

# EigenMaps: Algorithms for Optimal Thermal Maps Extraction and Sensor Placement on Multicore Processors

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## ABSTRACT

Chip designers place on-chip sensors to measure local temperatures, thus preventing thermal runaway situations in multicore processing architectures. However, thermal characterization is directly dependent on the number of placed sensors, which should be minimized, while guaranteeing full detection of all hot-spots and worst case temperature gradient. In this paper, we present EigenMaps: a new set of algorithms to recover precisely the overall thermal map from a minimal number of sensors and a near-optimal sensor allocation algorithm. The proposed methods are stable with respect to possible temperature sensor calibration inaccuracies, and achieve significant improvements compared to the state-of-the-art. In particular, we estimate an entire thermal map for an industrial 8-core industrial design within  $1^\circ\text{C}$  of accuracy with just four sensors. Moreover, when the measurements are corrupted by noise (SNR of 15 dB), we can achieve the same precision only with 16 sensors.

## ACM Categories & Subject Descriptors

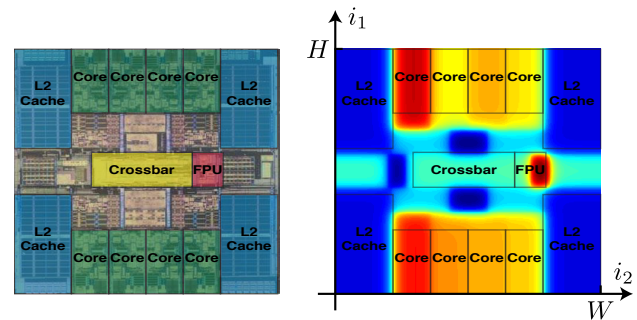
B.7.1 [Integrated Circuits]: Types and Design Styles.

**General Terms:** Design, Performance, Algorithms

**Keywords:** Thermal characterization, principal component analysis, sensor allocations, least-square estimation

## 1. INTRODUCTION

The continuous evolution of process technology enables the inclusion of multiple cores, memories and complex interconnection fabrics on a single die [7]. Although many-core architectures potentially provide increased performance, they also suffer from increased IC power densities and thermal issues are a serious concern in latest designs with deep submicron process technologies [5, 8]. In particular, it is key to design many-core designs that prevent hot spots and large on-chip temperature gradients, as both conditions severely affect system's characteristics. In fact, thermal stress increases the overall failure rate of the system [13], reduces performance due to an increasing operating temperature [1], significantly increases leakage power consumption (due to its exponential dependence on temperature) and cooling costs [13, 3]. Designers organize the floorplan to limit these thermal phenomena, for example, by placing the highest power density components closer to the heat sink [5]. However, the workload execution patterns are fundamental to determine the transient on-chip temperature distribu-



**Figure 1:** Left: a simplified floorplan of the Ultrasparc T1, the considered 8-core processor. Right: an example of thermal map [3].

tion in multicore designs and, unfortunately, these patterns are not fully known at design time. Furthermore, these issues are amplified in many-core designs, where thermal hot-spots are generated without a clear spatio-temporal pattern due to the dynamic task set execution nature, based on external service requests, as well as the dynamic assignment to cores by the many-core OS [3, 4].

Therefore, latest many-core designs include dynamic thermal management approaches that incorporate thermal information into the workload allocation strategy to obtain the best performance while avoiding peaks or large gradients of temperature. Nowadays, a few sensors are already deployed on the chip to obtain this thermal information. However, their number is limited by area/power constraints and their optimal placement to detect all the worst case temperature scenarios is a very complex problem that has received significant attention in the last years [2, 10, 12, 14, 15].

Unfortunately, the reconstruction of the entire thermal map from a limited number of thermal sensors poses many — and still unresolved — questions. In particular, for a specific many-core architecture, the two fundamental questions are the possible trade-offs regarding the number of sensors to place and the reachable degree of temporal and spatial thermal precision, as well as the sensor placement to maximize the reconstruction performance.

In this paper, we propose to recover the entire thermal map using a new method, which we call *EigenMaps*. Our method estimates an entire temperature map using a limited set of measurements collected by sensors, as inspired by [12]. First, we reduce the complexity of the thermal map by considering an optimal low-dimension approximation. Then, we use the sensors measurements to estimate the parameters of the approximated thermal map. Specifically, our contributions are:

- A reliable low-dimensional approximation of thermal maps based on a thermal analysis done at design time.
- A reconstruction algorithm that recovers the thermal map approximation from the sensors measurements. The quality of reconstruction can be adjusted according to the number of

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sensors and the quality of the measurements by adapting the precision of the aforementioned approximation.

- A theoretical derivation of a sensor allocation method minimizing the reconstruction error.
- An algorithm that finds a near-optimal allocation in polynomial time.

