# Historical development of side-channel spillway in hydraulic engineering

W.H. Hager[1] and M. Pfister[2]
[1]Laboratory of Hydraulics, Hydrology and Glaciology (VAW)
ETH Zürich
CH-8092 Zurich, Switzerland
E-Mail: hager@vaw.baug.ethz.ch
[2]Laboratory of Hydraulic Constructions (LCH)
Ecole Polytechnique Fédérale de Lausanne (EPFL)
CH-1015 Lausanne, Switzerland

***Abstract:*** *Side-channels are a typical element of earth or rockfill dams to discharge floods sideways instead frontally, such as normally adopted for concrete dams. The side channel received attention from the hydraulics community from the 1920s, once the first large mentioned dams were erected, and its use culminated at Hoover Dam in the mid 1930s, based on a large test program. This paper highlights the major advancement of this basic element of hydraulic structures up to the late 1950s, both describing key installations over the world, and the actors who have furnished the governing design equations for water flow with spatially-increasing discharge. Short summaries on the biographical background are also provided, along with a discussion of the main literature that allowed for the current knowledge in this fascinating field of open channel flow.*

***Keywords:*** *Biographical background, History, Lateral inflow, Side channel, Spillway*

## 1. INTRODUCTION

Roof drainage is possibly the most evident application of side-channels in daily life. More generally, whenever water has to be collected from a surface on which rainfall is discharged, then side-channels may often be found. Examples include reservoirs that are controlled by earth or rockfill dams, final sedimentation basins in wastewater schemes or swimming pools from which a small portion of water is constantly renewed using a side-channel as supply facility. The advantages of side channels include, among others, small flow velocities upstream of the inlet to minimize erosion and a minor increase of the reservoir level for large discharges due to the long structures. Disadvantages are the abrupt increase of reservoir level if the channel is submerged and strong vortices propagating into the adjacent chute. Figure 1 shows the side-channel of the Kárahnjúkar hydraulic scheme in Iceland as an example where a side-channel was recently erected (Pfister *et al.* 2008).

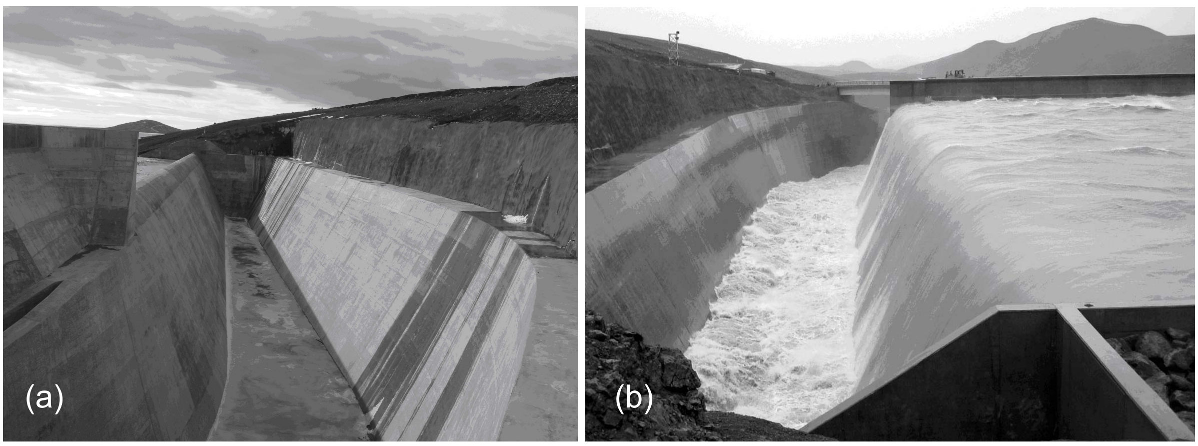



**Figure 1. Kárahnjúkar side-channel, view (a) against (VAW), and (b) in streamwise direction (Courtesy Prof. Dr. S.M. Garðarsson)**

## 2. DEVELOPMENTS UNTIL HOOVER DAM

Despite open channel flows were hydraulically analysed from the 18th century, including unsteady and abruptly-varied flows, side-channel flow was first investigated only by Hinds (1926). The design of Arrowrock Spillway (Fig. 2a) in Idaho was model-tested at Bellevue Laboratory and at the Hydraulic Laboratory of the Bureau of Reclamation USBR, Denver CO.

