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Game Theory Video Streaming

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Abstract

We consider the problem of peer selection for multimedia transmission over a peer-to-peer (P2P) network to find a strategy model for peer selection for each lost packet. Our problem consists of two parts one related to the P2P network that is a peer selection method, and the other is related to the uplink bandwidth that each peer in the network receives from a licensed spectrum holder. Moreover, if a peer wants to download a desired packet from another peer in the network, it should pay a certain amount of money to the source to download a packet. Also, if a node in the P2P network wants to use its uplink rate to send a packet to another peer in the network, it should pay some money to the primary user in order to use the primary user bandwidth. In addition, in peer selection method every node in the P2P network decides its peers based on a model that maximizes its utility function. On the other hand, the interaction between the P2P network and spectrum holder is cast in the framework of Stackelberg games and we use game theory model for this part of the report. It is worth mentioning that we consider the P2P network in different situations based on peers' characteristics in the network.

1 Introduction

Nowadays multimedia applications that use P2P network have become increasingly attractive. Each peer in the P2P network can act as both server and receiver, this characteristic of the P2P network makes them even more attractive. Which means that every peer can send packets to other peers, while receiving a packet from the others. This feature also makes them more attractive for video streaming applications.

One of the files sharing application that use P2P network is BitTorrent[1]. BitTorrent use tit-for-tat algorithm for peer selection in which every node in the network selects those nodes with the highest uplink rate as its peers and in return, it sends its contents to these peers. Tit-for-tat algorithm dose not take into account simultaneously sharing of multiple multimedia files and also cannot make any differentiation for different packets which may have different preference for peers. Thus, tit-for-tat does not fully reveal the joint manners of peers.

In this report we will try to find a solution for video streaming over P2P network in which uplink and downlink rate have important interest. We will consider our model in different situations based on connectivity of peers and buffer size of peers in the network. Each peer tries to be in coalition with nodes that have higher bandwidth. Thus, we can model our problem in game theory with autonomous and rational players, struggling to have the highest possible utility value.

We use Pareto optima to find the best possible utility for the peers in the network. Moreover, we will take into account the cost that the uplink rate can have for each node in the network, as the result, there will be a provider who charges nodes based on the uplink rate that they have which means a higher uplink rate would have a higher cost. Thus, there is another game between nodes in the P2P network and the uplink rate provider which casts in the format of Stackelberg game method.

This report is organized as follow: in section 2 we will speak about game theory tools that we used in our problem, in section 3 we will find a model for our problem and use mathematical methods to solve this problem, in section 4 we will have simulation for different situation of the P2P network and also for the bandwidth provider, in section 5 there is a discussion about a related work to our model, and this section is followed by a conclusion which is the last section of this report.

2 Game Theory

Game theory is a branch of applied mathematics that is used in the social sciences. Game theory is the study of problems of conflict and cooperation among independent decision-makers.

2.1 Essential Concepts in Game Theory

A game is defined by the triplet $G = (P, S, U)$

- Player
  A player is an agent who makes decisions in a game.
- Strategy
  In a game in strategic form, a strategy is one of the given possible actions of a player.

- Utility
  A utility is a number, which reflects the desirability of an outcome to a player, for whatever reason.

- Rationality
  A player is said to be rational if he seeks to play in a manner, which maximizes his own utility. It is often assumed that the rationality of all players is common knowledge.

We concentrate on dynamic games of complete information, mostly on Stackelberg model. [6]

2.2 Nash Equilibrium

Suppose that players forecast the strategies of their opponents. In order for this prediction to be correct, it is essential that each player be willing to select the strategy predicted by the theory. Thus, each player's predicted strategy must be that player's best response to the predicted strategies of the other players. Such a forecast could be called strategically stable, because no single player wants to deviate from his or her predicted strategy. We call such a prediction a Nash equilibrium: In the n-player normal-form game $G = \{S_1, \cdots, S_n; u_1, \cdots, u_n\}$, the strategies $(u_1^*, \cdots, u_n^*)$ are Nash equilibrium if s_i^* is the player i best response to the strategies specified for the n-1 others, $(s_1^*, \cdots, s_{i-1}^*, s_{i+1}^*, \cdots, s_n^*)$

$$u_i(s_1^*, \cdots, s_{i-1}^*, s_i^*, s_{i+1}^*, \cdots, s_n^*) \geq u_i(s_1^*, \cdots, s_{i-1}^*, s_i, s_{i+1}^*, \cdots, s_n^*) \quad (1)$$

In other words:

$$\max_{s_i \in S_i} u_i(s_1^*, \cdots, s_{i-1}^*, s_i, s_{i+1}^*, \cdots, s_n^*) \quad (2)$$

It follows that, by definition, for a strategy combination $(s_1, \cdots, s_n)$ that does not form a Nash equilibrium, at least one player will have an motivation to differ to another strategy [2].

2.3 Pareto Optimumality

A strategy profile is Pareto-optimal if it is not possible to increase the utility of any player without decreasing the utility of another player.That is, a Pareto Optimal utility cannot increase without decreasing at least the utility of one of the players.

2.4 Dynamic Games of Complete Information

In this subset players' utility is a common knowledge for all. This branch consists of two kinds of games:

1. Perfect Information

   At each turn the player owing the turn knows the full history of the game. In other words, a player knows the strategy of its opponents in the past.

2. Imperfect Information

   Players do not know the full history of the game or simply, a player may not know exactly previous choices.

It is worth noticing that the main issue in this kind of game is credibility [2], which means that in every move that any player takes he thinks players previous moves were based on maximization of their utilities.

