Robust Smith Predictor Design for Time-Delay Systems with H_∞ Performance

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Abstract: A new method for robust fixed-order H_∞ controller design for uncertain time-delay systems is presented. It is shown that the H_∞ robust performance condition can be represented by a set of convex constraints with respect to the parameters of a linearly parameterized primary controller in the Smith predictor structure. Systems with uncertain dead-time and with multimodel and frequency-domain uncertainty can be considered straightforwardly in the proposed approach. Furthermore, it is shown that the design method can also be extended to design of robust gain-scheduled dead-time compensators. The performance of the method is illustrated by simulation examples.

1. INTRODUCTION

Most industrial processes present dead time in their dynamics. Generally, dead times are caused by the time needed to transport energy, mass or information, but they also can be caused by processing time or by accumulation of time lags in a sequence of simple dynamic systems interconnected in series (Normey-Rico and Camacho [2007]).

The presence of dead times in the control loops has two main consequences: it greatly complicates the analysis and the design of feedback controllers and it makes satisfactory control performance more difficult to achieve (Palmor [1996]). Dead-time compensators can be used to improve the closed-loop performance of classical controllers (PI or PID controllers) for processes with delay. The Smith Predictor (SP) (See Fig.1), proposed in the late 1950s by Smith [1957], was the first dead-time compensation structure used to improve the performance of the classical controllers and became the most known and used algorithm to compensate dead time in the industry.

Although the SP offers potential improvement of the closed-loop performance of process with large dead-time, it requires a good model since small modeling errors can lead to very poor performance. In fact, even if the SP is nominally stabilizing, the closed-loop system may become unstable for infinitesimal change in the process dynamics (Palmor [1980]). For this reason, research efforts have been focused on robustness issues of the SP. A tuning method for models with one uncertain parameter is proposed by Brosilow [1979]. Easy tuning rules for SP in the presence of dead-time uncertainty is addressed in Santacesaria and Scattolini [1993] and a guideline for selection the closed-loop bandwidth based on the dead-time uncertainty bound is proposed. In Palmor and Blau [1994], a robust tuning rule is developed which considers the modeling error in the dead time. Robust PID tuning for SP considering model uncertainty is proposed by Lee et al. [1999]. In particular, first and second order plus dead-time systems which may contain uncertainty in multiple parameters of the model are considered. In Meinsma and Zwart [2000], tuning guidelines are presented for setpoint tracking considering model mismatches in the dead-time.

Recently, many researchers are interested in the optimal control of dead-time systems, especially H_∞ control, i.e., to find a controller to internally stabilize the system and to minimize the H_∞-norm of an associated transfer function. Many relevant results have been presented in this framework using modified versions of the SP. See, for instance, Mirkin [2003], Meinsma and Zwart [2000] and Zhong [2003a]. Nonetheless, as pointed out by Zhong [2003b], these controllers are too involved and the predictor always includes additional unstable hidden modes even for stable plants. As the author argues, these hidden modes are not safe as they tend to destabilize the system when implemented. Therefore, the practical significance of these results are limited.

This paper presents a new method to design fixed-order SP controllers that considers uncertainty simultaneously in the dead-time and in the rational part of the model. The performance specification, like the standard H_∞ control problem, is a constraint on the infinity norm of the weighted sensitivity function and is represented by a set of convex constraints in the Nyquist diagram. A line search on the upper bound γ of the infinity norm of the weighted sensitivity function can be used in association to a convex feasibility problem to determine the primary controller parameters. The proposed method can be used for PID controllers, which are of great practical interest, as well as for higher order linearly parametrized controllers in discrete or continuous time. The main idea is introduced for LTI-SISO systems and will be extended to MIMO systems in future works.

It is important to point out that a method to design fixed-order controllers for standard control loops based on convex constraints in the Nyquist diagram has been firstly proposed in Karimi and Galdos [2010]. This paper uses

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the same framework to design dead-time compensators for time-delay systems. The extension to MIMO systems will be based on the idea presented in Galdos et al. [2010b] for designing H_∞ decoupling MIMO controllers.