Hinds realized that side-channel flow may be considered as a generalized form of gradually-varied flow, with an increasing discharge due to lateral discharge addition. The momentum equation was applied to account for hydraulic losses due to lateral discharge addition. For these flows, the discharge increases linearly as $Q=qx$, in which $q$ is the lateral discharge per unit length along the streamwise channel coordinate $x$, with $x=0$ at the side-channel start. Hinds (1926) further assumed that the streamwise average velocity is related to $x$ as $V=ax^n$, where $a$ and $n$ are constants. The momentum equation furnishes a relation among flow depth $h$, velocity $V$ and discharge $Q$ as

$$\frac{\mathrm{d}h}{\mathrm{d}x}=\frac{V}{g}\frac{\mathrm{d}V}{\mathrm{d}x}+\frac{qV^2}{gQ} \qquad (1)$$

where $g$ is gravitational acceleration. This system of equations can be solved by a step-by-step method starting from a boundary condition. The latter must be selected downstream from the side-channel for typically subcritical flow. For the usual case in which the flow in the tailwater channel is supercritical, Hinds correctly stated that the hydraulic control is at the critical flow section, which is (generally) at the downstream end of the side-channel. Experiments generally support the theoretical approach except that predictions were slightly below observations due to air entrainment into the flow increasing the mixture flow surface close to the downstream side-channel end. A peculiar 'rope' was also observed and attributed to the helicoidal flow structure in the side-channel due to lateral flow, later referred to as tornado vortex (Hager 1990) (Fig. 2b). Hinds' paper involved a number of well-known hydraulicians of the era, including John L. Savage (1879-1967), Ivan E. Houk (1888-1972), Ralph L. Parshall (1881-1959) and Joseph W. Ellms (1867-1950) conducting parts of the experiments, and John C. Stevens (1876-1970), among others as a discusser.

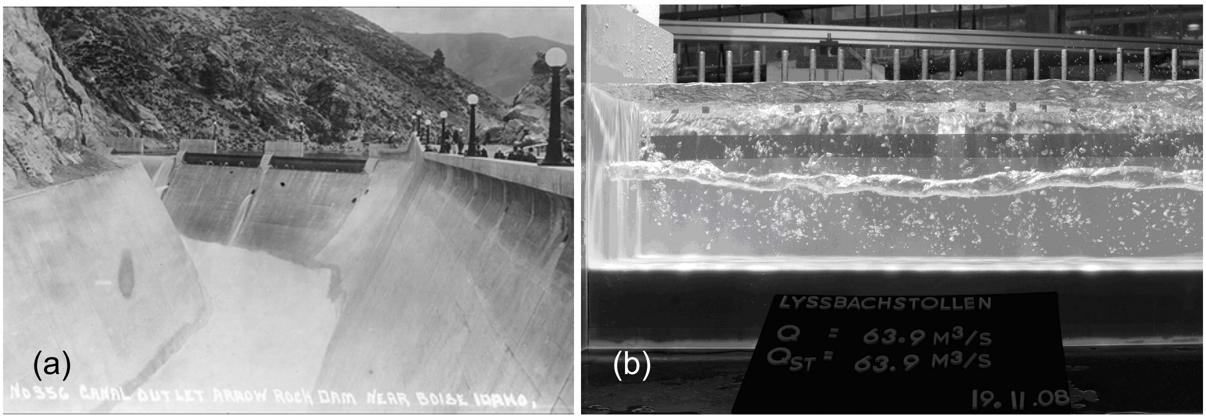



**Figure 2. (a) Postcard dated Sep. 9, 1918, of Arrowrock Dam Spillway. (flickr.com/PokyTom), (b) tornado vortex in Lyssbach side channel model (VAW)**

Julian Hinds was born on 22.12. 1881 in Warrenton AL/USA and passed away on 15.7. 1977 in Santa Barbara CA/USA. He obtained in 1908 the civil engineering degree from University of Texas, Austin TX. He was then there instructor of civil engineering until 1911, when joining the design office of the US Bureau of Reclamation USBR, Denver CO, for the Elephant-Butte and the Yakima-Sunnyside projects. From 1926 to 1929 Hinds was resident engineer for the Calles project, Aguascalientes MX, joining then as hydraulic engineer the Department of Water & Power, Los Angeles CA. He was transferred to the Metropolitan Water District in 1931, becoming assistant chief engineer in 1933 and general manager in 1941 until retirement in 1951. From then to 1956 he was chief engineer of the United Water Conservation District then becoming a private consultant.

