Optimal Design of Primary User Spectrum Management Using Stackelberg Games

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Abstract

We consider a cognitive radio network in which a base station provides opportunistic unlicensed spectrum access for secondary base stations to transmit data to their subscribers. The primary user may decide to release some parts of its bandwidth for the use of secondary users. As a result, secondary users pay a fee to the primary user based on the interference they make. Considering cognitive radios, we propose and analyze a framework, whereby a primary user has the possibility to release its channel to a secondary network in exchange for money. On one Hand the primary user attempts to maximize its payoff, while on the other hand, secondary users try to minimize the money they pay to the primary user and maximize their own payoff. The investigated model is conveniently cast in the framework of Stackelberg games. Our simulation consists of two major parts. First there is a negotiation among the secondary network nodes about the distribution of secondary channels. In this part, we use social optimum of secondary network as the negotiation result. Second, we consider a Stackelberg game between the primary user and the secondary network in which the primary user wants to maximize its payoff by increasing its cost or the number of channels available for the secondary network.

1 Introduction

Recently the FCC reported that there are huge temporal and spatial variations in the usage of the allocated spectrum. This stimulates the notion of *opportunistic unlicensed spectrum access*, which lets the secondary cognitive radio networks opportunistically make use of the underutilized spectrum. Although by allowing opportunistic spectrum access, the spectrum exploitation will get better, transmission from a cognitive radio network can interfer with the primary user as well. Thus, important design criteria for cognitive radio comprise of maximizing the spectrum exploitation and minimizing the interference caused to primary user.

To alleviate the problem, we consider a cognitive radio network that consists of multiple cells. Within each cell, there is a cell head (CH) supporting a set of fixed cognitive radios (CRs). We consider the downlink scenario and the spectrum of interest is divided into a set of non-overlapping channels divided into two parts by the primary user. The first part is for use of the primary user itself and the second part can be shared with the cognitive radio network. Each CR can be either active or idle and a CH needs exactly one channel to serve each active CR.

For each cognitive radio in the secondary network the received signal to interference plus noise ratio (SINR) must exceed a predefined threshold, which depends on the radio module of cognitive radio to be able to reconstruct data from the received radio message. Here, we assume the primary user release some channels for CHs. The payoff of a CH is the number of CRs that can be supported by using at least one of its allocated channels. The primary user will gain profit based on the interference that CHs make at a specific point in the network field. We use two game theory concepts: one in the secondary network where we devise social optimum and a Stackelberg game between secondary network and the primary user to find the optimum number of channel released by the primary user in forms of revenue.

What we have down is comparably a new way of using Stackelberg game in such networks. Previously [5],in Stackelberg games were used in the case of licensed bandwidth where the secondary network makes use of a distributed space-time coding (DSTC)[3] or the primary user, instead they could gain amount of bandwidth for its inter-network communication. They used Stackelberg game to find the optimum fractions of bandwidth that should be released for the secondary network; moreover, in the secondary network, nodes play a non-cooperative power control game and choose their transmitting power according to Nash equilibrium. The Stackelberg model that they used is the same as ours, but our cooperation model is different from them.

The rest of this report is structured as follows; we first introduce basic concepts of game theory and Stackelberg games and explain how to solve such games. Then, we examine a cognitive radio as described earlier.

2 Game Theory

Game theory is a branch of applied mathematics that is used in the social sciences. Game theory is the study of problems of conflict and cooperation among independent decision-makers.

2.1 Essential Concepts in Game Theory [6]

A game is defined by the triplet $G = (P, S, U)$

- Player
  A player is an agent who makes decisions in a game.

- Strategy
  In a game in strategic form, a strategy is one of the given possible

actions of a player.

- Payoff
  A payoff is a number, also called utility, which reflects the desirability of an outcome to a player, for whatever reason.

- Rationality
  A player is said to be rational if he seeks to play in a manner, which maximizes his own payoff. It is often assumed that the rationality of all players is common knowledge.

We concentrate on dynamic games of complete information, mostly on Stackelberg model, since it is the main part of our cooperation model.

2.2 Dynamic Games of Complete Information

In this subset players' payoff are common knowledge among them. This branch consists of two kinds of games:

1. Perfect Information

   At each turn the player owing the turn knows the full history of the game. In other words, a player knows the strategy of its opponents in the past.