Our methods are stable in the presence of noise: the error corrupting the measurements is not amplified by the reconstruction algorithm. Moreover, the allocation algorithm can easily integrate sensor location constraints, such as the unfeasibility of placing a sensor into the cache [11]. The reconstruction algorithm achieves the highest precision when compared to the available literature. The price of this precision is the necessity of storing a matrix in the memory/cache. The size of this matrix, and therefore the necessary space in memory, grows linearly with the desired precision.

We substantiate our theoretical findings with extensive numerical evidence, based on thermal simulation of an eight-core Niagara T1 Ultrasparc processor, shown in Fig. 1. These experiments show that EigenMaps achieve a significant improvement when compared to the state-of-the-art. In particular, we can estimate an entire thermal map with high precision ( $<1^\circ$ ) with only four sensors. Moreover, we can achieve the same precision with measurements that have a SNR of 15 dB using only 16 sensors.

## 2. BACKGROUND AND RELATED WORK

The thermal map of a processor can be estimated using two dual strategies:

- Solution of the direct problem, given the heat sources and the physical model of the temperature diffusion (e.g. a non-linear diffusion equation),
- Solution of the inverse problem, given the value of the temperature in some locations and some a-priori information about the thermal map.

The first approach is limited by its requirements: the knowledge of the heat sources can be ascribed to the knowledge of the detailed power consumption of the different components. This information cannot be known exactly at run time and moreover the computation of a solution would require an excessive computational power.

On the other hand, it is impossible to solve the inverse problem from few, spatially localized, imprecise measurements without some a-priori constraints on the thermal map, such as limited bandwidth [2]. The performance is significantly impacted by the small number of available sensors and the *structure* we consider for the thermal map, i.e. the a-priori information. Nowroz et al. [12] proposed a low-pass approximation strategy to reduce the number of sensors that are placed using an energy-oriented algorithm. This sensor allocation algorithm has been improved by Reda et al. [14] using a heuristic iterative approach to approximate an NP-hard problem. The authors in [9] proposed a grid-based uniform sensor placement followed by interpolation to approximate the temperature. These works estimated entire thermal maps, but the precision of the estimates is limited by the sub-optimality of the proposed a-priori information.

Other works have notable performance but are not focused on the estimation of the entire thermal map. Namely, the approach in [19] employs the correlation in power distribution to estimate the expected value of temperature at different locations of the chip using a dynamically tuned Kalman filter. The problem of noisy measurements has also been already considered. In particular, the correlation between the different sensor measurements has been exploited to denoise the measurements [18].

The remainder of the paper is organized as follows. In Section 3, we describe our three main findings: the optimal approximation, the reconstruction algorithm and the sensor allocation algorithm.

Then, the experimental setup is described in Section 4, followed by the experimental results in Section 5.

## 3. RECOVERY OF THERMAL MAPS

In this work, we use the two dual estimation strategies to optimize the reconstruction. First, we use the direct problem to define an optimal low-dimensional approximation. Then, we use this approximation to recover thermal maps solving a simpler inverse problem. The inverse problem is simplified because the approximation reduces the number of parameters that must be estimated by the reconstruction algorithm.

The performance of the reconstruction algorithm depends on the approximation quality and the conditioning of the inverse problem—a complicated function of the sensor locations. Therefore, we conclude with a sensor allocation algorithm that minimizes the conditioning of the inverse problem.

We consider a processor with an  $N$ -dimensional discrete temperature map  $\mathbf{t}$ . The temperature at coordinates  $i_1$  and  $i_2$  is defined as  $t[i_1, i_2]$ , for  $0 \leq i_1 \leq H-1$  and  $0 \leq i_2 \leq W-1$ , where  $W$  and  $H$  are the width and the height of the discretized thermal map, respectively. We vectorize the thermal map as  $x[i]$ , for  $0 \leq i \leq N-1$  and  $N = WH$ , that is

$$x[i] = t \left[ i \bmod H, \left\lfloor \frac{i}{W} \right\rfloor \right].$$

In other words, we stack the columns of the discrete thermal map to transform the matrix into a vector. For the remainder of the paper, a bold symbol,  $\mathbf{x}$ , indicates vectors or matrices while the normal symbols,  $x$  are reserved for scalars or elements of vectors.

### 3.1 Approximation of thermal maps

In this section, we derive the approximation method as a projection onto the low-dimensional linear subspace that minimizes the mean squared error (MSE). It allows us to describe the  $N$ -dimensional thermal map with only  $K$  coefficients, where  $K \ll N$ .