Hinds was the first formulating correctly the equation of spatially-varied flow, for which he was awarded the ASCE Norman Medal. He was also a co-author of various important books in dam engineering, given his practical experiences in hydraulic structures. In parallel he developed a number of hydraulic structures, including stilling basins, Venturi flumes or gate elements. Hinds was awarded ASCE Honorary Membership in 1959 for his achievements in irrigation, reclamation and water supply, particularly the development of Southern California's Colorado River supply. The Julian Hinds Award, an annual ASCE distinction, is presented from 1974 for outstanding research in the field of water resources development.

Henry Favre joined in 1930 Versuchsanstalt für Wasserbau, ETH Zurich, today's VAW. One of his first projects (Favre 1933) dealt with spatially-varied flows in which the general equations of steady open channel flows were derived using the energy and momentum equations. He then proceeded to investigate flows with spatially-increasing discharge, discussing the solution more rigorously than Hinds (1926). He obtained an exact solution for frictionless fluid in a horizontal rectangular channel and compared the general solution with laboratory observations. As an application he further analysed the flow conditions at Tieton Dam on Yakima River, Washington State, obtaining substantial agreement with observations. Favre stated that spatially-varied flow generates an additional head loss proportional to the local velocity, the lateral approach flow direction and the ratio between the lateral and the local channel discharges.
Meyer-Peter and Favre (1934) applied the previous results to Boulder Dam side-channel (Fig. 3a), today's Hoover Dam on Colorado River. A hydraulic model was set up at ETH Zurich to test the proposed side-channel (Fig. 3b). It was again observed that the agreement with computations was excellent, yet standing surface waves were observed in addition in the vicinity of the hydraulic control.

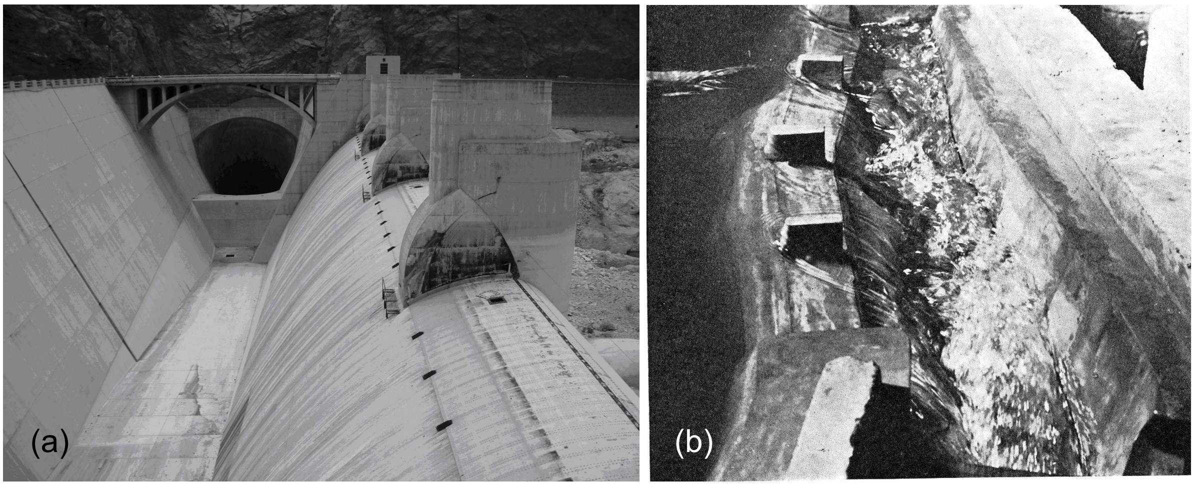
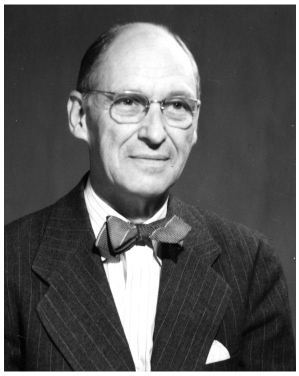