We focus on Stackelberg game, which is a branch of dynamic game of complete and perfect information game, but before going deeply in that area, we cast a look at backward induction, since it has an important role in Stackelberg games.

2.4.1 Backward Induction

Backward induction is a technique to solve a game of perfect information. It first considers the last moves of the game, and determines the best move for the player in each case. Then, taking these as given future actions, it proceeds backwards in time, again determining the best move for the respective player, until the beginning of the game is reached [6]. The key features of a dynamic game of complete and perfect information are:

- The moves are in sequence
- All pervious moves are known before the next move is taken place
- For each combination of players moves, utilities are common knowledge[2]

Games with these characteristics are solved by backward induction. Backward induction is used to find Stackelberg games' equilibriums, thus we describe how to solve a two-level backward induction.

- Player 1 chooses an action a_1 from the A_1.
- Player 2 chooses an action a_2 from the A_2.

- Utilities are $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$.

When it is player 2's action time, he will face following problem, given the action a_1 previously chosen by player 1:

$$\max_{a_2 \in A_2} u_2(a_1, a_2)$$

$R_2(a_1)$ is player 2's best reaction tworad player 1 . Since both players can predict each other action, player 1's action at the first stage is

$$\max_{a_1 \in A_1} u_1(a_1, R_2(a_1))$$

If we assume player 1's best action is a_1^*, we call $(a_1^*, R_2(a_1^*))$ the *backwards-induction outcome* of the game.
The backward induction is again based on credibility. That is player 1 knows player 2 moves is in a way that player 2 will recieve maximum utility based on player one moves.

2.5 Stackelberg Game

The Stackelberg model in economy consists of a leader firm which moves first and a follower firm which moves after. The Stackelberg model is solved by finding the subgame perfect Nash equilibrium[1] of the game. To calculate SPNE we first need to find the best reaction of follower to any quantity of its leader, thus we use backward induction to solve this kind of game. In a Stackelberg game the leader announces its strategy and follower responds to it *rationally*, as far as the leader knows the follower cost function, it can compute follower's reaction to all of its strategies. The timing of a Stackelberg game is as follows: [2]

1. Leader chooses a quantity $q_1 \geqslant 0$
2. Follower observes q_1 and then chooses a quantity $q_2 \geqslant 0$
3. Payoff for player i is:

$$u_i(q_i, q_j) = [P(q_i + q_j) - C_i(q_i)]q_i \tag{3}$$

[1] A subgame perfect Nash equilibrium (SPNE) is a set of strategies $\{s_i, i = 1, \ldots, n\}$ such that for each subgame g, the set of induced strategies $\{s_i(g), i = 1, \ldots, n\}$ forms a Nash equilibrium for this subgame

Price for firms is $P(q_1 + q_2)$ which is simply the function of total output. Moreover, we suppose that firm i has cost function as $C_i(q_i)$.
We use backward induction to solve a Stackelberg game, thus first we need to calculate the follower best response to an arbitrary quantity of leader.

$$\max_{q_2 \geqslant 0} u_2(q_1, q_2) = [P(q_1 + q_2) - C_2(q_2)]q_2 \tag{4}$$

The values of q_2 satisfying this response are follower's best response. For the best responses of the leader we need to find the follower best responses as a function of the leader possible actions, $R_2(q_1)$, and then maximize the leader utility:

$$\max_{q_1 \geqslant 0} u_1(q_1, R_2(q_1)) = [P(q_1 + R_2(q_1)) - C_1(q_1)]q_1 \tag{5}$$

These two maximizations can easily be found by just a derivation of each utility with respect to its given quantity and put the result equal to zero and find the respective value that satisfies the resulting expression. To have a better understanding of the problem we bring an example.

Supposing that the cost functions of both leader and follower are zero that is $C_1(q_1)$ and $C_2(q_2)$ are equal to zero; moreover, the inverse demand function is $P(q_1 + q_2) = A - B(q_1 + q_2)$ (A and B are constants). q_1^* and q_2^* are the leader and follower best answers.

$$u_2(q_1, q_2) = (A - Bq_1)q_2 - Bq_2^2 \tag{6}$$

and player 2 best reaction is:

$$\frac{\partial u_2}{\partial q_2} = 0 \longrightarrow R_2(q_1) = \frac{A - Bq_1}{2B} \tag{7}$$

firm 1's best answer will be:

$$u_1 = \frac{A}{2}q_1 - \frac{B}{2}q_1^2 \tag{8}$$

so;

$$\frac{\partial u_1}{\partial q_1} = 0 \longrightarrow q_1^* = 2q_2^* = \frac{A}{2B} \tag{9}$$

As a result in a two-player Stackelberg game we have these characteristics;

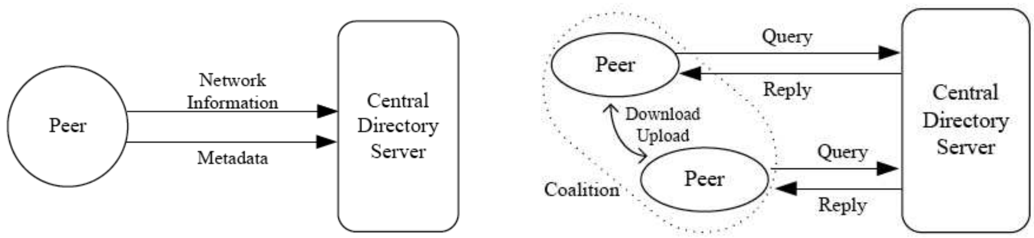
- Cost function of each player depends on the both players strategies.
- Each player tries to minimize its cost function.