We define the subspace at design time, exploiting the set of  $T$  thermal maps generated by a numerical solution of the direct problem. These are considered as realizations of the  $N$ -dimensional random vector  $\mathbf{x}$ . We assume that the elements of  $\{\mathbf{x}_j\}_{j=0}^{T-1}$  have zero mean to keep a simple notation<sup>1</sup>. Any vector  $\mathbf{x}$  can be represented using a basis  $\Phi$  as,

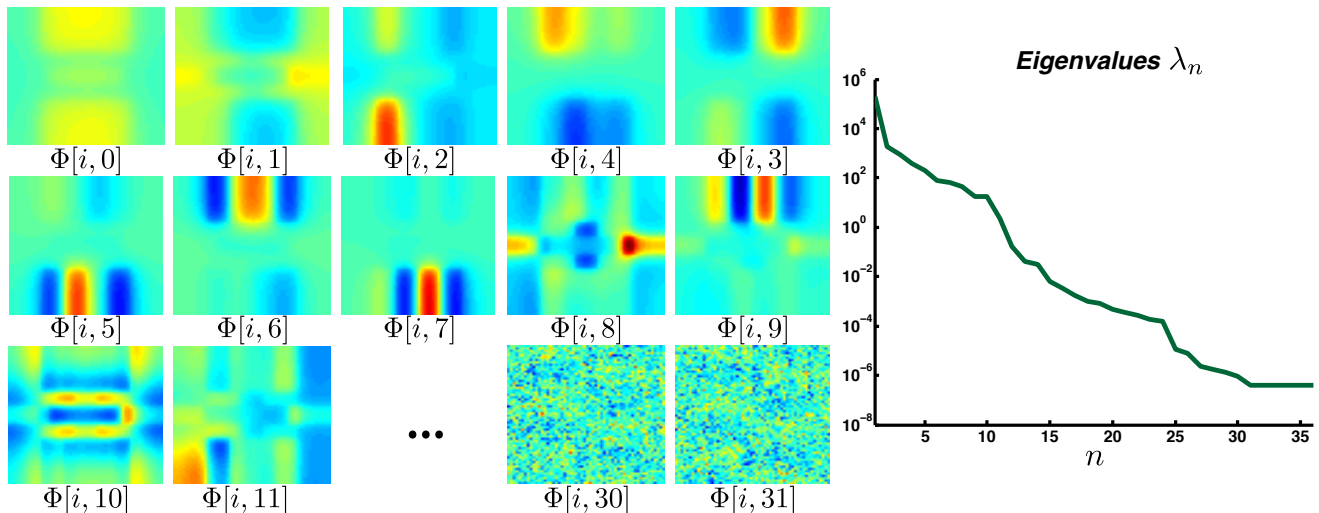
$$x[i] = \sum_{n=0}^{N-1} \Phi[i, n] \alpha[n],$$

where  $\alpha[n]$  are the coefficients of the expansion over the basis  $\Phi$ . Note that once we define a basis for the data, knowing the coefficients  $\alpha$  is equivalent to knowing the thermal map  $\mathbf{x}$ . We can describe the approximated thermal maps with a linear combination of  $K$  columns of  $\Phi$  with  $K$  elements of  $\alpha$  out of  $N$  as coefficient. More precisely, the approximated thermal map  $\hat{\mathbf{x}}$  is given by the following overdetermined system of equations

$$\begin{aligned} \hat{\mathbf{x}} &= \begin{bmatrix} \Phi[0, 0] & \cdots & \Phi[0, K-1] \\ \vdots & \ddots & \vdots \\ \Phi[N-1, 0] & \cdots & \Phi[N-1, K-1] \end{bmatrix} \begin{bmatrix} \alpha[0] \\ \vdots \\ \alpha[K-1] \end{bmatrix} \\ &= \Phi_K \alpha_K, \end{aligned} \quad (1)$$

where the subscript  $K$  indicates the selection of the first  $K$  columns for a matrix or the first  $K$  elements for a vector. This approximation is equivalent to a projection onto the  $K$ -dimensional subspace

<sup>1</sup>Note that we can always subtract the mean to get zero-mean vectors.



**Figure 2:** Left: a selection of the first 32 EigenMaps for the Niagara T1 Ultrasparc. Note that the informative content decays rapidly to just noise (the last two EigenMaps). This analysis is confirmed by the decay of the eigenvalues given in the right plot.

spanned by the columns of  $\Phi_K$ . We suggest that the optimal subspace is the  $K$ -dimensional one introducing the smallest error in the MSE sense. We define the following optimization problem to find this basis and the relative optimal subspace we are looking for.

**PROBLEM 1.** Find the set of basis vectors in  $\Phi$  such that the approximation  $\hat{\mathbf{x}}$  with the first  $K < N$  components,  $\hat{\mathbf{x}} = \Phi_K \alpha_K$ , minimizes the following error

$$\xi = \mathbb{E} [\|\mathbf{x} - \hat{\mathbf{x}}\|^2] = \mathbb{E} \left[ \left\| \sum_{n=K}^{N-1} \Phi[i, n] \alpha[n] \right\|^2 \right].$$

This dimensionality reduction technique is well known in other fields under different names, such as *Principal Component Analysis* (PCA) and *Karhunen-Loeve Transform*. It has an analytic solution and it requires the covariance matrix  $\mathbf{C}_x$ , that is defined for real zero-mean random variables as

$$C_x[i, j] = \mathbb{E} [x[i]x[j]].$$

We estimate this matrix using the set of  $T$  thermal maps simulated at design time. The quality of the available dataset impacts the quality of the estimate  $\mathbf{C}_x$ . This estimation is a well studied topic [6] and will not be discussed here.