**Figure 3. Hoover Dam side-channel, (a) prototype, (b) model of Meyer-Peter and Favre (1934)**

Henry Favre was born on 10.06. 1901 in Genève, Switzerland and passed away on 30.05. 1966 in Zürich. He obtained the ETH civil engineering diploma in 1924, joined an engineering office and submitted a PhD thesis on photo-elasticity in 1929. He joined Eugen Meyer-Peter (1883-1969) at the newly opened Versuchsanstalt für Wasserbau at ETH in 1930, becoming his deputy director. Right from the beginning, mainly under the scientific leadership of Favre, this laboratory presented excellent works on surge waves in tunnels, spatially-varied flow phenomena, sediment transport of uniform sediment, water hammer or unsteady open channel flows, with a theoretical approach confirmed with experiments. Favre may be regarded a great engineer in mechanics, and one of the most gifted Swiss hydraulicians working only during a limited period in this branch.
Favre was appointed professor of mechanics at ETH in 1938, working in theoretical mechanics related to civil engineering, including resistance problems or elasticity. He also published on engineering history, including Euler or the Swiss general and engineer Henri Dufour. From 1951 to 1953, he was ETH rector, organizing ETH's centennial in 1955. He received honorary doctorates from Universities of Poitiers, Lausanne and Liège.

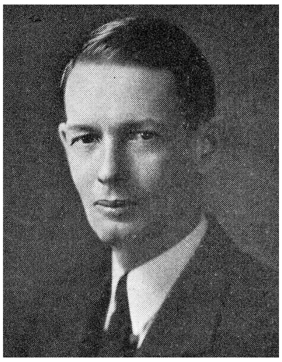
Beij (1934) considered flow in roof gutters, a particular application of side-channels. General surface profiles for semi-circular, triangular and rectangular cross-sections were determined for nearly horizontal gutters, and the governing equation of the free surface profile was re-derived using the momentum equation, following essentially Hinds (1926). The transition from the side-channel to the vertical shaft was further investigated, thereby considering two outflow types. These include free outflow from the channel similar to an end overfall, and flow through bottom outlet and leader connection. It was further observed that the maximum flow depth occurs at the upstream channel end, corresponding for frictionless fluid in a horizontal channel to $3^{1/2}h_c$, and $(5/2)^{1/3}h_c$ for the rectangular and triangular cross-sections, respectively, with $h_c$ as critical flow depth. The discharge capacity of roof gutters was also studied, as a design guide line for roof designers.



Karl Hilding Beij was born on 10.11. 1893 at Hartford CT/USA and passed away on 26.02. 1986 at Franklin NH/USA. He was junior engineer in Hartford from 1914 to 1916, then surveyor of the Rock Creek & Potomac Parkway Community WA until 1920. Beij then joined as associate engineer the Aero Instrument Section of the National Bureau of Standards NBS, Washington DC, and was there from 1930 hydraulic engineer in its Hydraulic Laboratory. He was Member of the Philosophical Society of Washington, the American Meteorological Society, the International Association of Hydraulic Research IAHR, and the American Geophysical Union AGU, serving there as Secretary of the Section of Hydrology from 1933 to 1947, and of the General Section from 1947 to 1953.

After Beij had joined the NBS staff, he designed aircraft instruments. In 1942, he became assistant of the chief, NBS Hydraulic Laboratory. His numerous papers include topics as aircraft instruments, copper roofings, and hydraulics. He edited 5 volumes of Bibliography in hydrology for the United States, AGU. In hydraulics, he was known for his works on flows in pipe bends, spillways, stilling basins, weirs and dry docks. In 1941 he model-tested thermal effects of hot water into a navy yard. He also analysed filling systems for dry docks at Pearl Harbour and at Hunter's Point CA. He collaborated with in the 1930s with Garbis H. Keulegan (1890-1989).