2. Imperfect Information

   Players do not know the full history of the game or simpley, a player may not know exactly previous choices.

It is worth noticing that the main issue in this kind of game is credibility[1], which means that in every move that any player takes he thinks players previous moves were based on maximization of their utilities.

We focus on Stackelberg game, which is a branch of dynamic game of complete and perfect information game, but before go deeply in that area, we cast a look at backward induction, since it has an important role in Stackelberg games.

2.2.1 Backward Induction

Backward induction is a technique to solve a game of perfect information. It first considers the last moves of the game, and determines the best move for the player in each case. Then, taking these as given future actions, it proceeds backwards in time, again determining the best move for the respective player, until the beginning of the game is reached [6]. The key features of a dynamic game of complete and perfect information are:

- The moves are in sequence
- All pervious moves are known before the next move is taken place
- For each combination of players moves, payoffs are common knowledge[1]

Games with these characteristics are solved by backward induction. Backward induction is used to find Stackelberg games' equilibriums, thus we describe how to solve a two-level backward induction.

- Player 1 chooses an action a_1 from the A_1.
- Player 2 chooses an action a_2 from the A_2.
- Payoffs are $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$.

When it is player 2's action time, he will face following problem, given the action a_1 previously chosen by player 1:

$$\max_{a_2 \in A_2} u_2(a_1, a_2)$$

$R_2(a_1)$ is player 2's best reaction tworad player 1 . Since both players can predict each other action, player 1's action at the first stage is

$$\max_{a_1 \in A_1} u_1(a_1, R_2(a_1))$$

If we assume player 1's best action is a_1^*, we call $(a_1^*, R_2(a_1^*))$ the *backwards-induction outcome* of the game.
The backward induction is again based on credibility. That is player 1 knows player 2 moves is in a way that player 2 will recieve maximum payoff based on player one moves.

2.3 Stackelberg Game [1]

The Stackelberg model in economy consists of a leader firm which moves first and a follower firm which moves after. The Stackelberg model is solved by finding the subgame perfect Nash equilibrium[1] of the game. To calculate SPNE we first need to find the best reaction of follower to any quantity of its leader, thus we use backward induction to solve this kind of game. In a Stackelberg game the leader announces its strategy and follower responds

[1]A subgame perfect Nash equilibrium (SPNE) is a set of strategies $\{s_i, i = 1, \ldots, n\}$ such that for each subgame g, the set of induced strategies $\{s_i(g), i = 1, \ldots, n\}$ forms a Nash equilibrium for this subgame

to it *rationally*, as far as the leader knows the follower cost function, it can compute follower's reaction to all of its strategies. The timing of a Stackelberg game is as follows: [1]

1. Leader chooses a quantity $q_1 \geqslant 0$
2. Follower observes q_1 and then chooses a quantity $q_2 \geqslant 0$
3. Payoff for player i is:

$$u_i(q_i, q_j) = [P(q_1 + q_2) - C_i(q_i)]q_i$$

Price for firms is $P(q_1 + q_2)$ which is simply the function of total output. Moreover, we suppose that firm i has cost function as $C_i(q_i)$.
We use backward induction to solve a Stackelberg game, thus first we need to calculate the follower best response to an arbitrary quantity of leader.

$$\max_{q_2 \geqslant 0} u_2(q_1, q_2) = [P(q_1 + q_2) - C_2(q_2)]q_2$$

The values of q_2 satisfying this response are follower's best response. For the best responses of the leader we need to find the follower best responses as a function of the leader possible actions, $R_2(q_1)$, and then maximize the leader payoff;

$$\max_{q_1 \geqslant 0} u_1(q_1, R_2(q_1)) = [P(q_1 + R_2(q_1)) - C_1(q_1)]q_1$$

These two maximizations can easily be found by just a derivation of each payoff with respect to its given quantity and put the result equal to zero and find the respective value that satisfies the resulting expression. To have a better understanding of the problem we bring an example.

