1, \ldots, N \quad i = 1, \ldots, m \end{aligned} \tag{15}$$

4.1 Gain-scheduled controller evaluation

A gain-scheduled PI controller will be designed in way that its parameters are second-order polynomial functions ($\delta = 2$) of the water flow rate Q. The controller will take the form

$$C(s,Q) = \frac{K_P(Q)s + K_I(Q)}{s} \tag{16}$$

The problem is now to find the polynomial coefficients of each gain of the controller. Here, the set of identified models shown in Table 1 will be used with the corresponding scheduling parameter values $Q = 3, 4, 5, 6, 7$ and 8 l/min. Using the same constraints on phase and gain margins as in the fixed controller case ($\varphi_m = 60°$ and $g_m = 2$) we solve the linear optimization problem in (15) by `linprog` to get the following controller parameters:

$$K_p(Q) = 6.1529Q^2 - 33.7901Q + 170.9287$$
$$K_i(Q) = 0.0624Q^2 + 0.8932Q - 2.2132$$

Note that using this control structure, we are able to increase the closed-loop bandwidth. Figure 7 shows the step response of the gain-scheduled control system. It can be observed that the settling time has been significantly improved in all operating points. Figure 8 depicts a simulated experiment in which the performance of the gain-scheduled controller is compared to that of a fixed controller for $Q = 8$ l/min. At time 10s a set-point step change is applied. The settling time using the gain-scheduled controller and robust controller are, respectively, 13s and 208s. Note that although the fan speed computed by the gain-scheduled controller is much higher, it is still far from the maximum admissible fan speed of 5350 rpm.

It can be seen that the most critical conditions in the control design are those with small water flow rate because of the higher gains and time constants. This means that boundary of feasibility for the controller design is defined by the models in those conditions. When a robust controller is designed it will respect this boundary. However, this solution is too conservative in other operating points with higher flow rates which deteriorates the performance. If a gain-scheduled controller is used, it will respect the constraints by the most critical models and at the same time will have room for performance improvement in different conditions. However, such an enhancement on performance comes with a cost: to implement the gain-scheduled strategy a water flow-meter should be installed in order to provide the flow rate information to the controller. Alternatively, an estimator of the water flow rate could be implemented to provide the information. Such estimator would need the information regarding the temperature of the incoming water. Variation of temperature of DHW is measured as well as the variation of temperature

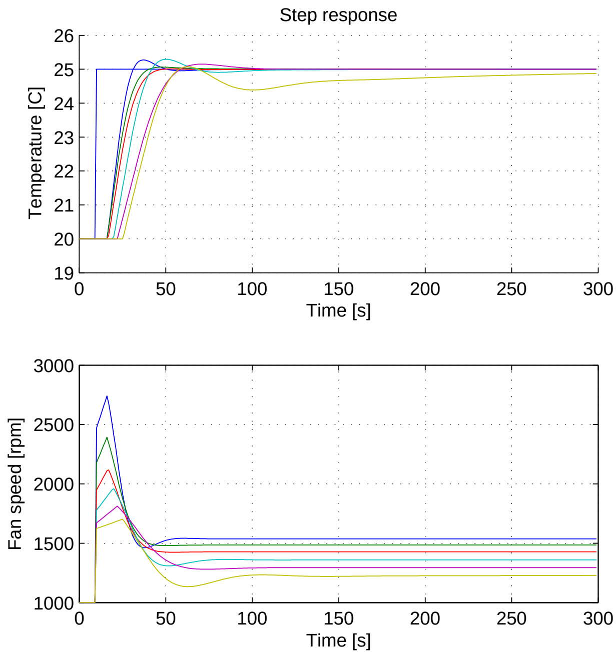

Fig. 7. Step response for the gain-scheduled control system

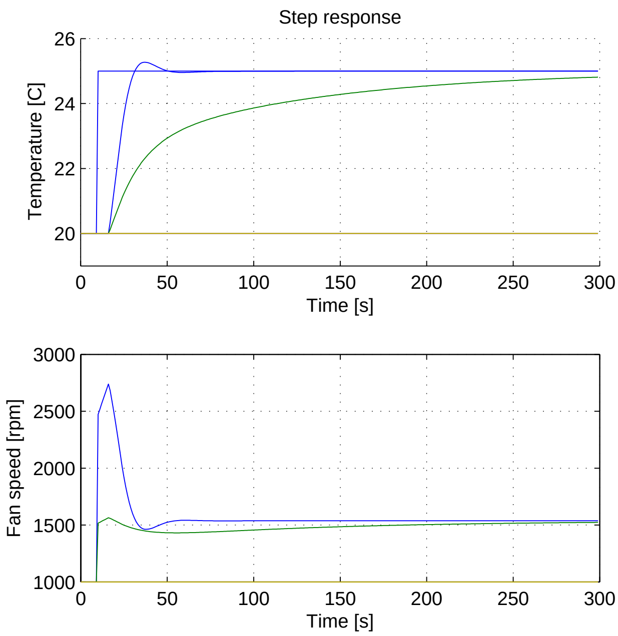

Fig. 8. Controller comparison (Q = 8 l/min), blue solid line: gain-scheduled; green solid line: robust controller.

of CH water when running in the closed-loop. The flow-rate of CH is fixed and is known a priori. Using simple mass and energy balance equations allows one to estimate the water flow rate in the DHW circuit. This is economically advantageous over the flow-meter solution since normally temperature sensors are cheaper than flow-meters.

5. CONCLUSIONS

In this paper a methodology to robust controller design for boiler temperature control was presented. With the proposed approach it has been possible to cope with large variations in the operating conditions providing good performance for the whole range of demanded flow rate. Furthermore, it was shown that the performance of the closed-loop can be improved even further by using a gain-scheduled strategy, where the gains of the controller are polynomial function of the water flow rate.

6. ACKNOWLEDGEMENTS

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