Under the leadership of John C. Page model studies for the Boulder Canyon Project were conducted at the US Bureau of Reclamation USBR. Bulletin 1 relates exclusively to hydraulic investigations, including an enormous number of tests. After a historical introduction to the dam site, the particular Boulder Canyon side-channel is described involving a trapezoidal cross-section along lateral discharge addition, a transition zone which then passes into the circular tunnel shape. Of particular note is a step-like tunnel entry to 'stabilize the flow' (Fig. 3a). The model tests and their analysis were conducted by individuals of the calibre of Emory W. Lane (1891-1963), Jacob E. Warnock (1903-1949), Joseph N. Bradley (1903-1993), Whitney M. Borland (1905-2001), James W. Ball (1905-2001) and Charles W. Thomas (1906-1979). Tests relate mainly to the free surface profiles in a trapezoidal side-channel, which were again compared with computations using the momentum approach, resulting in satisfactory agreement. Guide vanes on the overflow section were also tested, yet not considered in the final design. A particular attention was devoted to the strong helocoidal flow pattern mainly under small discharges due to the large elevation difference between the spillway crest and the side-channel bottom. Coping baffles along the outer side-channel bottom were abandoned. Reservations by hydraulic engineers against hydraulic modeling were from this study practically removed because they were from then considered as a precious means to investigate complex flow features, and an important tool for improving flow conditions.



John Chatfield Page was born on 12.10. 1887 at Syracuse NE/USA, and passed away on 23.03. 1955 in Denver CO/USA. He obtained the BS degree from University of Nebraska, Lincoln NE, and the MS degree from Cornell University, Ithaca NY in 1911. After work as a topographer with the US Bureau of Reclamation USBR he became in 1909 city engineer of Grand Junction CO. From 1911 to 1925 Page was junior engineer with USBR, Denver CO, then until 1930 superintendent of the Grand Valley Project in Colorado, involved in the design of Boulder Dam until 1935, from when he was acting commissioner until 1943, then retiring as a consulting engineer in Denver CO.

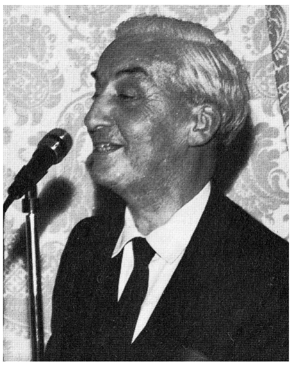
Page had been connected to irrigation work in Western US almost all through his professional career. In 1935 he was transferred to Washington DC to head up the USBR Engineering Division following Elwood Mead (1858-1936). Under Page's direction the Bureau carried forward the Colorado River Project as well as such large work as the Central Valley Project in California, and the Columbia Basin irrigation and power projects in Washington State. The main dams in which Page was involved were Hoover Dam, Grand Coulee Dam and Marshall Ford Dam. Due to ill health, Page resigned in 1943 as commissioner, but continued as a consultant until 1947. He was elected Honorary Member ASCE in 1953. A memorial to Page was unveiled at his namesake city Page AZ in 1964, two miles from Glen Canyon Dam, consisting of a low concrete platform, a flagpole and a bronze mounted on the block of polished granite.

## 3. THROUGH WAR TO THE LATE 1950s

A popular work on side-channel flow was published by Camp (1940). Using again the momentum equation and simplifications relative to the friction slope, a closed-form solution for constant width channels was provided. Both rectangular and non-rectangular channel shapes received interest. The friction factor was experimentally determined and found essentially similar to that of constant-discharge flows. His findings were particularly addressed to wastewater treatment stations, such as rapid sand filters. The discussion of this paper is more than double of the paper length, having attracted individuals such as John K. Vennard (1909-1969), Fred Knapp (1901-1971), Carl Rohwer (1892-1959), or Harold A. Thomas, Jr. (1913-2002).

Thomas Ringgold Camp was born on 05.11. 1895 in San Antonio TX/USA and passed away on 15.11. 1971 in Boston MA/USA. He was known affectionately throughout his life as 'Tom Camp'. He graduated in 1916 in architectural engineering from Texas A&M University. After World War I he designed pumping stations, filtration works and sewer systems before enrolling for graduate studies in sanitary engineering at MIT. After receiving a Master's degree, he specialized from 1925 in water works and sewerage. Camp became in 1929 instructor at MIT's Sanitary Engineering Department, where he coupled teaching with research for the next 15 years.
Camp was one of the first realizing that sanitary engineering was interdisciplinary, currently referred to as environmental engineering. He mastered all the associate disciplines. Both theoretical and pragmatic new concepts were generated in water and wastewater treatment, and soon he became a national authority. By 1944, he decided to enter full-time consulting with an associate. In his office, hundreds of collaborators dealt with questions in water resources, water and wastewater works, solid waste disposal, water and air pollution abatement, and flood control. He was president of the New England Water Pollution Control Federation, the New England Water Works Association, and the Boston Society of Civil Engineers. Camp's name is particularly associated with the design of sedimentations basins, for which he developed a design based on the particle fall velocity. He was awarded ASCE Honorary Membership in 1965.