2.5.1 One leader many followers

Most Stackelberg games consist of a leader and a follower that replies to leader strategy “rationally” by selecting a strategy that minimizes its cost function. But if we have more than one follower, we cannot distinctively reveal what is meant by “rational” response of the followers. As a result, the leader should know not only the followers cost function but also their “mood of play” which can be of two kinds non-cooperative or cooperative. In non-cooperative mode among followers we can consider their Nash equilibrium as their strategy in response to a leaders strategy. In case of cooperation we can use Pareto optimal as a case of modeling [4].

3 Problem Definition

The P2P network that we assume is a kind of overlay hybrid decentralized system model. All peers are connected to a centralized server, which has all the information about the peers in the network (IP, connection bandwidth, files, multimedia content, etc.). Peers can use this information to exchange multimedia content with each other. A typical decentralized P2P architecture is depicted in Fig. 1.



(a) (b)

Figure 1: An architecture of hybrid decentralized P2P network [5](1a) Registering process (1b) Interacting process

In P2P network, uplink rate is the most important source [3], thus peers ask for cost to contribute their uplink rate to others in the network. On the other hand, the uplink rate is not the only resources that has effect on the utility of the peers, thus it is essential that a peer select other peers in order to have cooperation in a way that it would have the highest possible utility.

We consider our problem in three different cases for the P2P network based on difference of connectivity and buffer state of peers. But before going to simulation we firstly, try to present a model for our system and formulate that model and after all we will use a mathematical method to find a solution for our model.

3.1 System Model

Our system consists of two separate parts: the primary user or bandwidth provider for nodes in the P2P network, and the secondary network, which is our P2P network. The primary user is the real owner of the bandwidth that peers use to send packets to each other and can give them the permission to use the bandwidth so that they can upload files each other. The secondary network is a group of peers that want to use the bandwidth of the primary user to share their video packets and try to receive their lost packet form each other in order to increase their quality of service.

The primary user based on the bandwidth that releases for the secondary network will receive money form them. So, it will try to release bandwidth in a way that it would receive more money from the secondary network. While the secondary network tries to use bandwidth in an efficient way, which means that it will try to use it in a way that the network will have the highest possible utility and pays the least possible amount of money to the primary user.

As we can see there is an interaction between the primary and the secondary network, thus this interaction will lead us to game theory methods for our model.

At first we will try to find a model that is suitable for the secondary network, which is a P2P video streaming network, and we simulate this network for three different cases, which are:

- Case A: Secondary network is a completely connected network and at time zero[2] buffer contents are the same.
- Case B: Secondary network is a completely connected network and at time zero buffers have different packets.

[2]time zero is the time that simulation is started

- Case C: Secondary network is not completely connected which means that from a node might have no direct link to another node in the network, moreover, buffers contains different packets at time zero.

Before going to simulation part for different cases, we describe our mathematical model and use Lagrange theorem to solve our model analytically.

3.1.1 Utility Model

In order to formulate a utility model for a video streaming node in a P2P network, firstly we should know that the utility consists of two major parts: one related to the attributes of node i and the other related to connectivity model of node i which means to which nodes in the network it is connected.

As we know some characteristics like what a node's buffer contains, what packet it has missed, preference of each packet, downlink and uplink rate, the cost that it will pay to a neighbor to download a missed packet from it, and uplink cost are amongst most important characteristics that can be effective in modeling the utility in a P2P network. Thus, utility of a given node depends on a set as:

$$\mathbf{A}_i = \{\mathbf{P}_i, \mathbf{D}_i, \mathbf{p}_i, R_i^U, R_i^D, c_i\} \tag{10}$$

$\mathbf{P}_i$ is the set of packet that node i owns, and $\mathbf{D}_i$ is the set of packets that node i is missed, $\mathbf{p}_i$ is the preference factor set which is ordered by popularity of packets it means that a more popular packet has higher weight, also we have $\sum_{j=1}^{|\mathbf{p}_i|} p_{ij} = 1$. In addition, R_i^U, R_i^D are uplink and downlink rate respectively. c_i consists of two part one related to the uplink cost of node i (c_i^U), the other is related to the money that it will pay to one of its neighbor to download a missed packet from him (c_i^p).

We assume that if a user download a lost packet j form a peer, then its multimedia quality will increase as a function of downlink rate and p_{ij}(the preference factor) of that packet. We use function $Q(R^D)$ to represent the achieved quality by a given downlink rate equal to R^D.

Therefore, we define utility function for user i as:

$$U_i = \sum_{j \in \mathbf{D}_i} p_{ij} Q(R_l^D) + \sum_{k \in \mathbf{P}_i} (c_i^{p_{ik}} - c_{ik}^U) R_i^U - \sum_{j \in \mathbf{D}_i} c_l^{p_{ij}} R_l^D \tag{11}$$

As we see in Eq. 11 utility of user i depends on $\mathbf{P}_i, \mathbf{D}_i, p, R^D, c_i^p, c_{ik}^U, R^U$, and l is the peer from whom lost packet j is downloaded. It means that utility of a given node depends on packets that it owns and also packets that it needs with their importance weighted with the so-called preference factor (p_{ij}).

Moreover, when a node sends a packet to a peer it will ask form the receiver a certain amount of money depending on the preference factor packet and also the rate of that packet, thus if a node can send a packet with a better quality, it will ask for more money. The cost that a node should pay in order to receive packet number j form peer l is $c_l^{p_{ij}}$ and the benefit that node i can gain when it sends a packet to a node is denoted by $c_i^{p_{ik}}$.

Additionally, the sender of the packet must pay some amount of money to the primary user (c_{ik}^U) because of using the uplink bandwidth, which is originally owned by the primary user, to send a packet which is denoted by c_{ik}^U. It is worth mentioning that all cost are factor of rate so there are multiplied by the rate of up\downlink rate.