Supposing that the cost functions of both leader and follower are zero that is $C_1(q_1)$ and $C_2(q_2)$ are equal to zero; moreover, the inverse demand function is $P(q_1 + q_2) = A - B(q_1 + q_2)$ (A and B are constants). q_1^* and q_2^* are the leader and follower best answers.

$$u_2(q_1, q_2) = (A - Bq_1)q_2 - Bq_2^2$$

and player 2 best reaction is;

$$\frac{\partial u_2}{\partial q_2} = 0 \longrightarrow R_2(q_1) = \frac{A - Bq_1}{2B}$$

firm 1's best answer will be;

$$u_1 = \frac{A}{2}q_1 - \frac{B}{2}q_1^2$$

so;

$$\frac{\partial u_1}{\partial q_1} = 0 \longrightarrow q_1^* = 2q_2^* = \frac{A}{2B}$$

As a result in a two-player Stackelberg game we have these characteristics;

- Cost function of each player depends on the both players strategies.
- Each player tries to minimize its cost function.

2.3.1 One leader many followers [4]

Most Stackelberg games consist of a leader and a follower that replies to leader strategy "rationally" by selecting a strategy that minimizes its cost function. But if we have more than one follower, we cannot distinctively reveal what is meant by "rational" response of the followers. As a result, the leader should know not only the followers cost function but also their "mood of play" which can be of two kinds non-cooperative or cooperative. In non-cooperative mode among followers we can consider their Nash equilibrium as their strategy in response to a leaders strategy. In case of cooperation we can use Pareto optimal as a case of modeling.

3 Problem Definition

3.1 System Model

We consider an opportunistic spectrum access scenario depicted in Fig. 1. The primary user divides the spectrum of interest to K channels, and primary user releases some of the channels for secondary network. In the same area, a cognitive radio network is set up. This cognitive network consists of B cells. Within each cell, there is a cell head (CH) serving a number of fixed cognitive radios. We consider the downlink scenario in which data are transmitted from CHs to CRs. Our objective is to find a channel/power allocation scheme that maximize both CHs and BS payoff.

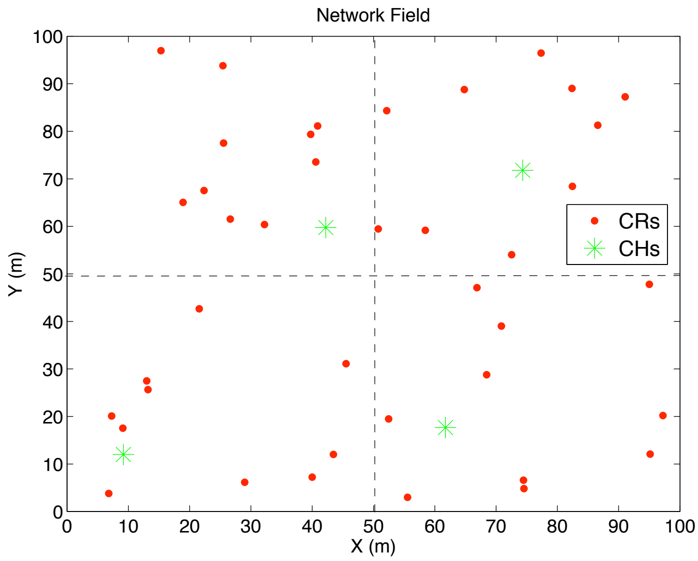

Figure 1: Deployment of a cognitive radio network. Dashed lines are cells' boundary.

3.2 Operational Requirements

3.2.1 SNR

Let G_i^c be the channel power gain from the CH serving CR_i on channel c. Let P_i^c be the transmit power toward CR_i on channel c. If channel c is not assigned for the transmission toward CR_i, $P_i^c = 0$ but if it is assigned we have [2]

$$\gamma_i^c = \frac{G_i^c P_i^c}{N_0}$$

Where N_0 is noise spectral density of each CR. For a reliable transmission toward CR_i it is essential;

$$\gamma_i^c \geq \gamma_{min}$$

In practice, γ_{min} is the minimum SNR required to reach a certain bit error rate (BER) for each CR.