We give the solution to Problem 1 in the following proposition. The proof is a well-known result.

**PROPOSITION 1. Optimal Approximation** Let us consider a set of  $T$  thermal maps  $\{\mathbf{x}_j\}_{j=0}^{T-1}$  with zero mean and covariance matrix  $\mathbf{C}_x$ . The orthonormal basis  $\Phi_K$  that defines the approximation  $\hat{\mathbf{x}}$  with the minimum error  $\xi$ , is formed by the  $K$  eigenvectors of  $\mathbf{C}_x$  with the largest eigenvalues  $\{\lambda_n\}_{n=0}^{K-1}$ . Moreover, the approximation error is decreasing when  $K$  grows as

$$\xi = \sum_{n=K}^{N-1} \lambda_n. \quad (2)$$

The connection between  $\mathbf{C}_x$  and the optimal basis has an intuitive explanation. In fact, if the temperatures at different spatial points are statistically correlated, then  $\mathbf{C}_x$  has nonzero elements outside its diagonal. These elements can be used to infer the temperature at points without sensors. Moreover, if the correlation is strong, then the eigenvalues  $\lambda_n$  of  $\mathbf{C}_x$  decay fast and we can precisely approximate the temperature  $\mathbf{x}$  with a smaller  $K$ , see (2). Recall that  $K$  is the number of parameters we need to estimate from the sensor measurements; having the optimal approximation

with the minimum  $K$  is fundamental to have a truthful reconstruction with just few sensors. Note that this optimal approximation pays the increased precision with more space occupied in memory; namely, we need to store the matrix  $\Phi_K$  in the memory, with an occupation that is proportional to the desired resolution. Other methods, such as the one proposed in [12], use standard basis, avoiding the occupation of memory but obtaining a lower precision.

Inspired by a classical work in computer vision [17], we call *EigenMaps* the eigenvectors of  $\mathbf{C}_x$ . An example for EigenMaps of the Ultrasparc T1 multicore architecture is given in Fig. 2. Note that each EigenMap represents a particular structure of the processor, such as cores, FPU and cache.

### 3.2 Reconstruction of thermal maps

Thermal maps are now defined only by their  $K$  coefficients  $\alpha_K$  in the basis  $\Phi_K$ . Here, we explain how to estimate them from the sensors measurements.

In principle, we can find the coefficients by inverting the over-determined linear system of equations given in (1). However, this would require the knowledge of the temperature  $x[i]$  at every spatial location  $i$ . Let us assume that we can place only  $M$  sensors at locations  $\mathcal{S} = \{j_i\}_{i=1}^M$ . Considering (1), it is equivalent to

$$\mathbf{x}_S = \begin{bmatrix} \Phi[j_1, 0] & \cdots & \Phi[j_1, K-1] \\ \vdots & \ddots & \vdots \\ \Phi[j_M, 0] & \cdots & \Phi[j_M, K-1] \end{bmatrix} \begin{bmatrix} \alpha[0] \\ \vdots \\ \alpha[K-1] \end{bmatrix} = \tilde{\Phi}_K \alpha_K, \quad (3)$$

where  $\tilde{\Phi}_K$  is a matrix formed by the rows of  $\Phi_K$  corresponding to the sensor locations  $\mathcal{S}$ ,  $\mathbf{x}_S$  is a vector containing our sensor measurements and  $\alpha_K$  is the unknown vector.

Before we characterize the solution of (3), we need to introduce the concept of noise into the model. More precisely, we have two different noise sources affecting our measurements. First, we have the approximation error  $\epsilon = \hat{\mathbf{x}} - \mathbf{x}$  that is systematic and is due to the approximation on the  $K$ -dimensional subspace. Second, the measurements are corrupted by a significant amount of noise due to many factors, such as thermal noise, quantization and calibration inaccuracies [15]. Therefore, we consider the following modification of (3),

$$\mathbf{x}_S + \mathbf{w} = \tilde{\Phi}_K \alpha_K, \quad (4)$$

where  $\mathbf{w}$  is the noise term. There is no exact solution to (4). How-

ever, we can find the coefficients  $\hat{\alpha}_K$  such that the error w.r.t. the measured temperature is minimized. Namely, we solve the following least square problem,

$$\min_{\hat{\alpha}_K} \|x_S - \tilde{\Phi}_K \hat{\alpha}_K\|_2^2.$$

We reconstruct thermal maps using the  $K$ -dimensional approximation given in (1). This leads to the theorem for the reconstruction of a thermal map from noisy measurements.