As we can deduct, the sum of the paid cost of the secondary network to the primary user makes the body of the primary user utility. Thus, the utility of the primary van be modeled as:

$$U^{pri} = \sum_{\substack{i \\ \forall i \in \text{ secondary network}}} \sum_{k \in \mathbf{P}_i} c_{ik}^U R_i^U \tag{12}$$

Here we can conclude that there is an interaction between the primary user and the secondary network users. We summarized all of our modeling parameters in Table 1 to have a better over view of the model.

3.1.2 Utility Analysis

The primary user addition to the uplink bandwidth, also he decides on the relationship between the cost and rate of each node in the secondary network. The constraint of the primary user is modeled as:

$$aR_i^U - bc^{p_{ij}} = C \tag{13}$$

In Eq. 13 C is a constant and a, b are weights of rate and cost respectively. When a user wants to download a lost packet from one of his peers, he will

Symbol	Definition
p_{ij}	Denotes the preference factor of peer i on the packet j
$Q(.)$	Quality of service function
R_i^U	Uplink rate of node i
R_i^D	Downlink rate of node i
c_U^j	Cost of uplink to peer j
$c_i^{p_{ik}}$	Benefit that node i gains from sending packet k to a peer
c_{ik}^U	Cost of uplink from node i to its k^{th} peer
$c_l^{p_{ij}}$	Cost that node i pays to download packet k from peer l
$\mathbf{P}_i$	Set of shared packets of peer i
$\mathbf{D}_i$	Set of missed packets of peer i
U^{pri}	Utility function of the primary user
U_i	Utility function of peer i

Table 1: List of symbols and definitions in our utility models.

choose a peer who has less cost and higher rate, in the other word, it chooses one who can maximize its utility function for that specific packet. So, when node i wants to choose its peer l for a lost packet j it comes up with the following:

$$\max_l U_{ij} \approx \max_l (p_{ij} Q(R_l^D) - c_l^{p_{ij}} R_l^D) \tag{14}$$

To maximize Eq. 14 with constraint (Eq. 13) we have:

$$\begin{aligned} G &= U_{ij} - \lambda(aR_i^U - bc^{p_{ij}} - C) \\ G &= (p_{ij} Q(R_l^D) - c_l^{p_{ij}} R_l^D) - \lambda(aR_i^U - bc^{p_{ij}} - C) \end{aligned} \tag{15}$$

We use Lagrange method to maximize G with respect to $R^U, c^{p_{ij}}, \lambda$; moreover, $Q(.)$ is well estimated with $\ln(.)$, thus we have:

$$\frac{\partial G}{\partial R_l^D} = p_{ij} \frac{1}{R_l^D} - c^{p_{ij}} - a\lambda = 0 \tag{16}$$

$$\frac{\partial G}{\partial c^{p_{ij}}} = -R_l^D + b\lambda = 0 \tag{17}$$

$$\frac{\partial G}{\partial \lambda} = aR_i^U - bc^{p_{ij}} - C = 0 \tag{18}$$

From the Eq. 17 we find that $R_l^D = b\lambda$ and using Eq. 16 we can find $c^{p_{ij}} = \frac{p_{ij}}{b\lambda} - a\lambda$ and from Eq. 18 we find that $\lambda = \frac{C+\sqrt{C^2+8abp_{ij}}}{4ab}$ as the result the optimal value of cost and rate can be found as:

$$R_l^{*D} = \frac{C + \sqrt{C^2 + 8abp_{ij}}}{4a} \tag{19}$$

$$c^{*p_{ij}} = \frac{4ap_{ij}}{C + \sqrt{C^2 + 8abp_{ij}}} - \frac{C + \sqrt{C^2 + 8abp_{ij}}}{4b} \tag{20}$$

4 Numerical Example

Now that we could mathematically model our problem and found a maximization technique, we are going to have simulation for our 3 aforementioned cases of the secondary network. After that we will analyze the whole secondary and the primary network using Stackelberg game for the rational response of the secondary network as a whole.

4.1 Secondary Network Simulation

Before going to simulation of different cases, we define parameters that we will use for our simulation model.

The number of packets that each node reads at each time stamp for the video stream is called buffer read and it is equal for all nodes in all-different cases. Furthermore, depending on the simulating case there would be delays among buffer contents of peers. In addition, based on the case that we simulate, there might be difference in buffer contents of peers (some have more packets and some have less packets). D is randomly chosen for the secondary network nodes, beside that every node knows what packets its neighbors have and need, also the up\downlink rate of its neighbors. In the other word, all nodes have a complete information about their neighbors.

In order to verify the efficiency of the aforesaid peer selection algorithm based on Lagrange maximization, every node, instead of following the peer selection algorithm, will also choose a potential peer randomly. We are supposed to have a better result for the case using Lagrange theorem.

We run the case simulation for 100 times, for each iteration we change the cost that node i would ask from its peers upon sending packet j with the preference factor of p_{ij} to him. This cost distribution will also fulfill the primary user forced constraint. Our cost distribution is based on considering; a better node in the sense of quality (higher uplink rate) has a higher cost. After all iterations we will compute Pareto optima of the game and we find which cost distribution is a Pareto optimal for our simulation model. As mentioned before we characterize our different cases depending on the connectivity of the nodes and buffer contents of our nodes 3 different cases and we simulate both the primary and the secondary network together for case C, since this is a more realistic case. Table 2 summarizes numerical value of each case.