This paper is organized as follows: In Section II the class of models, controllers and the control objectives are defined. Section III introduces the control design methodology based on convex constraints in the Nyquist diagram. In Section IV the results are extended to unstable time-delay systems. Gain-scheduled SP is designed for time-delay systems in Section V. Finally the concluding remarks are given.

2. PROBLEM FORMULATION

2.1 Class of models

Consider the class of stable time-delay LTI-SISO systems with bounded infinity norm. It is assumed that the plant model can be represented by:

$$P(s) = G(s)e^{-\tau s} \tag{1}$$

where the time delay τ is unknown but belongs to a finite set $\{\tau_1, \tau_2, \ldots, \tau_q\}$ and the dead-time free part of the model has unstructured multiplicative uncertainty described as:

$$G(s) = G_n(s)[1 + \Delta(s)W_2(s)] \tag{2}$$

where $W_2(s)$ is a known stable uncertainty filter, $G_n(s)$ the nominal dead-time free model and $\Delta(s)$ an unknown stable transfer function with $\|\Delta\|_\infty < 1$. Therefore, we can assume that $P(s)$ belongs to a set $\mathcal{P}$ of q models given by:

$$\mathcal{P} \triangleq \{P_i(s) = G(s)e^{-\tau_i s};\ i = 1, \ldots, q\} \tag{3}$$

2.2 Class of controllers

The SP control structure shown in Fig.1 is considered. The nominal model $P_0(s) = G_n(s)e^{-\tau_n s}$ with a fixed τ_n belongs to $\{\tau_1, \tau_2, \ldots, \tau_q\}$ is used for the implementation of the controller.

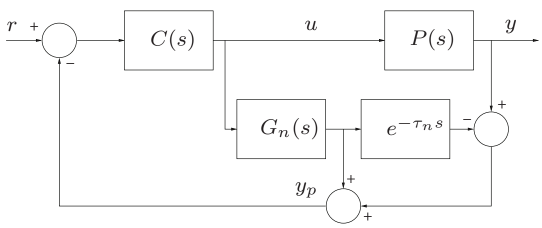

Fig. 1. Smith Predictor

The primary controller $C(s)$ is linearly parametrized by

$$C(s) = \rho^T \phi(s) \tag{4}$$

where $\rho^T = [\rho_1, \rho_2, \ldots, \rho_{n_c}]$ is an n_c dimensional vector of the controller parameters and

$$\phi^T(s) = [\phi_1(s), \phi_2(s), \ldots, \phi_{n_c}(s)] \tag{5}$$

is a vector of basis functions with $\phi_i(s)$ transfer functions with no RHP poles. For instance, a PID controller could be linearly parametrized by

$$\rho^T = [K_p, K_i, K_d] \quad , \quad \phi^T(s) = [1, \frac{1}{s}, \frac{s}{1 + T_f s}]$$

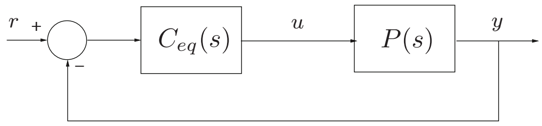

Fig. 2. Equivalent control diagram

2.3 Design specifications

The SP can be represented by a classical feedback loop (see Fig. 2) where $C_{eq}(s)$ is the equivalent controller given by

$$C_{eq}(s) = C(s)[1 + C(s)H(s)]^{-1} \tag{6}$$

with $H(s) = G_n(s) - P_0(s) = G_n(s)(1 - e^{-\tau_n s})$.

Let the sensitivity function of the closed-loop system with the i-th plant model $P_i(s)$ be defined as

$$S_i(s) = \frac{1}{1 + P_i(s)C_{eq}(s)} = \frac{1 + C(s)H(s)}{1 + C(s)[H(s) + P_i(s)]} \tag{7}$$

and the complementary sensitivity function as

$$T_i(s) = \frac{P_i(s)C_{eq}(s)}{1 + P_i(s)C_{eq}(s)} = \frac{C(s)P_i(s)}{1 + C(s)[H(s) + P_i(s)]} \tag{8}$$

A standard robust control problem is to design a controller that satisfies $\|W_1 S_i\|_\infty < 1$ for a set of models where $W_1(s)$ is the performance weighting filter. If the model is described by unstructured multiplicative uncertainty, the necessary and sufficient condition for robust performance is given by (Doyle et al. [1992]):

$$\||W_1 S_i| + |W_2 T_i|\|_\infty < 1 \quad \text{for} \quad i = 1, \ldots, q \tag{9}$$

The goal of the proposed approach is to design the primary controller $C(s)$ in the SP structure to guarantee robust performance of the closed-loop system.

