Cycling dynamics of the internal kink mode in non-linear two-fluid MHD simulations

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September 27th, 2011
Sawtooth oscillations are marked by sudden, periodic relaxations of the plasma core profiles. Reconnecting internal kink mode with $q = m/n = 1/1$ helicity can lead to heat, current, momentum, and fast particle redistribution during the reconnection event, which takes place on a 100 $\mu$s timescale. Long, quiescent ramps take place between crashes. This is important for reactor operation, yet not fully understood. The experiments show somewhat perplexing behavior, such as “mini-crashes,” snakes, and helical states, which indicate partial magnetic reconnection.
Introduction

- Sawtooth oscillation are marked by sudden, periodic relaxations of the plasma core profiles
  - Reconnecting internal kink mode with $q = m/n = 1/1$ helicity
  - Heat, current, momentum, fast particles are redistributed during reconnection event taking place in $100\,\mu s$ timescale
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  - Long, quiescent ramp takes place between crashes
- Important for reactor operation, yet not fully understood
- The experiments show somewhat perplexing behavior
  - ”Mini-crashes”, snakes, helical states, partial magnetic reconnection...
Objectives

We aim to:

- Improve the understanding of the cyclic behavior of sawteeth using three-dimensional, fully non-linear fluid simulations.
- Characterize the steady-state ($\tau_\eta$, $\omega_*$) cyclic regimes of the internal kink with respect to $\tau_\eta$, $\omega_*$ to find diamagnetic thresholds for sawtoothing.
- Attempt to respect some of the experimental timescales set by plasma heat and current sources.
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- We will attempt to respect some of the experimental timescales set by plasma heat and current sources
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- This behavior is only tractable with numerical simulations!
Outline

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  Cyclic regimes in $S - \omega_*$ phase space
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  Thresholds for cyclic regimes
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Discussion
The simulations are carried out using the XTOR-2F code

The system evolved is a subset of the Braginskii two-fluid equations

\[
\begin{align*}
\partial_t \rho &= -\rho \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla \rho - \alpha \nabla p_i \cdot \nabla \times \mathbf{B} / B^2 + \\
&\quad \nabla \cdot D_\perp \nabla (\rho - \rho_{t=0}), \\
\rho \partial_t \mathbf{v} &= -\rho (\mathbf{v} + \mathbf{v}_{*i}) \cdot \nabla \mathbf{v} + J \times \mathbf{B} - \nabla p + \nu \nabla^2 \mathbf{v}, \\
\partial_t p &= \Gamma p \nabla \cdot \mathbf{v} - \mathbf{v} \cdot \nabla p - \alpha \Gamma \frac{p}{\rho} \nabla p_i \cdot \nabla \times \mathbf{B} / B^2 + \\
&\quad \nabla \cdot \chi_\perp \nabla_\perp (\rho - \rho_{t=0}) + \nabla \cdot \chi_\parallel \nabla_\parallel p \\
\partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \alpha \nabla \times \nabla_\parallel \mathbf{p}_e / \rho - \nabla \times \eta J \\
\mathbf{v}_i &= \mathbf{v}_E \times \mathbf{B} + \mathbf{v}_\parallel + \mathbf{v}_{*i}, \ J = e n_e (\mathbf{v}_i - \mathbf{v}_e), \\
\alpha &= (\omega_{ci} \tau_a)^{-1} = \frac{c}{\alpha \omega_{pi}}, \ \mathbf{v}_* \propto \alpha
\end{align*}
\]

Terms in red are corrections due to \( \omega_* \) effects
Plasma equilibrium

- Equilibrium computed using CHEASE code
- Circular equilibrium, $A = \epsilon^{-1} = 2.7$, $\beta_p = 0.22$, $\partial_r \beta_p \approx 0$
- Parabolic $q$ profile, $q_0 = 0.77$, $q_a = 5.2$, $(\psi/\psi_s)^{1/2}(q=1) \approx 0.4$
- Warning: Initial equilibrium never recovered after first crash

![Graph showing q and pressure profiles]
Simulation setup

- Simulations must be advanced until the cycle period and amplitude stabilizes or until cycles stop.
- Retained toroidal harmonics have $n = 0, 1, 2, 3$, with $n - 4 \leq m \leq n + 7$ for $n = 1, 2, 3$. 
Internal kink timescales

- The internal kink cycles are affected by the interplay between:
  - \( S = \tau_\eta = 1/\eta = 10^6 - 10^7 \) (resistive time)
  - \( \tau_\eta = 30\tau_{\chi_\perp}, \chi_\parallel/\chi_\perp \approx 10^7 \) (energy diffusion times)
  - \( \omega_*'s \) introduce additional timescale through growth rate of internal kink (\( \gamma_\eta \sim S^{-1/3} - \alpha \)), we consider \( \alpha = 0-0.2 \)
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- Pressure dynamics follows magnetic field lines
  - Parallel temperature perturbations are strongly damped
    $\nabla_\parallel T \approx 0$, so $\omega_{*i} \approx 9\omega_{*e}$
Cyclic regimes