Case	A	B	C
Number of Secondary Network Nodes	30	30	30
Minimum Uplink Rate [kbps]	60	60	60
Minimum Number of Connection	Fully Connected	Fully Connected	10
a, b, C	1, 50, 50	1, 50, 50	1, 50, 50
Buffer Read	30	30	30
Maximum Buffer Delay[3]	0	10	10
Window Size	30	Randomly Chosen	Randomly Chosen
Uplink Cost	Randomly Chosen	Randomly Chosen	Randomly Chosen
Number of Iterations	100	100	100

Table 2: Summarization of simulation parameters for different cases of the secondary network model

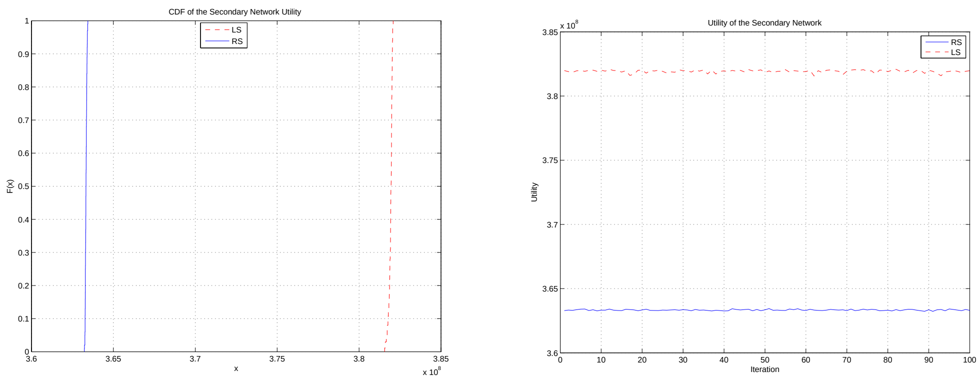
4.1.1 Case A

In this model we assume that all nodes are connected together and the video stream is started at the same time for all nodes, thus peers receive the same packets in every time stamp, no delay among peers. Moreover, number of the packets that reaches a node is equal to the number of packets that it

[3]All nodes have the same video stream, but some nodes might have started their stream sooner than others. The maximum number of packets because of this asynchronization of peers is denoted by *maximum buffer delay*

reads in each time stamp and is equal to 30.

Fig. 2 shows the CDF of the secondary network utility and the utility value of the secondary network during the simulation time. Utility of the secondary network is equal to sum of the utility of each node per simulation time. As we see in this figure when we use our suggested peer selection algorithm, the secondary network could gain higher utility value comparing to just randomly select a potential a neighbor.



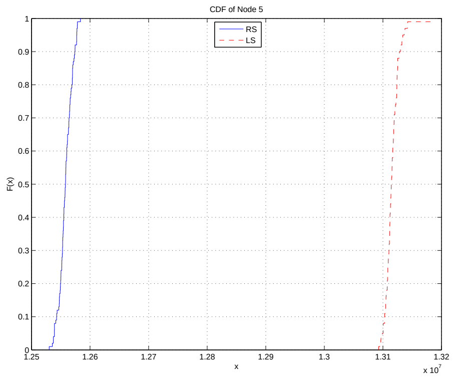
(a) (b)

Figure 2: Utility of the secondary network (Case A), LS: Lagrange selection method, RS: Random selection method (2a) Utility CDF (2b) Utility value for each iteration

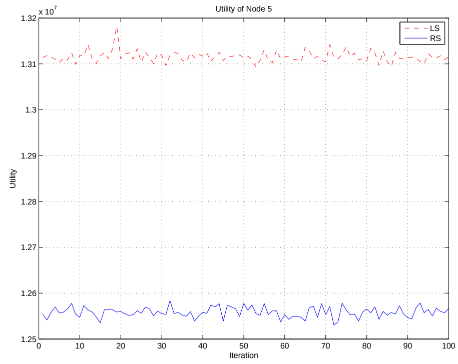
Moreover, in Fig. 3 we randomly choose a node in the secondary network (node number 5) and its utility CDF, and value for the whole simulation time. We can see in both figures that our algorithm works extremely better than the random selection method.

4.1.2 Case B

In this model the secondary network is fully connected but peers are not synchronized, thus there is buffer delay among them and the maximum buffer delay is equal to 10 packets, which is randomly assigned to the nodes. Besides the buffer delay, we assume that the video streaming downlink rates of peers are not the same. These rates difference lead to variation in nodes buffer contents and it does not contain the same number of packets, some peers will have more packets and some less. If a node has bigger buffer contents,



(a)



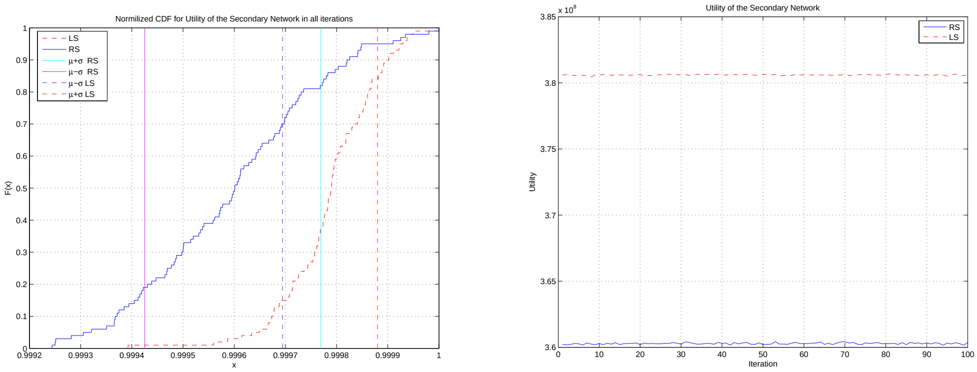
(b)

Figure 3: Utility of node 5 (Case A), LS: Lagrange selection method, RS: Random selection method(3a) Node 5 utility CDF (3b) Utility value of node 5 for the whole simulation time

in each time stamp it can increase its quality of service more comparing to a node, which has fewer packets in its buffer. Since at each time stamp it can look for more lost packets in his buffer and download them if they exist in the network; furthermore, it can share more packet for other peers and gain money from them. Buffer contents size of node (video stream downlink rate) of nodes is randomly distributed among all secondary network nodes.