3. PROPOSED METHOD

The robust performance condition (9) can be written as:

$$|W_1(j\omega)S_i(j\omega)| + |W_2(j\omega)T_i(j\omega)| < 1, \quad \forall\omega \tag{10}$$

for $i = 1, \ldots, q$. The dependency on frequency ω and s will be omitted for brevity but the dependency on the controller parameter ρ will be highlighted.

Let $L_i(\rho)$ be defined as $L_i(\rho) = C(\rho)(H + P_i)$. Note that this transfer function is not equal to the open loop transfer function $L_{eq_i}(\rho) = P_i C_{eq}(\rho)$ of the equivalent control loop in Fig. 2. The main properties of $L_i(\rho)$ are:

(1) linearity with respect to the controller parameter ρ;
(2) the closed loop poles are the roots of $1 + L_i(\rho) = 0$.

The first property is clear because $C(\rho)$ is linearly parameterized. To show the second property, let $C(\rho) = \frac{N_c}{D_c}$, $G_n = \frac{N_g}{D_g}$ be defined. The equivalent controller can be written as

$$C_{eq}(\rho) = \frac{N_c D_g}{D_c D_g + N_c N_g(1 - e^{-\tau_n s})} \tag{11}$$

Then, we have:

$$1 + L_{eq_i}(s) = \frac{D_c D_g + N_c N_g(1 - e^{-\tau_n s} + e^{-\tau_i s})}{D_c D_g + N_c N_g(1 - e^{-\tau_n s})} \tag{12}$$

Note that the numerator of $1 + L_{eq_i}(s)$ is a quasi-polynomial and its zeros are the closed-loop poles of the system. On the other hand, we have

$$1 + L_i(\rho) = \frac{D_c D_g + N_c N_g (1 - e^{-\tau_n s} + e^{-\tau_i s})}{D_c D_g} \quad (13)$$

It is clear that $1 + L_i(\rho)$ shares the same zeros with $1 + L_{eq_i}(s)$. This property will be used to prove the closed loop stability.

3.1 Main Result

The main result of this section is given in the following theorem.

Theorem 1. Consider the set of models $\mathcal{P}$ in (3) with multiplicative uncertainty filter $W_2(j\omega)$, then the linearly parametrized controller in (4) in the SP structure guarantees closed-loop stability and satisfy the following robust performance condition:

$$\| |W_1 S_i| + |W_2 T_i| \|_\infty < 1 \quad \text{for } i = 1, \ldots, q \quad (14)$$

if

$$\left[\left| W_1(j\omega)[1 + C(j\omega,\rho)H(j\omega)] \right| + \left| W_2(j\omega) C(j\omega,\rho) P_i(j\omega) \right| \right] |1 + L_d(j\omega)| - Re\{[1 + L_d^*(j\omega)][1 + L_i(j\omega,\rho)]\} < 0 \quad \forall\omega \quad \text{for } i = 1, \ldots, q \quad (15)$$

where $L_d(j\omega)$ is a strictly proper transfer function which does not encircle the critical point and $L_d^*(j\omega)$ is its complex conjugate.

Proof : Since the real part of a complex number is less than or equal to its magnitude, we have

$$Re\{[1 + L_d^*][1 + L_i(\rho)]\} \leq |[1 + L_d^*][1 + L_i(\rho)]| \quad (16)$$

Then, using (15) and the fact that $|1 + L_d| = |1 + L_d^*|$, one obtains

$$\left| W_1(1 + C(\rho)H) \right| + \left| W_2 C(\rho) P_i \right| - |1 + L_i(\rho)| < 0 \quad \forall\omega \;\; \text{for } i = 1, \ldots, q \quad (17)$$