3.2.2 Payoff

We have two payoff functions one for primary user and another for secondary users. CH in the Secondary network choose those payoff functions that stasify

$$U_i^{sec} \geq \alpha$$

Where U_i^{sec} is payoff function for secondary network, α is a minimum payoff for a secondary CH to accept this payoff as an accountable channel allocation, and x is CH_i membership fee for each of its user. Payoff functions for the whole network are:

$$U_i^{sec} = \frac{CR_i}{N} \times X_i - \frac{\sum_{j=1}^{CR_i} P_j \times G_{BSi}}{\sum_{j=1}^{CH_i^{CRs}} P_j \times G_{BSi}} \times Y$$

$$U^{pri} = \sum_{i=1}^{B} \frac{\sum_{j=1}^{CR_i} P_j \times G_{BSi}}{\sum_{j=1}^{CH_i^{CRs}} P_j \times G_{BSi}} \times Y$$

where CH_i^{CRs} is total number of CRs in cell number i, N is total number of CRs in the whole network, B is total number of cells in the network, CR_i is the umber of CRs that CH_i is supported, P_j denotea the transmit power for the transmission toward CR_j form its respective CH, U_i^{sec} is payoff function of CH_i, U^{pri} is payoff function of primary user, X_i is membership cost of CH_i, Y is primary user cost, which is based on interference that a secondary network makes at a certain point, and G_{BSi} is channel gain from CH_i to the primary user. Parameters are described in Table 1.

4 Stackelberg model based channel allocation

In this section, we provide some insight into the performance and analysis of the channel allocation. In particular we are interested in condition in which it is advantageous for the primary user to release its channels for the secondary network.

Symbol	Definition
CH_i^{CRs}	Total number of CRs in cell number i
N	Total number of CRs in the whole network
B	Total number of cells in the network
CR_i	Number of CRs that CH_i is supported
P_j	Denote the transmit power for the transmission toward CR_j form its respective CH
U_i^{sec}	Payoff function of CH_i
U^{pri}	Payoff function of primary user
X_i	Membership cost of CH_i
Y	Cost for attaining a channel from primary
G_{BSi}	Channel gain from CH_i to primary user

Table 1: List of symbols and definition in our system model.

4.1 Performance

Channel allocation is based on Stackelberg and Social optimum equilibriums.[2]. Timing of channel allocation is as follow. Firstly, BS releases some channels for the use of secondary network and secondary networks CHs negotiate with each other and based on their negotiation they choose CRs that make the whole secondary network pay minimum possible cost to the primary user. They use social optimum equilibrium of the game to maximize their payoffs and as the result minimize the cost function of the whole network. On the other hand, primary user tries to maximize its payoff so, it changes the cost that it receives from secondary network for the power that each CH puts at a certain point in the network. Totally, primary user maximizes its payoff by finding the Stackelberg equilibrium of the game.

4.2 Analysis

In this part, we determine conditions in which the primary user can maximize its payoff while secondary user can also take part in a game and maximize their payoff. The problem is solved by noticing that

$$\left.\begin{array}{l} U^{sec} = \sum_{i=1}^{B} U_i^{sec} \\ U_i^{sec} \geq \alpha \end{array}\right\} \Longrightarrow U^{sec} \geq B \times \alpha$$

[2]Allocation in which is Nash equilibrium and Pareto optimum happen simultaneously and the secondary network payoff is maximum.

$$K \geq CR_i$$

Thus

$$U^{sec}(CR_i, Y, P_j) = \frac{\sum_{i=1}^{B} CR_i \times X_i}{N} - U^{pri}(CR_i, Y, P_j)$$

Lemma At the social optimum point, the secondary network uses all the channels available and support CRs that are close to their respective CHs.

Proof From the payoff of the secondary network, we can understand that if the number of supported CRs is increased, the payoff of the secondary network will be increased. So, the secondary network uses all released channels, because the total number of supported CRs is equal or less than released channels. Moreover, the secondary networks CHs negotiate with each other to select CRs that are closer to CHs, since they want to decrease the interference of supporting CRs. As a result, all channels will be used and the closest CRs will be chosen.

$$U^{sec} \geq B \times \alpha \rightarrow \frac{\sum_{i=1}^{B} CR_i^{opt} \times X_i}{N} - U^{pri}(CR_i^{opt}, Y, P_j^{opt}) \geq B \times \alpha$$

Thus

$$U^{pri}(CR_i^{opt}, Y, P_j^{opt}) \leq \frac{\sum_{i=1}^{B} CR_i^{opt} \times X_i}{N} - B \times \alpha$$

$U^{pri}(CR_i^{opt}, Y, P_j^{opt})$ is maximized if it is equal to the right hand side so,

$$Y^{opt} = \frac{\frac{\sum_{i=1}^{B} CR_i^{opt} \times X_i}{N} - B \times \alpha}{\sum_{i=1}^{B} \frac{\sum_{j=1}^{CR_i^{opt}} P_j^{opt} \times G_{BSi}}{\sum_{j=1}^{CH_i^{CRs}} P_j \times G_{BSi}}}$$

It is worth to mention that in the secondary network there is a trade of among P_j, X_i for each CH, since CHs negotiate with each other to choose CR that make all these parameter less.