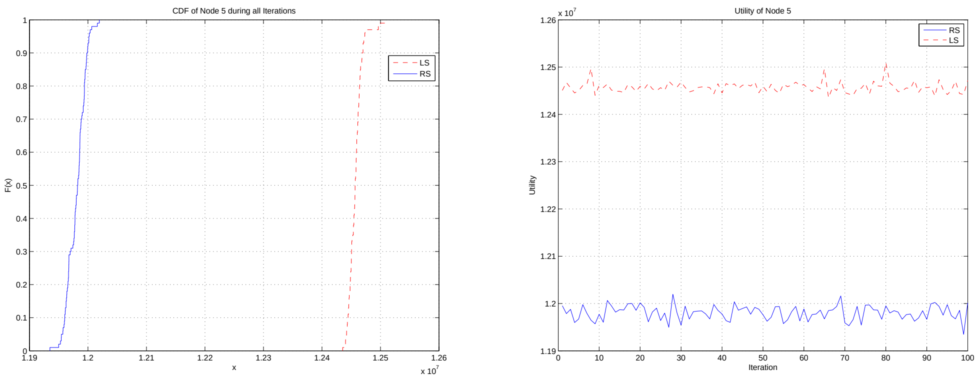
In Fig. 4a we plotted the normalized CDF of the secondary network utility by the highest utility value. In addition, Fig. 4b shows the secondary network utility value of the secondary network for the entire simulation. In addition, to have a better understanding of the secondary network utility distribution the variance and mean it have great importance so, we also plotted $\mu - \sigma$ and $\mu + \sigma$ for both random and suggested algorithm for neighbor selection method . If we compare $\mu \pm \sigma$ in both cases, we can deduce that our suggested algorithm works better than just randomly select a peer.

In Fig. 5 we again chose node 5 and plot its CDF and utility value for both algorithms during all iterations. In Fig. 5b we can also see that node 5 has higher utility value for the suggested algorithm comparing to random selection algorithm. These figures gain show that our suggested algorithm works better.



(a) (b)

Figure 4: Utility of the secondary network (Case B), LS: Lagrange selection method, RS: Random selection method (4a) Utility CDF (4b) Utility value for each iteration



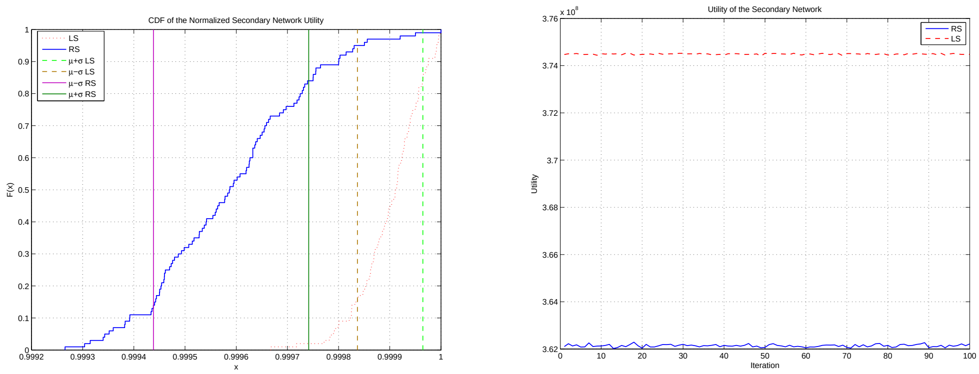
(a) (b)

Figure 5: Utility of node 5 (Case B), LS: Lagrange selection method, RS: Random selection method(5a) Node 5 utility CDF (5b) Node 5 utility value

4.1.3 Case C

In this model not only the original video downlink rates of peers and as the result buffers contents are not the same but also the secondary network is not fully connected. Connectivity of node is randomly chosen and each node has at least 10 peers. The other simulation parameters of this model is the same as other cases

In Fig. 6a we plot the normalized CDF of the secondary network utility and $\mu \pm \sigma$ for both algorithms. Furthermore, Fig. 6b shows utility value of the secondary network for different iterations of both algorithms.



(a) (b)

Figure 6: Utility of the secondary network (Case C), LS: Lagrange selection method, RS: Random selection method (6a) Utility CDF (6b) Utility value for each iteration

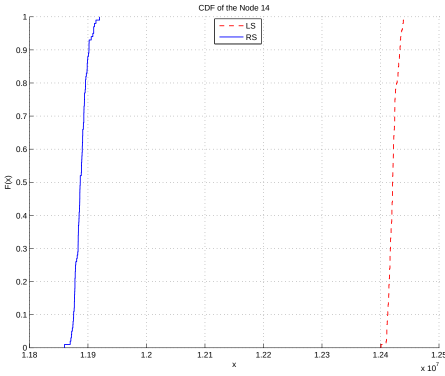
In Fig. 7 we chose a random node in the secondary network (Node 14^{th}) and plotted its CDF and utility for both peer selection methods. As we can see again our algorithm works better comparing to random selection method.