Using $L_i(\rho) = C(\rho)(H + P_i)$ we have

$$\frac{|W_1(1 + C(\rho)H)| + |W_2 C(\rho) P_i|}{|1 + C(\rho)(H + P_i)|} < 1 \quad \forall\omega \;\; \text{for } i = 1, \ldots, q \quad (18)$$

that leads directly to (14). To prove that all closed-loop transfer functions are stable, consider (15) which gives:

$$Re\{[1 + L_d^*(j\omega)][1 + L_i(j\omega,\rho)]\} > 0 \quad \forall\omega \quad (19)$$

or, alternatively,

$$wno\{[1 + L_d^*(j\omega)][1 + L_i(j\omega,\rho)]\} = 0 \quad (20)$$

where *wno* stands for winding number around the origin. Since both $L_d^*(j\omega)$ and $L_i(j\omega,\rho)$ are constant or zero for the semi-circle with infinity radius of the Nyquist contour the *wno* depends only on the variation of s in the imaginary axis. Thus,

$$wno\{[1 + L_d(j\omega)]\} = wno\{[1 + L_i(j\omega,\rho)]\} \quad (21)$$

Since $L_d(j\omega)$ satisfies the Nyquist stability criterion $L_i(j\omega,\rho)$ will do so and all zeros of $1 + L_i(j\omega,\rho)$ will be in the left-hand side of the complex plan. Since the zeros of $1 + L_i(j\omega,\rho)$ are the closed-loop poles, the system will be internally stable. □

Remark I: The constraint in (15) is an inner convex approximation of the non convex constraint in (14) or (10). The quality of this approximation depends on the choice of L_d. It can be shown that better approximation is achieved if L_d is close to $L_i(\rho)$ (Karimi and Galdos [2010]). Note that $L_i(\rho)$ can be written as:

$$L_i(\rho) = C(\rho)G_n + C(\rho)(P_i - P_0) \quad (22)$$

Therefore, L_d can be seen as the desired open loop transfer function of the system without dead-time, i.e., $C(\rho)G_n$. This will be a good approximation if τ_i is close to τ_n or the model mismatch $P_i - P_0$ is small. For systems with large uncertainty in the time delay, for each model P_i different L_d can be considered. In this case, L_{d_i} can be computed as follows:

$$L_{d_i} = L_d + \frac{L_d}{G_n}(P_i - P_0) \quad (23)$$

Note that L_{d_i} should not encircle the critical point. An iterative algorithm can be used to improve the results. In this algorithm L_d in the $(k+1)$-th iteration is computed using the controller of the last iteration $C_k(\rho)$ and (22):

$$L_{d_i} = C_k(\rho)G_n + C_k(\rho)(P_i - P_0) \quad (24)$$

3.2 Primary controller design

The problem of minimizing the upper bound γ of the infinity norm of the weighted sensitivity function is considered. Therefore, the primary controller should be obtained from the following optimization problem:

$$\min_{\rho} \gamma$$
$$\text{Subject to:} \quad (25)$$
$$\| |W_1 S_i| + |W_2 T_i| \|_\infty < \gamma \quad \text{for } i = 1, \ldots, q$$

This optimization can be convexified using Theorem 1 and solved by an iterative bisection algorithm. At each iteration j, γ_j is fixed and W_1 and W_2 are replaced by W_1/γ_j and W_2/γ_j. Then, a feasibility problem is solved under the convex constraints (15). If the problem is feasible, γ_{j+1} is chosen smaller than γ_j. Otherwise γ_{j+1} is increased.

Notice that the condition (15) is defined for every frequency ω leading to infinite number of constraints. In practice, a frequency grid can be used with a sufficiently large number of frequency points N (a finer grid can be used around the crossover frequency). The effect of gridding on the stability and performance of the closed loop system has been studied in Galdos et al. [2010a].

Example 1 Consider the process described by (1) with multiplicative uncertainty as in (2) with

$$G_n(s) = \frac{1}{(5s + 1)(10s + 1)} \quad (26)$$

and

$$W_2(s) = \frac{-s^2 - 2s}{s^2 + 2s + 1} \quad (27)$$

The unknown time delay τ belongs to the set $[4.5, 5, 5.5]$. The nominal model used in the SP structure is chosen as $P_0(s) = G_n(s)e^{-5s}$. The performance specification is defined by the following filter:

$$W_1(s) = 2/(30s + 1)^2 \quad (28)$$

A PID primary controller with $T_f = 0.01$ that minimizes $\| |W_1 S_i| + |W_2 T_i| \|_\infty < \gamma$ for $i = 1, 2, 3$ should be computed.