Distribution of cyclic regimes in $S - \omega_*$ parameter space:

We now describe briefly each regime...
$m/n = 1/1$ helical states

- First regime: Equilibrium due to low-shear saturated kink (axisymmetric boundary and $m/n = 1/1$ helical core)
  - [Internal kink: Waelbrock, Phys.Fluids 31, 1217 (1988)]
  - [Equilibrium state: Cooper, NF 51 072002 (2011)]
Resistive kink cycles (Kadomtsev’s sawteeth)

- Diamagnetic stabilization allows access to cycling regime
- They are characterized by slow, collisional crashes \( (\tau_{\text{crash}} \sim S^{-1/2}) \) [Baty et al., Phys.Fluids B 5, 1213 (1993)]
- The ramp is never quiescent, large \( m/n = 1/1 \) island present
Sawtooth cycles

- Cycles have quiescent ramps, precursor and postcursor modes
- Fast, collisionless crashes (weak scaling of $\tau_{\text{crash}}$ vs $S$)
- Sometimes a "mini-crash" is observed
Magnetic field cross sections
Diamagnetic thresholds for internal kink cyclic regimes

Critical diamagnetic stabilization thresholds have the form

$$\alpha_{\text{crit},1} = \alpha_1 S^{-0.34}$$
$$\alpha_{\text{crit},2} = \alpha_2 S^{-0.60}$$
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- Transition at \( \alpha_{\text{crit},2} \): Stabilization of deep-ideal-MHD-stable branch of internal kink with \( \gamma \sim S^{-3/5} - \alpha \) (tearing like)
Diamagnetic thresholds for internal kink cyclic regimes

Instability regimes appear to inhabit different regions of stability diagram during the ramp:

\[ \frac{\lambda H}{\gamma \eta} \sim -1, \text{ move toward } \frac{\lambda H}{\gamma \eta} = 0 \]

\[ \text{Precursor} \]

\[ \text{Ramp} \]

\[ \omega_n * e/\gamma \eta \]

\[ S = 10 \]

\[ S = 6.66 \times 10^6 \]

\[ S = 3.33 \times 10^6 \]

\[ (\omega_n * e \omega_i)*^{1/2}/\gamma \eta \]
Diamagnetic thresholds for internal kink cyclic regimes

Instability regimes appear to inhabit different regions of stability diagram during the ramp:

- Kink cycles have $\lambda_H/\gamma \eta \sim -1$, move toward $\lambda_H = 0$
- Sawteeth have more strongly negative $\lambda_H/\gamma \eta$
- Compare to [Migliuolo, NF 33 (1993) 1721]:

![Graph showing diamagnetic thresholds for internal kink cyclic regimes](image)
Measuring ramp, precursor, and crash times
Ramp, precursor, crash timescales

$\tau_{\text{ramp}}, \tau_{\text{precursor}}, \tau_{\text{crash}}$ are shown for cases with $S = 10^7$
Role of diamagnetic stabilizations at crash onset

Well within the "sawtooth" regime:

\[ r_{\text{max}}(q=1) \text{ at } t=2.856 \times 10^5 \tau_a \]
Role of diamagnetic stabilizations at crash onset

Just below the diamagnetic threshold:

- The crash time is increasing, with $\tau_{\text{crash}} + \tau_{\text{precursor}} \approx \tau_{\text{ramp}}/2$
- Rate of energy release accelerates, without any effect on the crash time

![Graph showing pressure and kinetic energy over time]
Interpretation of results

Regime transitions can be described as a competition between relaxation timescales of pressure, current, reconnection drive, and $\omega^*$ stabilization.

- Ramp: Quiescence is determined by $\omega^*$ stabilization of $m/n = 1/1$ mode with $\gamma \sim S^{-3/5}$ (similar to resistive tearing).

- Precursor stage: Competition between resistive tearing instability and $\omega^*$. If resistive instability is strongly stabilized, fast crash takes place.

- Postcursor stage: Pressure must increase fast enough to overcome reconnection drive, slow enough not to destabilize pressure-driven flat $q$ mode.

Access to sawtoothing regime requires that all three conditions are fulfilled.
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Summary

XTOR-2F simulations reveal a pattern of 3 cyclic regimes:

- (Non-cyclic) equilibria with $m/n = 1/1$ helicity component
- Resistive kink cycles (Kadomtsev's sawteeth)
- Sawtooth cycles

Established $\eta$ scaling of critical diamagnetic stabilization:

- $\alpha_{\text{crit}}, 1 = \alpha_1 S^{-1/3}$
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In a two-fluid model with realistic $S$ and $\omega^*$, sawtooth cycles should have a quiescent ramp and a crash in the 100 $\mu$s scale.
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