**THEOREM 1. Noisy Reconstruction** Consider an  $N$ -dimensional thermal map  $x$ , with zero mean and covariance matrix  $C_x$ . Choose a basis  $\Phi$ , such as the one in Proposition 1. Define a new matrix  $\tilde{\Phi}_K$  according to (3) to represent the approximation on the  $K$ -dimensional subspace and the sensing with  $M$  noisy sensors located at  $S$  as in (4). If  $M \geq K$  and  $\text{rank}(\tilde{\Phi}_K) = K$ , then the reconstruction  $\tilde{x}$  of the thermal map  $x$  is **unique** and equal to

$$\tilde{x} = \Phi_K \left( \tilde{\Phi}_K^* \tilde{\Phi}_K \right)^{-1} \tilde{\Phi}_K^* x_S.$$

Moreover, the reconstruction MSE is bounded by the condition number  $\kappa$  of  $\tilde{\Phi}_K$  and the noise energy  $\|w\|_2$  as

$$\frac{\|\tilde{x} - x\|_2}{\|x\|_2} = O\left(\kappa^2(\tilde{\Phi}_K)\right) \|w\|_2. \quad (5)$$

This theorem highlights a focal point of our work: given  $M$  sensors and an optimal  $K$ -dimensional subspace  $\Phi_K$ , the optimal sensor location is the one that minimizes the condition number of  $\tilde{\Phi}_K$ . If this condition number is minimal, the reconstruction error  $\epsilon_r = \tilde{x} - x_S$  is minimal for the given amount of noise  $w$ . In other words, the condition number is an excellent metric to evaluate different sensing patterns and find the optimal one. Note that, once we have fixed  $M$ , increasing  $K$  will in general increase the reconstruction error  $\epsilon_r$  (worse conditioning) and decrease the approximation error  $\epsilon$  (better approximation). Therefore, we should pick an optimal  $K$  such that the sum of  $\epsilon$  and  $\epsilon_r$  is minimal.

### 3.3 A greedy algorithm for sensor allocation

In what follows, we present a greedy polynomial algorithm to find the solution to the sensor allocation problem, i.e. choosing  $M$  rows from  $\Phi_K$  such that the resulting  $\tilde{\Phi}_K$  is full rank and has minimal condition number.

Intuitively, we are looking for  $M$  rows that form the best orthonormal basis for the  $M$ -dimensional subspace. The optimal solution can be obtained by computing the condition number of all possible sets of  $M$  rows out of the  $N$  original rows. This is equivalent to computing  $\binom{N}{M}$  singular value decompositions, which is computationally impossible, as this number is proportional to  $N!$ .

We propose a sensor allocation algorithm that has polynomial complexity and achieves the best performance when compared to the state-of-the-art. First, we compute the correlation between all rows of  $\tilde{\Phi}_K$ , then we remove one by one the rows that show the highest correlation with the other ones. Intuitively, we do not consider the sensor locations that would add the least informative content. Eventually, the remaining  $M$  rows indicate where we should place the sensors. The structure of the algorithm is given in Algorithm 1 and an example of the sensor allocation algorithm output is given in Fig. 6 (a).

## 4. EXPERIMENTAL SETUP

We tested the proposed methods on an Ultrasparc T1 and we simulate its thermal behavior using 3D-ICE [16]. This simulator is based on a compact transient thermal model; it can be used for thermal simulations of 2D or 3D chips cooled with conventional or liquid cooling. The simulator has been validated against

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### Algorithm 1 Sensor allocation

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**Require:** Subspace  $\Phi_K$ , number of sensors  $M$

**Ensure:** Sensing matrix  $\tilde{\Phi}_K$ , sensor locations  $S$

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1. Normalize the rows of  $\Phi_K$  such that they are unit-norm. Call  $U$  the normalized matrix.
  2. Compute  $G = UU^* - I$ , where  $I$  is the identity.
  3. **Repeat until  $M$  rows are left:**
    - (a) Find the maximum element, i.e.  $G[i, j] = \max G$ .
    - (b) Remove the  $i$ -th row and column from  $G$ .
    - (c) Build  $S$  from the remaining rows, build the sensing matrix  $\tilde{\Phi}_K$ .
    - (d) **If  $\text{rank}(\tilde{\Phi}_K) < K$ , then**
      - i. Restore  $\tilde{\Phi}_K$  from the previous iteration.
      - ii. Break.
- 

computational fluid dynamics simulations, it is easily configurable and publicly available. The input of the simulation are the power traces given in [7]. These traces describe the power consumed by the elements of the processor while running different scenarios/workload. The output is a set of thermal snapshots at each time interval: namely, we have  $T = 2652$  thermal maps with  $N = 3360$ , since the thermal maps are discretized with  $W = 60$  and  $H = 56$ . Note that our thermal maps are rather coarse-grained since we consider large blocks having the same average power consumption, but we expect to obtain similar performance on more detailed thermal maps.

As a reference, we choose in the literature the reconstruction algorithm that shows the best performance for the entire thermal reconstruction. Specifically, we consider the  $k$ -LSE algorithm for reconstruction and the energy-center technique for sensor allocation, both from [12].  $k$ -LSE is also one of the first algorithms estimating the entire thermal map from few measurements. Moreover, our reconstruction methods are conceptually close to their approach, the main difference being the choice of the approximation subspace.