Since the controller has an integrator, L_d is chosen as $L_d(s) = \omega_c/s$ where $\omega_c = 0.1$ rad/s which is 20% higher than open loop bandwidth. Then, the optimization problem (25) is solved considering $N = 100$ equally spaced frequency points between 10^{-3} and 10^3 rad/s. The resulting primary controller is:

$$C(s) = \frac{12.3s^2 + 3.28s + 0.2201}{0.01s^2 + s} \tag{29}$$

and leads to $\gamma = 0.313$. This controller is compared to that proposed in Kaya [2001]. Kaya's controllers performs better than other controllers presented in the literature (Palmor and Blau [1994], Hagglund [1992] and Hang et al. [1995]). Fig. 3 depicts the performance of both controller on unitary step setpoint change considering the time-delay $\tau = 4.5s$, $\tau = 5.0s$ and $\tau = 5.5s$. As it can be seen, both controller performed well, however, the proposed controller achieves faster response.

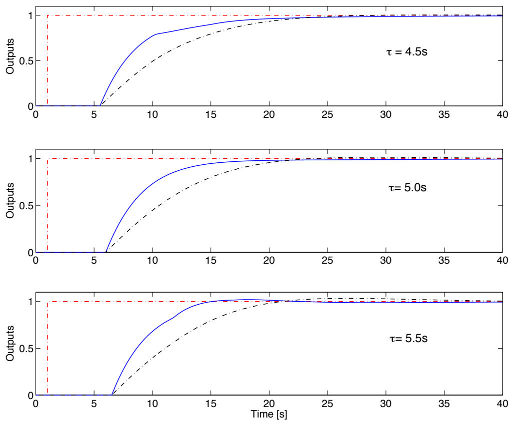

Fig. 3. Example 1: Blue solid line: proposed; black dot-dashed line: ref Kaya [2001]

4. EXTENSION TO UNSTABLE SYSTEMS

The SP in the scheme shown in Fig. 1 cannot be used for unstable plants since the controller will contain zeros in right-hand side of the s-plan which cancel the unstable poles in the plant and leads to instability. To avoid this unstable zero-pole cancellation, the control structure shown in Fig.1 should be changed. Several alternatives are available in literature to cope with unstable processes with dead-time (see, for example, De Paor [1985], Majhi and Atherton [1998], Liu et al. [2005], Normey-Rico and Camacho [2007, 2009]). Consider, for instance, the SP with modified dead-time free model depicted in Fig. 4 which is discussed in Normey-Rico and Camacho [2007]. In this case, the dead-time free model is defined as $G_m = \frac{N_m}{D_n}$ and the equivalent controller becomes:

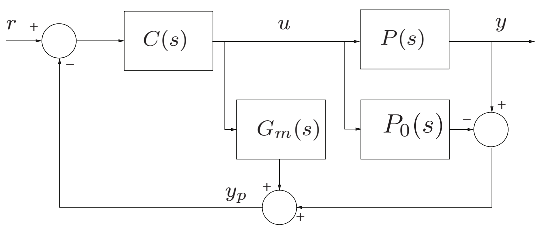

Fig. 4. Smith Predictor with modified dead-time free model

$$C_{eq_m}(\rho) = \frac{C(\rho)}{1 + C(\rho)H} \tag{30}$$

where

$$H = G_m - P_0 = (N_m - N_n e^{-\tau s})\frac{1}{D_n}$$

Note that the poles of H are zeros of $C_{eq_m}(\rho)$. Therefore, N_m must be tuned such that the zeros of $N_m - N_n e^{-\tau s}$ cancel the unstable poles in D_n. Once N_m has been properly designed, the primary controller can be obtained by solving the optimization problem in (25) redefining $H = G_m - P_0$ and $L_i(\rho) = (G_m - P_0 + P_i)C(\rho)$.