From the early 1940s, Europeans added significantly to the topic. Known for his studies on side-weir flow, Giulio De Marchi as one of the outstanding hydraulic engineers of post-war Europe dealt with side-channel flow. Applying the momentum equation to spatially-varied flow, the basic flow profiles were discussed including entirely sub- and super-critical flows, and mixed flows with a hydraulic jump along the lateral approach flow zone. The surface profile for a rectangular prismatic side-channel was also presented and compared with hydraulic test data.
De Marchi's student and later successor at the Politecnico of Milan, Duilio Citrini, investigated side-channel flows for increasing channel width along the lateral discharge addition (Citrini 1942). An extra term was added to the equation of the free surface profile depending on the width increase and the local Froude number. Following De Marchi, a general discussion of the solution was attempted indicating that the upstream flow depth for sub-critical flow was highest for triangular, and lowest for rectangular plan shape. Therefore, from freeboard considerations, channel width increase along the lateral discharge zone is a disadvantage. Hydraulic jumps in side-channels were also considered. Citrini (1948) investigated the effect of lateral flow direction on the free surface profile, and the associated head loss by using this particular design.



Duilio Citrini was born on 26.04. 1913 in Milano/I and there passed away on 06.01. 2006. He graduated from Politecnico di Milano as civil engineer and there gained the PhD title in 1936. He was appointed in 1949 professor of hydraulics at Cagliari University, returning to Milan in this position in 1959. He was then the principal collaborator of Giulio De Marchi (1890-1972).
Citrini was one of the prime Italian hydraulicians in the 1940s and 1950s, on whom based the reputation of the Milan hydraulics school. With De Marchi, he was at the forefront of research making the Milan Hydraulic Institute to Italy's leading establishment. Citrini's 1939 paper on hydraulic jumps was a basic state-of-the-art, to which experimental data were added. The 1940 paper was an alternative approach to supercritical flows in chute curves, for which Arthur Ippen (1907-1974) made a theoretical basis two years earlier. The diffusion of turbulent jets was studied in 1946, following the Göttingen School of Ludwig Prandtl (1875-1953). During his stay in Sardinia, Citrini turned to questions of applied hydraulics: He investigated unsteady sideweir flow phenomena as complimentary work of his mentor, who had developed the classic theory of steady sideweir flow in 1934. In 1952, Citrini analysed the flow pattern in porous conduits as used in groundwater technology.

Garbis H. Keulegan (1944, 1952), a well-known mathematician, was the first to apply the singular point analysis to side-channel flow, following essentially De Marchi. His 1952 paper deals with side-channel flow in a rectangular channel for which the equation of the surface profile may be expressed as (Keulegan 1952)

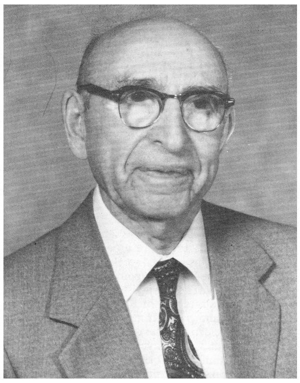
$$\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{S_o - S_f - \dfrac{2q^2 x}{gb^2 h^2}}{1 - \dfrac{q^2 x^2}{gb^2 h^3}} \tag{2}$$

where $S_o$ is bottom slope, $S_f$ friction slope and $b$ constant channel width. To lowest order, $S_f=(f/h)(V^2/2g)$ with $f$ as friction coefficient. At the critical point where the Froude number is $F=qx/[bh(gh)^{1/2}]=1$, both the nominator and denominator must simultaneously be zero, i.e.

$$S_o - \frac{f}{h}\frac{V^2}{2g} - \frac{2q^2 x}{gb^2 h^2} = 0 \tag{3}$$

$$1 - \frac{qx}{bh\sqrt{gh}} = 0 \tag{4}$$

This system of equations was solved to result in the location and the flow depth of the so-called singular point. The friction coefficient was then determined using the laboratory tests of Beij (1934). The results were applied to predict his free surface profiles for which a good agreement was found under extremely small flow depths. A similar approach was also presented by the Japanese Iwagaki (1951) for laminar sheet flow.