We consider two main figures of merit when comparing the reconstruction techniques. The MSE of the reconstruction, defined as the average error over all thermal maps, that is

$$\text{MSE} = \frac{1}{TN} \sum_{i=0}^{N-1} \sum_{j=0}^{T-1} |x_j[i] - \hat{x}_j[i]|^2,$$

where the index  $j$  points to all  $T$  thermal maps available in the dataset. We also consider the maximum error among the maps, that is defined as

$$\text{MAX} = \max_{i,j} |x_j[i] - \hat{x}_j[i]|^2,$$

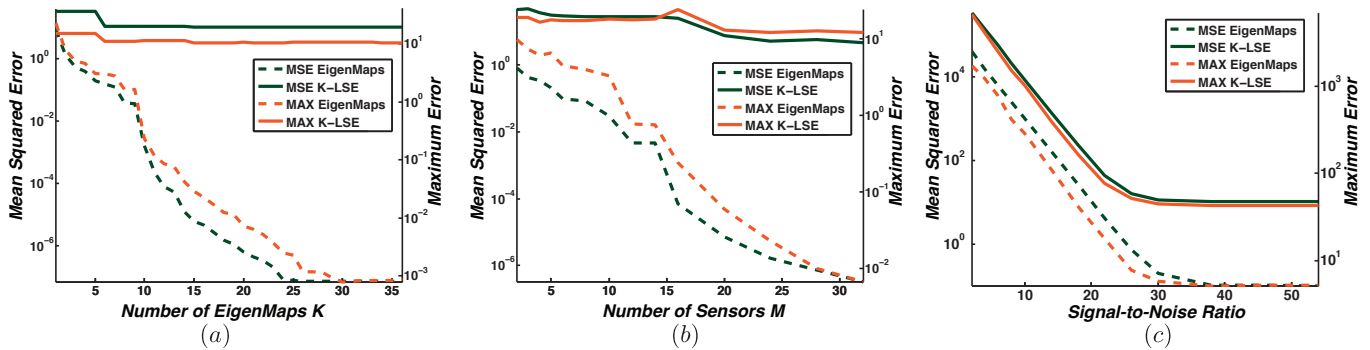
because localized peaks of error can lead to thermal runaway.

## 5. RESULTS

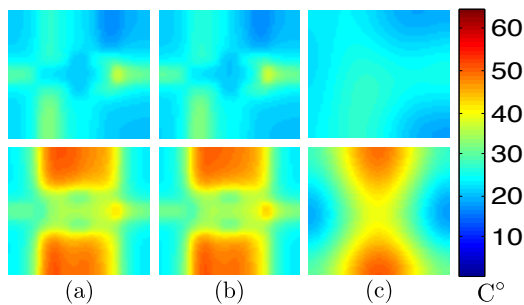
In what follows, we present and discuss the results of the numerical experiments. All the experiments have been run on all the 2652 thermal maps generated during the simulation phase.

### 5.1 Reconstruction Performance

First, we show the impact of the choice of the subspace. We compute the difference between a thermal map  $x$  and its approximation  $\hat{x}$  as a function of  $K$  for the two different methods,  $k$ -LSE and EigenMaps. The results are given in Fig. 3 (a). The theoretical optimality of the EigenMaps basis is confirmed by this experiment, where we note how the error is exponentially lower than for the DCT basis used in  $k$ -LSE. Note that this error has a strong impact



**Figure 3: Comparison between EigenMaps (dotted) and  $k$ -LSE (continuous). In all the plots, the MSE (dark green) is on the right and the MAX (orange) is on the left. Note the difference of scale between the two  $y$ -axes. (a) The approximation error as a function of the number of EigenMaps  $K$ . (b) The reconstruction error as a function of the number of sensors used. (c) The reconstruction error in presence of measurement noise as a function of the SNR using 16 sensors.**



**Figure 4: Visual comparison between EigenMaps and  $k$ -LSE two different thermal maps (top and bottom row) using 16 sensors. (a) The original thermal maps. (b) Reconstruction using EigenMaps. (c) Reconstruction using the  $k$ -LSE method.**

in the reconstruction phase. In fact, this error is noise for the reconstruction algorithm and is amplified by the conditioning of  $\Phi_K$  according to (5).

Therefore, we expect to have the same exponential decay for the reconstruction error. To numerically confirm this expectation, we present our first core result: a direct comparison of the reconstruction performance in terms of MSE and MAX between EigenMaps and  $k$ -LSE. The results are given in Fig. 3 (b). We observe that the reconstruction error is approximately decaying as fast as the approximation error. Here is our intuitive explanation: we can estimate the parameters of more EigenMaps by increasing the number of sensors  $M$ , while keeping a low conditioning. Therefore, the total error follows closely the approximation error.