Here, care should be taken in the choice of L_d. As it has been shown, the *wno* of $1 + L_i$ equals the *wno* of $1 + L_d$. Therefore, L_d should be chosen such that the number of encirclement of the critical point $(-1 + 0j)$ by its Nyquist plot is equal to the number of unstable poles in P_i.

Example 2 Consider the model studied in Meinsma and Zwart [2000] given by:

$$P(s) = \frac{k}{s - a}[1 + \Delta(s)W_2(s)]e^{-\tau s} \tag{31}$$

where $k = 1$, $a = 1$, $\tau = \tau_n \pm 0.02$ and $\tau_n = 0.2$. The interval of variation of τ is gridded using $q = 3$ equally spaced points. A finer grid just increases the number of constraints and for this example does not change significantly the final controller. The performance and uncertainty filters are respectively chosen as:

$$W_1(s) = 2\frac{s+1}{10s+1} \quad \text{and} \quad W_2(s) = 0.2\frac{s+1.1}{s+1} \tag{32}$$

Here, we use the SP with modified dead-time free model (Fig. 4) due to its simplicity. The dead-time free model $G_m(s)$ is chosen as

$$G_m(s) = \frac{T_m s + 1}{s - 1}. \tag{33}$$

T_m is computed in order to obtain $H(s) = G_m(s) - P_0(s)$ without a pole in $s = 1$. Since

$$H(s) = \frac{1}{s-1}[T_m s + 1 - e^{-0.2s}], \tag{34}$$

if $T_m = e^{-0.2} - 1$, then $s = 1$ is a zero of $H(s)$.

A PI as the primary controller is designed. The first step is to choose the transfer function $L_d(s)$, which must encircle the critical point in the Nyquist diagram once and must contain one integrator. Therefore, it is chosen as

$$L_d(s) = 10\frac{s+1}{s(s-1)}. \tag{35}$$

Optimization problem (25) is solved considering $N = 100$ equally spaced frequency points between $\omega = 10^{-3}$rad/s and $\omega = 10^3$rad/s and the following controller is obtained:

$$C_0(s) = (3.582s + 0.5838)/s \tag{36}$$

which yields $\gamma = 0.6854$. This result can be further improved by using a new $L_d(s)$ based on $C_0(s)$ in the optimization problem. With this new $L_d(s) = G_m(s)C_0(s)$ the optimal primary controller is:

$$C(s) = (2.994s + 0.4612)/s \tag{37}$$

and $\gamma = 0.6074$. Figure 5 depicts the function

$$\Gamma_i(j\omega) = |W_1(j\omega)S_i(j\omega)| + |W_2(j\omega)T_i(j\omega)|$$

where S_i and T_i are respectively given by (7) and (8) with $H = G_m - P_0$ and P_i is obtained by gridding of τ.

Note that the maximum value of the function is 0.6072, which occurs when $\tau = \tau_n + 0.02 = 0.22$, is close to the bound γ. It is worth to point out that, although the conditions given in Theorem 1 are only sufficient to guarantee $\|\Gamma_i\|_\infty < \gamma$, with a proper choice of L_d it is possible to obtain a solution with very low conservatism. Furthermore, the resulting controller is a standard PI which can be implemented in a straightforward manner and has great practical significance.

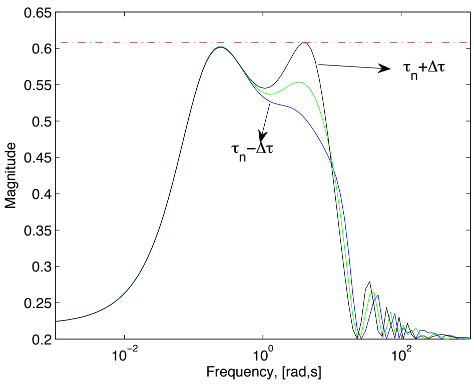

Fig. 5. Example 2: Blue solid line: Γ for $\tau = 0.18$; green solid line: Γ for $\tau = 0.2$; black solid line: Blue solid line: Γ for $\tau = 0.22$; red dot-dashed line: γ.