The Armenian Garbis H. Keulegan was born in today’s Turkey on 12.07. 1890 at Sebastia-Sivas/TR, and passed away on 28.07. 1989 at Vicksburg MI/USA. He left his home country in 1912 for the USA and started as an engineer at Ohio State University graduating as mathematician in 1915. He started as physicist at the National Bureau of Standards NBS in 1921. Until retirement in 1962, he was there primarily engaged as an expert in soil mechanics. He submitted a PhD thesis in 1928 to the Johns Hopkins University, Baltimore MD.
Keulegan’s interest in hydrodynamics started with the inauguration of the National Hydraulic Laboratory at NBS. He was one of the three staff members and greatly contributed with classic papers on turbulent flow in open channels, roll wave formation, water wave theory and flow in curved pipes. During World War II Keulegan was involved in the Beach Erosion Board in connection with the Allied Landing in France. He thus developed the theory of tides, the water wave theory and furnished information relating to the prediction of sea currents. After the war the NBS was asked to furnish information toward the basic laws of similitude involving density currents and the mixing of salt with fresh waters. Keulegan investigated both questions thoroughly and presented classic papers relating to lock exchange experiments. His results on wave propagation and density currents were published in the 1949 book of Rouse and the 1966 book of Ippen, respectively. Keulegan was awarded a number of prestigious decorations, including the National Medal of Science, ASCE Honorary Membership, and election to the National Academy of Engineering in 1979.

Li (1955) also considered one-dimensional flow in side-channels by applying again the momentum equation. He particularly investigated the effect of bottom slope and classified solutions according to the Froude number and a parameter $G=S_oL/h_o$, in which $L$ is the length of lateral discharge and $h_o$ the flow depth at its end. The analysis included next to the rectangular channel also channels with sloping side walls. Further, both sub- and super-critical flows were included in the analysis. The effect of wall friction was investigated with a simplified procedure, and it was demonstrated that this effect may often be neglected. Examples were then presented for applications in practice.

Wen-Hsiung Li was born on 05.11. 1918 Sin Hui/CN in Kwangtung Province, and passed away on 13.09. 2002 in Hartford CT/USA. He graduated as a civil engineer from Chiao Tung University, Shanghai in 1941, and earned his PhD in engineering from University of Manchester UK in 1947. He was then a post-doctoral fellow until 1950 at

the Johns Hopkins University, Baltimore MD, assistant professor there until 1955, and from 1959 until retirement professor of civil engineering at Syracuse University, Syracuse NY. Li was recipient of the Eddy Medal for noteworthy research, and he had research grants from the US Public Health Services and the US Navy.
Li mainly worked in mathematical physics, with a special interest in fluid transients in his early career. A first research project related to flow into gutter inlets, corresponding to the first study of flows in this common arrangement. His further papers include solutions to column separation as an effect of cavitation at low fluid pressure, which he dealt with in the early 1960s. Further topics include applied fluid mechanics in general, a summary of which is given in his 1964 book. His most significant work was published in 1972, dealing again with unsteady pipe flow, but also with groundwater flow and hydraulic dispersion. During his second professional period, Li thus took an interest in environmental engineering, investigating dispersion phenomena as occur in rivers due to pollution. His 1983 book covers the latter topic, and was one of the first in this then highly significant research issue.

A large experimental campaign on the free surface profiles of side-channel spillway flow was conducted by Sassoli (1959). A 10 m long and 0.50 m wide rectangular channel was used, along which the first 2.50 m supplied the lateral discharge. Bottom slopes of $S_o$=0.05 to 0.15 in steps of 1% were studied, covering a wide range of applications in practice. Velocities were measured using a Pitot tube, by which the momentum correction coefficient $\beta$ was determined. A large number of surface profiles is available by which computational models may be validated. This study is particular in the sense that an almost transverse horizontal water surface was generated by a grid structure inserted in the test channel. Also, free surface profiles both up- and downstream of the lateral discharge zone were measured. The agreement between observations and predictions is again noteworthy, except maybe close to the hydraulic control due to curvature effects.