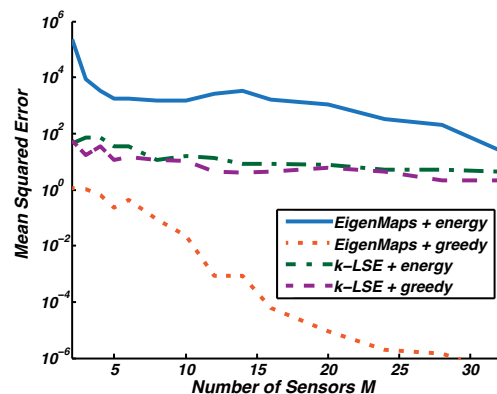
As further proof of the quality of our reconstruction, in Fig. 4 we give a visual comparison between the original thermal map and the reconstruction with the two methods using 16 sensors for each.

Lastly, we consider the realistic scenario of presence of noise and/or measurements errors. We compare the reconstruction performance using 16 sensors with EigenMaps and with  $k$ -LSE as a function of the SNR of the measurements, defined as

$$\text{SNR} = \frac{\|\mathbf{x}\|_2}{\|\mathbf{w}\|_2},$$

where  $\mathbf{w}$  is the noise vector. The results are depicted in Fig. 3 (c) and we note that EigenMaps performs better than  $k$ -LSE even when there is noise introduced by sensor calibration inaccuracies. We believe that the performance of  $k$ -LSE is negatively impacted by two main factors: we consider a very small number of sensors and the processor taken in consideration here (Ultrasparc T1 instead of Athlon dual-core) generates more high frequency content, which is not well-approximated by their choice of basis.

Among all the proposed results, we would like to underline the



**Figure 5: Comparison of the two sensor allocation techniques with both reconstruction algorithms. For both reconstruction algorithms, the greedy approach obtains better reconstruction performance in terms of MSE.**

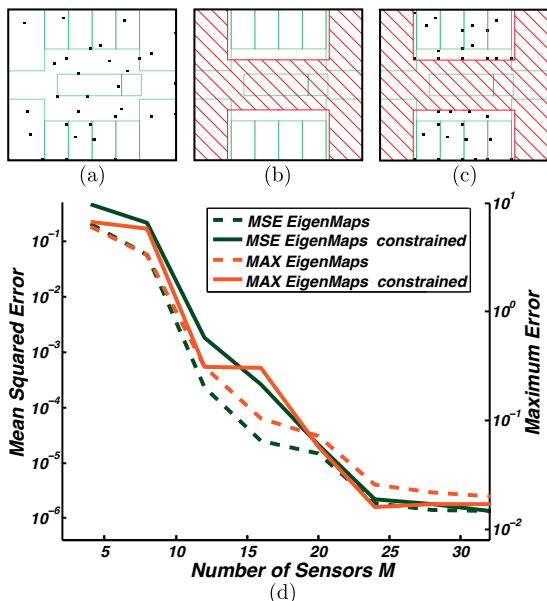
most important ones: we can recover with few sensors (4-5) entire thermal maps while keeping the MSE and the MAX below  $1^\circ\text{C}$ , see Fig 3 (b). Moreover, if we consider a very noisy environment, 15dB of SNR, we can keep the same excellent reconstruction performance with just 16 sensors, see Fig. 3 (c).

## 5.2 Sensor allocation performance

To underline the effectiveness of the sensor allocation algorithm, we propose the following experiment. We compute the MSE for four different combinations of reconstruction algorithms and sensor allocation algorithms. In particular, we consider EigenMaps,  $k$ -LSE, our greedy algorithm for sensor allocation and the energy-center algorithm [12] (referred as “energy” in the figures). The results are depicted in Fig. 5. Note that whichever reconstruction method is chosen, the greedy algorithm improves the performance w.r.t. to the energy-center algorithm. Hence, the greedy algorithm leads to a better condition number of the inverse problem.

Finally, we look at the stability of our sensor allocation algorithm when we have design constraints. This experiment is motivated by the fact that we cannot place sensors in a very regular and/or critical structure, such as a cache [11]. To analyze this scenario we compare the reconstruction performance between the unconstrained and the constrained cases. The constraints are defined using a mask of allowed zones (black) that is given in Fig. 6 (b). The reconstruction error is given in Fig. 6 (d), while examples of sensor allocations are given in Fig. 6 (a) and (c).

## 6. CONCLUSION



**Figure 6: Comparison of the reconstruction error of the proposed method when the sensor locations are free (dotted line) and constrained by design (continuous line). (a) Location of 32 sensors without constraint. (b) Mask of the constraint: sensors can not be placed on the striped red zone. (c) Location of 32 sensors with constraint. (d) MSE (dark green) and MAX (orange) as a function of the number of sensors.**

In this work, we proposed a framework to optimally reconstruct thermal maps of multicore processors using a small number of sensors. We defined an optimal approximation of thermal maps to reduce the number of parameters to estimate, without losing precision. We reconstructed the thermal maps using a least square approach and we exposed the critical role of the sensor location for the conditioning of the inverse problem. We concluded proposing a sensor allocation algorithm that minimizes the reconstruction error by minimizing the conditioning of the inverse problem.