5 Numerical Example

5.1 Simulation Model

The system model used in our simulation is as follows. We consider a square field of size $100 \times 100m$ in which cognitive radios are randomly distributed. The area is divided to $B = 4$ cells and within each cell there is a randomly distributed CH. The total number of CRs is $N = 40$. A sample network is shown in Fig. 1.

We model a frequency division multiple access (FDMA) system in which the entire bandwidth is divided into $K = 5, \ldots, 10$ channels. The path loss exponent is taken to be 4. The primary user measurement point for the power of the secondary network is $(50, 50)$. Each CH randomly chooses one channel and uses it.

The noise power spectrum density at each CR is $N_0 = -100dBm$. The required SNR at each CR is $50dB$. The minimum acceptable threshold for each CH is $\alpha = 0.00001$. X_i is a randomly selected number less than 1 and Y gets number less than 4.

5.2 Simulation Analysis

For each released channel numbers, the secondary network computes all the possible allocation, for example, if number of released channel is equal to 5, then possible allocation is equal to 126. For all these allocations the secondary networks CH compute their payoff for a fixed primary user rate. They find the Pareto optimum, and Nash equilibrium (NE) of these possible allocations. The equilibrium, which is not only NE, and Pareto optimum but also the secondary network payoff is maximum is social optimum of those allocations. The secondary network strategy, the chosen channel allocation, goes to the primary user. The primary user increases its cost in order to maximize its payoff for that specific released channels and declares its new cost to the secondary network. This algorithm continues until the secondary network does not choose any channel allocation, since the newly declared cost by the primary user passes the payoff threshold of the secondary network. Then the primary user changes the number of released channels and starts the algorithm for the recently released channels.

In Fig. 2 we plotted U^{pri} versus Y for a fixed number of released channels which is equal to 5. We deduct that primary user payoff will be increased linearly as primary cost is increased until the point that cost is too much that that the secondary network will not choose any allocation, which leads to the zero payoff of the primary user.

In Fig. 3 we plotted U^{sec} versus Y is like the last plot for a channel allocation equal to 5. As we expected the secondary network payoff decreases, when the primary user cost is increased and the secondary network payoff goes to zero, when the primary user cost is too high that causes the secondary network payoff to go below the threshold.

In Fig. 4 we plotted optimum U^{pri} versus the number of released channels. We see that as the channel number is increased, the primary optimum payoff augment as well. In addition, from channel number equal to 9 or higher primary user payoff is constant. We proved that the secondary network uses all the released channels, but when the number of released channels is growing, the secondary network's CHs support CRs that are farther from their respective CH. This support causes more interference on the primary user measurement point. So, the secondary network payoff goes below the threshold sooner comparing to when number of released channel is less. Totally, the primary user cannot increase its cost as much that it could for less number of released channels, which leads to decrease in the payoff of the primary user comparing to pervious number of released channels.

Fig. 5 shows the optimum primary user cost versus the number released channel. The optimum value for the primary user is constant, until the point that more released channels lead to support CRs that are too far from their respective CH. As the result the optimum primary user cost will decrease.

Although it seems that if the primary user releases more channels it can have higher payoff, the simulation results show that this is not always possible, releasing more channels in order to maximize the payoff, and depends on the position of CRs in the secondary network. So, having more released channels can decrease the primary user payoff as well. Totally, the primary user should release more number of channels until the point that the secondary network payoff does not go below its threshold. What we understand from the simulation results are in correspondence with the analysis that we modeled.