4.1.4 Stackelberg

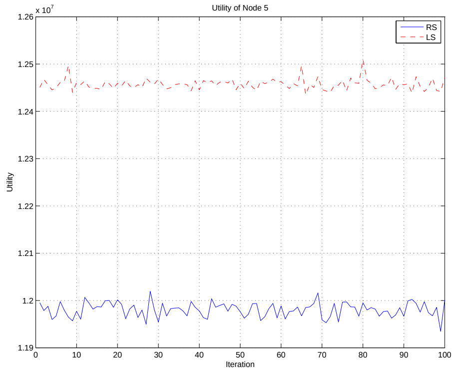
In this simulation part we select case C for the secondary network model, since it is more realistic to P2P video streaming network. Given that we have one leader (primary user), and many followers (secondary network peers) we need to decide on the mood of play for the secondary network. We use co-operative mood of play, thus we should use Pareto optimal output of the secondary network as a rational response to the primary user. In addition, to have a better utility result for the secondary network we choose social optimum[4] of the secondary network as the rational response.

As we can see in Eq. 12 the utility of the primary user depends on the rate, link cost, and the number of the secondary network nodes. In this part

[4]Social optimum is a Pareto optimum which is maximum among other Pareto optima of the game



(a)



(b)

Figure 7: Utility of node 14 (Case B), LS: Lagrange selection method, RS: Random selection method(7a) Node 14 Utility CDF (7b) The utility value of node 14 in each iteration

of simulation we decrease the number of the secondary network nodes to 10, beside that we also decrease the number of iterations for the secondary network utility calculation into 50 and run Stackelberg simulation for 100 times.

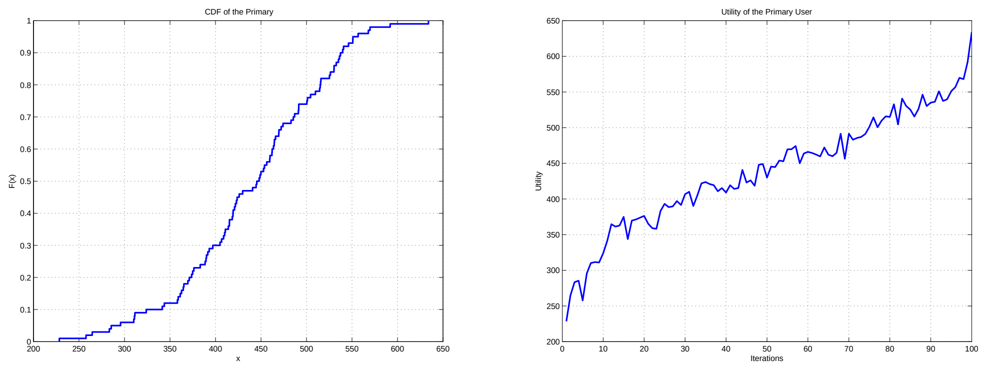
In addition, we will consider the effect of link cost (money that the secondary network's nodes should pay to the primary user in order to use the released bandwidth for uplink rate of nodes in the network) on the primary user utility. In Table 3 we summarized simulation parameters for the Stackelberg game of the primary-secondary players.

In Fig. 8a we plot the CDF of the primary user utility, while the secondary network is in case C and it runs suggested algorithm for peer selection. In Fig. 8b we plot the utility of the primary user for each iteration. It is worth mentioning that the link cost of the iteration number i is higher for the secondary network nodes comparing to the iteration number $i-1$.

In Fig. 9 we plot the utility of the primary user versus the total amount of link cost that the secondary network pays to the primary user in each iteration. We can see in the figure if the link cost is increased, the primary user utility will increase as well and this could be realized from the primary user utility model.

Stackelberg Parameter	Uplink Cost
Number of Secondary Network Nodes	10
Minimum Uplink Rate [kbps]	60
Minimum Number of Connection	3
a, b, C	1, 50, 50
Buffer Read	30
Maximum Buffer Delay	10
Window Size	Randomly Chosen
Number of Iter. for Sec. Net.	50
Number of Iter. for Primary. Net.	100

Table 3: Summarization of simulation parameters for Stackelberg game



(a) (b)

Figure 8: Utility of the primary user, the secondary network model is case C (8a) CDF of the primary user utility function (8b) Primary user utility of the for all iterations

Thus, if the primary user increases the link cost its utility will be increased as well. As the result of the link cost augment by the primary user, the secondary network should pay more money to the primary user per link

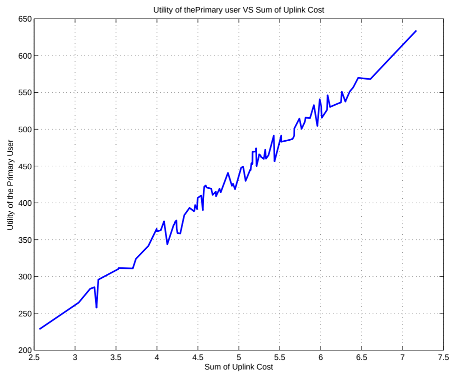


Figure 9: Utility of the primary user vs sum of the uplink cost for the secondary network

usage thus its utility value will be decreased. If we assume that the secondary network has a minimum acceptable utility, then the primary user can increase its link usage cost to reach this limit. At this point if the primary user increases its link cost, then the secondary network nodes will not share any packets with each other as a result the utility of the primary user goes to zero. Since there is no hard limit for the secondary network utility value in our simulation model, thus the primary user can increase its link cost without any limitations.

Totally, during all simulation results we observed that our suggested algorithm for peer selection works better than just random selection method and we could also prove it by using some mathematical methodology in analysis section. Moreover, by using Stackelberg game we could maximize the utility of the secondary and primary network.