We compared EigenMaps with  $k$ -LSE [12] using extensive numerical experiments. We demonstrated the higher fidelity of our reconstruction using a smaller number of sensors. We showed how the reconstruction performance is stable w.r.t. the noise introduced by the electronics or by sensor calibration inaccuracies. Moreover, even if we constrain the locations of the sensors, the reconstruction degrades only slightly. To the best of our knowledge, this is one of the first works that recovers the entire thermal map while considering noise measurements and constrained allocation.

To summarize, our work improves significantly the precision and the stability achievable for the thermal monitoring of a multicore processor. The price for the increased precision is the memory needed to store the matrix  $\Phi_K$ , that is the main limitation of the proposed method. We also introduce the concept of minimizing the condition number of the inverse problem for the sensor allocation problem, that is the critical figure of merit of the estimation problem.

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## 8. REFERENCES

- [1] BROOKS, D., DICK, R., JOSEPH, R., AND SHANG, L. Power, Thermal, and Reliability Modeling in Nanometer-Scale Microprocessors. *Micro, IEEE* 27, 3 (Jan. 2007), 49–62.
- [2] COCHRAN, R., AND REDA, S. Spectral techniques for high-resolution thermal characterization with limited sensor data. *Des. Aut. Con.* (June 2009), 478–483.
- [3] COSKUN, A. K., ROSING, T., WHISNANT, K., AND GROSS, K. C. Static and Dynamic Temperature-Aware Scheduling for Multiprocessor SoCs. *IEEE Trans. VLSI Syst.* 16, 9 (2008), 1127–1140.
- [4] COSKUN, A. K., ROSING, T. S., AND GROSS, K. C. Utilizing Predictors for Efficient Thermal Management in Multiprocessor SoCs. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* 28, 10 (Dec. 2009), 1503–1516.
- [5] HUANG, W., STAN, M. R., SANKARANARAYANAN, K., RIBANDO, R. J., AND SKADRON, K. Many-Core Design from a Thermal Perspective. In *DAC* (2008).
- [6] KAY, S. M. *Fundamentals of statistical signal processing: estimation theory*. Prentice-Hall, Inc., Mar. 1993.
- [7] LEON, A. S., TAM, K. W., SHIN, J. L., WEISNER, D., AND SCHUMACHER, F. A Power-Efficient High-Throughput 32-Thread SPARC Processor. *IEEE J. Solid-State Circuits* 42, 1 (Jan. 2007), 7–16.
- [8] LIN, S.-C., AND BANERJEE, K. Cool Chips: Opportunities and Implications for Power and Thermal Management. *IEEE Trans. Electron Devices* 55, 1 (Jan. 2008), 245–255.
- [9] LONG, J., MEMIK, S. O., MEMIK, G., AND MUKHERJEE, R. Thermal monitoring mechanisms for chip multiprocessors. *ACM Trans. Archit. Code Optim.* 5, 2 (Aug. 2008), 1–33.
- [10] MUKHERJEE, R., AND MEMIK, S. Systematic temperature sensor allocation and placement for microprocessors. In *DAC* (2006).
- [11] MUKHERJEE, R., AND MEMIK, S. O. Systematic temperature sensor allocation and placement for microprocessors. In *DAC* (2006).
- [12] NOWROZ, A. N., COCHRAN, R., AND REDA, S. Thermal monitoring of real processors: techniques for sensor allocation and full characterization. In *DAC* (2010).
- [13] PEDRAM, M., AND NAZARIAN, S. Thermal Modeling, Analysis, and Management in VLSI Circuits: Principles and Methods. *Proc. IEEE* 94, 8 (Aug. 2006), 1487–1501.
- [14] REDA, S., COCHRAN, R., AND NOWROZ, A. N. Improved Thermal Tracking for Processors Using Hard and Soft Sensor Allocation Techniques. *IEEE Trans. Comput.* 60, 6 (Nov. 2011), 841–851.
- [15] SHARIFI, S., AND ROSING, T.Š. Accurate Direct and Indirect On-Chip Temperature Sensing for Efficient Dynamic Thermal Management. *IEEE Trans. Comput.-Aided Des. Integr. Circuits Syst.* 29, 10 (Oct. 2010), 1586–1599.
- [16] SRIDHAR, A., VINCENZI, A., RUGGIERO, M., BRUNSCHWILER, T., AND ATIENZA, D. 3D-ICE: Fast Compact Transient Thermal Modeling for 3D ICs with Inter-tier Liquid Cooling. *IEEE ICCAD 1* 2010.
- [17] TURK, M. A., AND PENTLAND, A. P. Face recognition using eigenfaces. In *1991 IEEE Computer Society Conference on Computer Vision and Pattern Recognition* (Nov. 1991), IEEE Comput. Soc. Press, pp. 586–591.
- [18] ZHANG, Y., AND SRIVASTAVA, A. Accurate temperature estimation using noisy thermal sensors. In *DAC* (2009).
- [19] ZHANG, Y., AND SRIVASTAVA, A. Adaptive and autonomous thermal tracking for high performance computing systems. In *DAC* (2010).