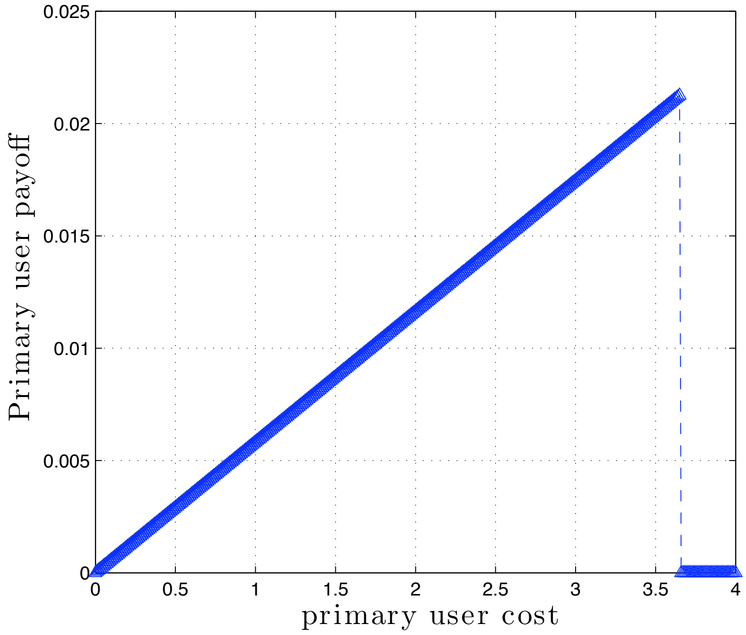

Figure 2: Primary user payoff versus the primary user cost, number of released channels equal to 5. The primary user measurement point is at $(50, 50)$.

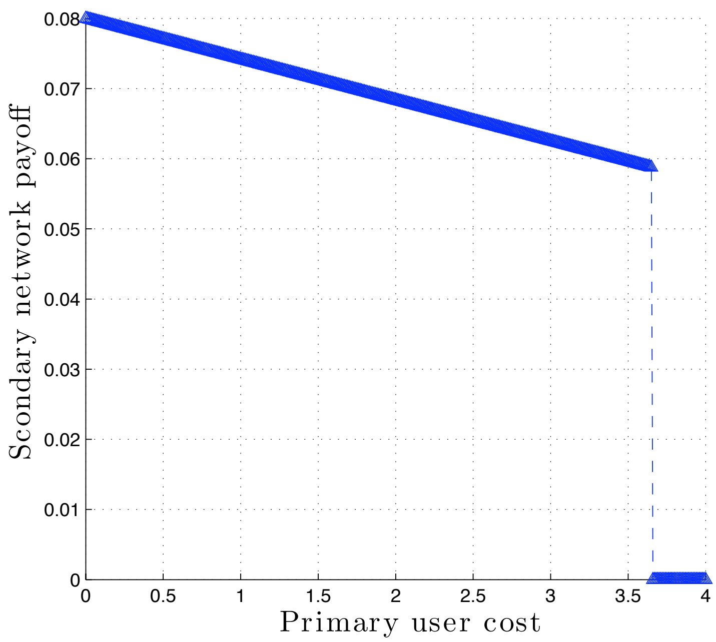

Figure 3: Secondary network cost versus the primary user cost. The Number of released channels is fixed and equal to 5. Measurement point is at $(50, 50)$.

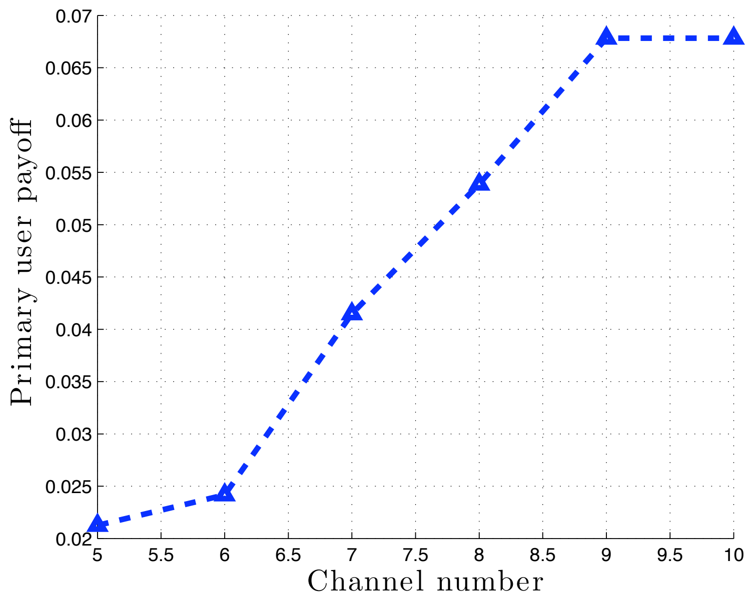

Figure 4: Optimum primary user payoff for each channel allocation versus the number of released channels. Measurement point is at $(50, 50)$.