For the same example, controller designed in Meinsma and Zwart [2000] leads to the optimal $\gamma = 0.9407$ which is 55% higher than the value obtained with the proposed method. It should be mentioned that the true robust performance criterion in (25) is not minimized in Meinsma and Zwart [2000]. Instead, the maximum singular value of $[W_1(j\omega)S(j\omega) \quad W_2(j\omega)T(j\omega)]$ for all ω is minimized by the H_∞ control theory.

5. GAIN-SCHEDULED CONTROLLER DESIGN

Consider an uncertain plant $P(s,\theta)$ belonging to the set:

$$\mathcal{P} = \{G(s,\theta)e^{-\tau_i(\theta)s},\ i = 1,\dots,q\} \tag{38}$$

where the dead-time free part of the model has unstructured multiplicative uncertainty and is described as:

$$G(s,\theta) = G_n(s,\theta)[1 + \Delta(s)W_2(s)] \tag{39}$$

and θ is a vector of scheduling parameters that belongs to a finite set $\Theta = \{\theta_1, \theta_2, \dots, \theta_m\}$ (corresponding e.g. to the different operating point parameters). It is assumed that the operating point does not frequently change (the stability and performance are achieved for the frozen scheduling parameter). The dead-time is also a function of the scheduling parameter and uncertain, so for a given value of θ it belongs to the set $\{\tau_1(\theta), \tau_2(\theta), \dots, \tau_q(\theta)\}$.

We will consider the SP shown in Fig. 6 where both, the nominal model $P_0(s,\theta) = G_n(s,\theta)e^{-\tau_n(\theta)s}$ and the primary controller $C(s,\theta)$ are functions of the scheduling parameter vector θ. The goal is to compute a primary gain-scheduled controller for this scheme that meets the H_∞ robust performance specification.

The primary controller $C(s,\theta)$ is linearly parametrized by: $C(s,\theta) = \rho^T(\theta)\phi(s)$, where the basis function vector $\phi(s)$ is defined in (5) and $\rho^T(\theta)$ is given by

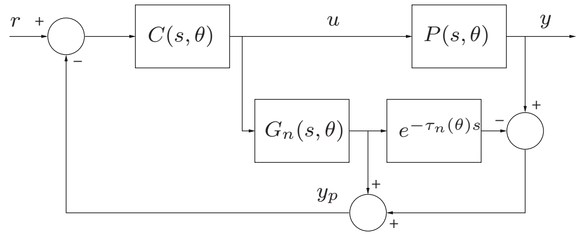

Fig. 6. Gain-Scheduled Smith Predictor

$$\rho^T(\theta) = [\rho_1(\theta), \rho_2(\theta), \dots, \rho_n(\theta)] \tag{40}$$

Every gain is a polynomial function of order δ of the scheduling parameters and is defined as

$$\rho_i(\theta) = (\nu_{i,\delta})^T\theta^\delta + \dots + (\nu_{i,1})^T\theta + \nu_{i,0} \tag{41}$$

and θ^k denotes element-by-element power of k of vector θ. The sensitivity and complementary sensitivity functions are respectively given by:

$$S_i(s,\theta) = \frac{1 + C(s,\theta)H(s,\theta)}{1 + C(s,\theta)(H(s,\theta) + P_i(s,\theta))} \tag{42}$$

$$T_i(s,\theta) = \frac{C(s,\theta)P_i(s,\theta)}{1 + C(s,\theta)(H(s,\theta) + P_i(s,\theta))},\ \forall\theta\in\Theta$$

where $H(s,\theta) = G_n(s,\theta) - P_0(s,\theta)$. The primary controller is obtained from the following optimization problem:

$$\min_{\rho} \gamma$$

$$\text{Subject to:} \tag{43}$$
$$\||W_1 S_i(s,\theta)| + |W_2 T_i(s,\theta)|\|_\infty < \gamma$$
$$\text{for } i = 1,\dots,q, \quad \forall\theta\in\Theta$$