5 Related Work

There are many applications of game theory in wireless communication and networking, researchers have tried to solve many problems in network optimization by using of game theory. For example; spectrum sharing for unlicensed band, video streaming in P2P networks, maximizing spectrum utilization of cognitive radio network using channel allocation and power control, and coalition based multimedia peer matching strategies for P2P networks. The most related one to our model is the last one, and we will explicitly explain it.

5.1 Coalition based Multimedia Peer Matching Strategies for P2P Networks

In this paper, they assumed that peers utility improves as they can download their desired packet and also there is uploading cost which decrease utility of peers. Hence, the utility function of peer i, which is downloading the demanded content D_{ik} at rate $R_{D_{ik}}$ and providing upload rate R_{U_i} to other peers is denoted as follow:

$$U_i(R_{D_{ik}}, R_{U_i}) = \begin{cases} 0 & \text{if } R_{D_{ik}} > R_{U_i} \\ p_{ik}Q_i(R_{D_{ik}}) - c_i R_{U_i} & \text{otherwise} \end{cases} \tag{21}$$

where p_{ik} denotes the preference factor of peer i on the multimedia content $D_{ik} \in D_i$ and a non-negative constant $R^{req}_{D_{ik}}$ represents the specifc minimum required rates to decode the video sequence.

In this paper they focused on a one-to many peers's interaction. Let $C_1 = \{1, \cdots, N\}$ be the coalition set of peer 1 with $(N-1)$ peers. Peer 1 download its desired packet form peers in this set and shares its possessed packets $(\mathbf{P}_1)$ also with this group of peers. Thus, the utility of peers in the coalition are expressed as:

$$U_1 = p_{11}Q_1\left(\sum_{i\in C_1\setminus\{1\}} R_{U_{i1}}\right) - c_1\left(\sum_{i\in C_1\setminus\{1\}} R_{U_{1i}}\right) \tag{22}$$

$$U_i = p_{ij}Q_i\left(R_{U_{1i}} + \sum_{l\in C_i\setminus\{1,i\}}\right) - c_i\left(R_{U_{i1}} + \sum_{l\in C_i\setminus\{1,i\}}\right) \tag{23}$$

p_{ij} is the preference factor of peer i on the j^{th} packet, and $R_{U_{1i}}$ denotes the upload rate of peer 1 to peer i. Utility of peer i depends on the uplink rate provided by peer 1 and other peers in coalition with peer i which is C_i.

Every node in the network should achieve a minimum utility, thus we have:

$$R_{U_{i1}} \geqslant \max\left\{R_{D_{11}}^{req}, Q_1^{-1}\left(\frac{c_1\left(\sum_{l\in C_1\backslash\{1\}} R_{U_{1l}}\right)}{p_{11}}\right)\right\} - \sum_{l\in C_1\backslash\{1,i\}} R_{U_{l1}} \tag{24}$$

$$R_{U_{i1}} \leq \frac{p_{ij}Q_i\left(R_{U_{1i}} + \sum_{l\in C_1\backslash\{1,i\}} R_{U_{li}}\right)}{c_i} - \sum_{l\in C_i\backslash\{1,i\}} R_{U_{il}} \triangleq R_{U_{i1}}^{MAX} \tag{25}$$

for all $i \in C_1\backslash\{1\}$. Eq. 24 can be expressed using $R_{U_{i1}}^{MAX}$ in Eq. 25 as:

$$R_{U_{i1}} \geq \max\left\{R_{D_{11}}^{req}, Q_1^{-1}\left(\frac{c_1\left(\sum_{l\in C_1\backslash\{1\}} R_{U_{1l}}\right)}{p_{11}}\right)\right\} - \sum_{l\in C_1\backslash\{1,i\}} R_{U_{l1}^{MAX}} \triangleq R_{U_{i1}^{min}} \tag{26}$$

Therefore, the upload rate $R_{U_{i1}}$ can be expressed as:

$$R_{U_{i1}} = \theta_i \cdot R_{U_{i1}}^{min} + (1-\theta_i) \cdot R_{U_{i1}}^{MAX} \tag{27}$$

with variable $(0 \leq \theta_i \leq 1)$. $R_{D_{ij}}^{req}$ is the minimum downlink rate required for the desired packet. We see that $R_{U_{i1}}^{MAX}$ and $R_{U_{i1}}^{min}$ depend on upload rate of peer 1 to its coalition peers. Thus, the achievable utilities in coalision C_1 can be expressed as a function of peer 1's upload rates to its coalition peers given the other coalition parameters. To resolve the problem based on game theoric appraoch they tried to map the problem to barganing set-up with N player, which is not in the same approach as we did in our problem so, we will not speak about this game theory methodology.

6 Conclusion

In this report, we considered the problem of peer selection for video streaming in P2P networks. We proposed a peer selection algorithm, which enables nodes in the P2P network to choose the best possible sets of peers for video packets sharing. Simulations revealed that our suggested algorithm for peer selection method could maximize the utility of the peers in the P2P network. Moreover, we considered the effect of the suggested algorithm for different cases based on the characteristics of a typical P2P network, in all of the cases our algorithm worked better than just random selection of the peers.

In addition, we toke into account the uplink rate cost that the peering nodes should pay for packet sharing to the link owner, and we proposed that this could be well modeled in the framework of Stackelberg game. Moreover, for this part we had a simulation and used Stackelberg game with the cooperative mood of play based on the Pareto optimal of the P2P network as their rational response and we could see the effect of link cost on the P2P network. Totally, the proposed peer selection strategies can enhance the performance of existing P2P network by efficiently choosing peers in the existing of a link holder network.

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