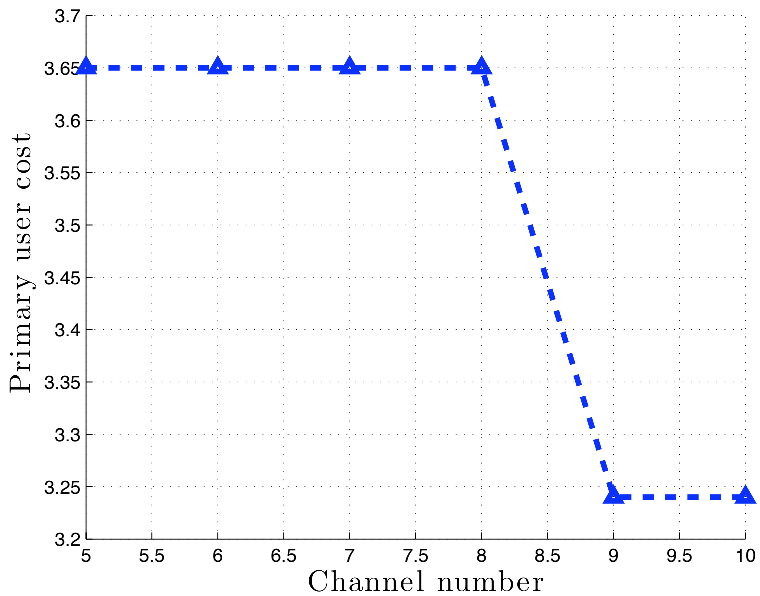

Figure 5: Maximum primary user cost for each channel allocation versus the number of released channels. Measurement point is at $(50, 50)$.

6 Related work

There are many applications of game theory in wireless communication, and researchers have tried to solve many problems in wireless communication by using of game theory. For example; spectrum sharing for unlicensed Band; maximizing spectrum utilization of cognitive radio network using channel allocation and power control, and spectrum leasing to cooperating secondary ad hoc networks are some of game theory application in wireless communication. The most related one to ours is the last one, and we will explicitly explain it .

6.1 Spectrum leasing to cooperating secondary ad hoc networks [5]

The primary user will release a fraction of its bandwidth, in exchange for enhancement in its equality of service by the secondary network. In turn, the secondary nodes can decide to cooperate or not with the primary user on the basis of the amount of cooperation required by the primary and the corresponding fraction bandwidth released for the secondary nodes.

The primary link may lease a fraction of its bandwidth for the secondary nodes in exchange the secondary network will have cooperation with the primary user in the form of transmission via distributed space-time coding.

The fraction of bandwidth released for secondary nodes is divided into two parts. The first part is used to relay primary node data to destination; the second part can be used for the secondary activity. In this subset secondary transmitters participate in a game for transmission to their respective receivers by performing decentralized power control. Each secondary node tries to maximize its utility function that account for the cost and benefit between the power needed for transmission and the quality of service. A small fraction of bandwidth may not worth for the secondary nodes to use it to transmit their data to their respective receivers, since they cannot overcome the expense that they should pay for these transmissions. The outcome of this decentralized game can be described by Nash equilibrium of the game.

The primary decides over the amount of its cooperation with the secondary network by observing the output of the secondary nodes. Stackelberg game can use to illustrate this kind of game.

Our goal has some similarities with this paper and it is based on a Stackelberg game as well, but there is some dissimilarity, we used FDMA for channel allocation, instead of CDMA. As the result there is no interference for channel allocation, additionally, the cooperation between two networks is by quality of service, alternatively the secondary nodes should pay money to the primary user, since they use the primary users channels.

7 Conclusion

In this report, we consider the problem of channel allocation to maximize the spectrum utilization of a primary user in exchange for negotiation with a cognitive radio network. As the result of this negotiation, the secondary network can use the unused spectrum of the primary user to have some services for its users and simultaneously pays to the primary user based on power that these services cause at a defined point. By casting the problem in the framework of Stackelberg games, we have provided analytical and numerical results that have confirmed the considered model as a promising paradigm for cognitive radio networks and could find the Stackelberg equilibrium of the game, in which the primary user can have its maximum payoff.

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