Optimization problem (43) is again solved using an iterative bisection algorithm as previously presented. At each iteration, a feasibility problem is solved with the following convex constraints:

$$\Big[|W_1(j\omega_k)[1 + C(j\omega_k,\theta_l)H(j\omega_k,\theta_l)| +$$
$$|W_2(j\omega_k)C(j\omega_k,\theta_l)P(j\omega_k,\theta_l)|\Big]|1 + L_d(j\omega_k)| -$$
$$Re\{[1 + L_d^*(j\omega_k)][1 + L_i(j\omega_k,\theta_l)]\} < 0$$
$$\text{for } k = 1,\dots,N, \quad i = 1,\dots,q, \quad l = 1,\dots,m \tag{44}$$

Example 3 The design method is applied on a simulated system having a resonance whose frequency changes as a function of a scheduling parameter θ. Consider the following plant model

$$P(s,\theta) = G(s,\theta)e^{-\tau s} \tag{45}$$

where $G(s,\theta) = G_n(s,\theta)[1 + \Delta(s)W_2(s)]$ and

$$G_n(s,\theta) = \frac{(2 + 0.2\theta)^2}{s^2 + 0.2(2 + 0.2\theta)s + (2 + 0.2\theta)^2} \tag{46}$$

$$W_2(s) = 0.8\frac{1.1337s^2 + 6.8857s + 9}{(s+1)(s+10)} \tag{47}$$

and $\theta \in [-1, -0.5, 0, 0.5, 1]$. Consider also that the dead-time is within the interval $\tau \in [2.7, 3.0, 3.3]$ but its exact value is unknown in runtime. The objective is to design a primary gain-scheduling PID controller for the Smith Predictor structure considering the performance filter $W_1(s) = \frac{2}{(20s+1)^2}$. The parameters ρ of the primary controller will be affine functions of the scheduling parameter θ. The filter of the derivative action is chosen to have a time constant of $T_f = 0.01$s.

Finally, optimization problem (43) is solved considering $L_d = 1/s$ and $N = 100$ equally spaced frequency points between 10^{-2} and 10^2 rad/s. The resulting gain-scheduled controller is given by: $K_p(\theta) = -0.0168\theta + 0.2152$, $K_i(\theta) = 0.0144\theta + 2.4736$, $K_d(\theta) = -0.1224\theta + 0.6424$.

This controller leads to:

$$\||W_1 S_i(s, \theta_l)| + |W_2 T_i(s, \theta_l)|\|_\infty < \gamma = 0.8928 \quad (48)$$
$$l = 1, \ldots, 5, \quad i = 1, 2, 3$$

The gain-scheduled controller is evaluated considering $\theta = -1, 0, 1$ and $\tau = 3.3$s. The performance is compared to a fixed-gain PID designed for the nominal case ($\theta = 0$ and $\tau = 3$s). Figure 7 shows the step response of the gain-scheduled controller in all conditions (blue, red and green solid lines) compared with the fixed PID controller (black dashed line, highly oscillating).

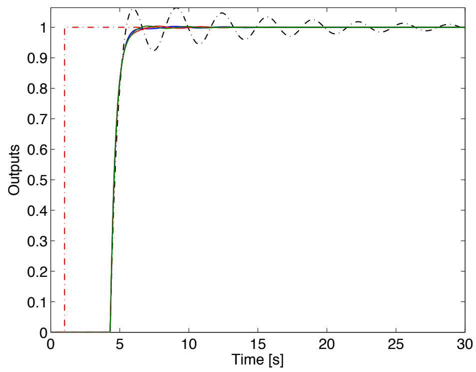

Fig. 7. Example 3: Blue, red and green solid line: gain-scheduled PID Smith Predictor and G_2 using $\theta_1 = -1$, $\theta_3 = 0$ and $\theta_5 = 1$ respectively; black dashed line: fixed PID Smith Predictor using $\theta = -1$.

6. CONCLUSIONS

This paper presents a new method to design robust Smith predictor for uncertain time-delay systems using convex optimization techniques. The proposed approached allows one to design PI/PID as well as higher order primary controllers in the Smith predictor structure which provide robust H_∞ performance for systems with uncertain dead-time and multiplicative or multimodel uncertainty in the dead-time free model of the system. The method is based on a convex approximation of the H_∞ robust performance criterion in the Nyquist diagram. This approximation relies on the choice of a desired open-loop transfer function L_d for the dead-time free model of the plant. The extension of the approach to MIMO systems is under investigation.

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