Franco Sassoli was born on 15.05.1923 in Pisa, Italy. He there received the PhD degree in 1951 from University of Pisa. He continued there as research assistant, taking over in 1962 the chair of applied hydromechanics. He was from 1966 until retirement in 1996 director of its Hydraulic Institute. In parallel he also taught fluid dynamics at the Naval Academy of Livorno, Italy.
His research included works on side-channel spillways, siphon spillways, discharge capacity of pressurized reinforced canals, damping effects of flood waves in nearly rectangular canals, urban water distribution networks, modeling of fluvial rivers including sediment transport, and hydraulic modeling of junction flows. Sassoli was a person of arts, playing excellently also the violin and enjoying painting.

## 4. CONCLUSIONS

Side channel spillways are currently a standard design in hydraulic engineering. The interest in this particular type of open channel flows started only in the 1920, but was almost completed by the late 1950s. Due to lateral flow addition, significant energy losses occur requiring to apply the momentum instead of the energy equation. The one-dimensional free surface profile may then be determined, provided proper boundary conditions are accounted for. The singular point analysis was particularly developed in hydraulics for these flows, allowing to initiate hydraulic computations.
This paper deals with the historical advances of an interesting topic in hydraulics in which mainly Americans, Swiss and Italians were involved. Their main contributions are detailed, along with a short biography of the individuals considered. Few additions in this topic are expected in the near future, given that questions of engineering concern are available from a one-dimensional approach. Further results may be expected in rainfall-runoff processes, a particular application of side-channels, involving two-dimensional effects and erosion patterns due to sheet flow.

## 5. ACKNOWLEDGEMENTS

The Authors would like to acknowledge the portrait of Prof. W.H. Li by his daughter Dr. Charlene Li. The portrait of Prof. Sassoli was provided by his former student, Prof. Dr. S. Pagliara, University of Pisa, Italy.

## 6. REFERENCES


Beij, K.H. (1934). Flow in roof gutters. *Journal of Research* NBS 12, 193-213.

Camp, T.R. (1940). Lateral spillway channels. *Trans*. ASCE 105, 606-637.

Citrini, D. (1942). Canali rettangolari con portata e larghezza gradualmente variabili (Rectangular channels with gradually-varied discharge and width). *L'Energia Elettrica* 19(5), 254-262; 19(6), 297-301 [in Italian].

Citrini, D. (1948). Canali rettangolari con apporto laterale di portata (Rectangular channels with lateral discharge*). L'Energia Elettrica* 25(4), 155-166 [in Italian].

De Marchi, G. (1941). Canali con portata progressivamente crescente. *L'Energia Elettrica* 18(6), 351-360 [in Italian].

Favre, H. (1933). *Contribution à l'étude des courants liquids*. Rascher, Zurich [in French].

Hager, W.H. (1990). Tornado-Wirbel im Wasserbau (Tornado vortices in hydraulic structures). *Wasser-Energie-Luft* 82(11/12), 325-330 [in German].

Hinds, J. (1926). Side channel spillways: Hydraulic theory, economic factors, and experimental determination of losses. *Trans*. ASCE 89, 881-939.

Iwagaki, Y. (1951). Theory of flow on road surface. *Memoir* 13, 139-147. Faculty of Engineering, Kyoto University, Kyoto Japan.

Keulegan, G.H. (1944). Spatially variable discharge over a sloping plane. *Trans*. AGU 6, 956-959.

Keulegan, G.H. (1952). Determination of critical depth in spatially variable flow. *Proc. 2nd Mid-western Conf. Fluid Mechanics*, Ohio State University, 67-80.

Li, W.-H. (1955). Open channels with non-uniform discharge. *Trans*. ASCE 120, 255-280.

Meyer-Peter, E., Favre, H. 81934). Analysis of Boulder Dam spillways made by Swiss laboratory. *Engineering News-Record* 113(Oct.25), 520-522; 114(Apr.4), 480-482; 115(Sep.19), 409.

Page, J.C. (1938). Model studies of spillways, Boulder Canyon Project: *Final Reports*. Bulletin 1, Part VI. USBR, Denver CO.

Pfister, M., Berchtold, T., Lais, A. (2008). Kárahnjúkar dam spillway: Optimization by hydraulic model tests. 3rd *IAHR International Symposium in Hydraulic Structures*, Hohai University, Nanjing VI, 2106-2111.

Sassoli, F. (1959). Canali collettori laterali a forte pendenza (Side-channels of large bottom slope). *L'Energia Elettrica* 36(1), 26-39 [in Italian].