Impact of the cost function in the optimal control formulation for an air-to-water heat pump system

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1. ABSTRACT

This paper investigates the influence of the choice of the cost function in the optimal control formulation for an air-to-water heat pump system. The aim is to minimize, under given thermal comfort requirements, the electricity consumption which is calculated as the ratio between the thermal power and the coefficient of performance (COP) at which the heat is produced. Because the COP depends on the supply water temperature, which in turn depends on the thermal power, the resulting optimization problem is non-convex. This study compares the results obtained for the original, non-convex optimal control problem formulation to the results obtained for convex approximations typically found in the literature. The study reveals that these convex approximations yield almost identical results as the original, non-convex one, namely a smooth profile for the thermal power which is correlated to the ambient air temperature profile. This is an interesting result since the considered convex optimization problems can be solved faster to (global) optimality compared to the considered non-convex problems which in general require significantly more computational effort in order to reach global optimality.

Keywords: Air-to-water heat pump, optimal control, COP, linear approximations

2. INTRODUCTION

The development of optimal control strategies for the application of building climate control is receiving growing attention due to the increasing search for intelligent operation of heating and cooling systems (e.g. Oldewurtel et al. 2010, Gyalistras and Gwerder 2010...). With a model predictive control (MPC) strategy, system operation is optimized by calculating at each control time step the control input profile which minimizes a given cost function, using a simplified dynamic system model, updated system information and disturbances predictions (e.g. weather forecast and occupancy prediction). The cost function is typically a weighted sum of the conflicting objectives of minimizing the energy cost and the thermal discomfort. For heating systems using gas or oil, the energy cost is proportional to the amount of thermal energy, whereas for a heat pump system, the energy cost is the electricity cost which depends on the amount of thermal energy delivered, on the coefficient of performance (COP) at which this heat is delivered as well as on the electricity price. However, the translation of thermal energy into electrical energy in the optimal control problem formulation is not straightforward because it is inherently nonlinear since to optimize the thermal power profile one needs the COP profile, which in turn depends on the supply water temperature profile and thus on the thermal power profile. Due to this nonlinearity, the optimization problem is non-convex and thus convergence to the global optimum is not guaranteed.

The solution of this non-convex optimal control problem (OCP) was first studied by Zaheer-Uddin et al. (1987) and Rink (1988) for the case of a heat pump connected to a water storage tank with a given heat demand profile. It was solved both analytically, necessitating important simplifications on the level of the boundary conditions, and numerically, with convergence found to be very sensitive to the choice of the initial trajectory. Later investigations of optimal control in the framework of MPC for heat pump systems by Wimmer (2004) and Bianchi (2006) use a convex approximation for the OCP. Convexity is achieved by using a predefined profile for the COP based on a prediction of the ambient air temperature and assuming a constant supply water temperature instead of taking the actual supply water temperature dependency into account. Moreover, the energy cost is defined as the square of the predicted electricity cost, contrary to most OCP formulations for building climate control which penalize the energy cost by a linear term (Rink 1988, Kummert 2001, Gyalistras and Gwerder 2010, Oldewurtel et al. 2010). Consequently, the resulting OCP boils down to a standard convex quadratic programming problem for which the global optimum can easily be found. The question arises to which extent the solution found with this convex approximation, approaches the solution of the original problem. Or, to put it the other way round, how much can be gained by solving the original non-convex problem instead of the convex approximation. This question is addressed in this paper.

The original non-convex problem is solved for the specific case of a modulating air-to-water heat pump connected to a floor heating system. First, this reference solution is compared to the solutions found with the convex approximations using a predefined COP profile and a constant COP. Hereby, the importance of discretization errors on the solution of the non-convex case is highlighted. Next, the influence of minimizing the square of the predicted electricity consumption instead of the absolute value itself, as encountered in previous investigations (Wimmer 2004, Bianchi 2006), is discussed.

3. PROBLEM DESCRIPTION

The reference OCP formulation to determine the optimal heat pump thermal power profile $\dot{Q}_{hp}^{*}$ can be presented as follows:

$$\min \int_0^{24h} \frac{\dot{Q}_{hp}(t)}{COP(T_{amb}(t), T_{w,s}(t))} dt \tag{1}$$

Subject to:

- Heat pump model
$$COP(T_{amb}(t), T_{w,s}(t)) = COP_0 + c_1 T_{amb}(t) + c_2 T_{w,s}(t) \tag{2}$$

- Building dynamics
$$C_{w,s}\dot{T}_{w,s} = \dot{m}_w c_{p,w}\left(T_{w,r} - T_{w,s}\right) + \dot{Q}_{hp}\ , \tag{3}$$

$$C_{w,r}\dot{T}_{w,r} = \dot{m}_w c_{p,w}\left(T_{w,s} - T_{w,r}\right) + UA_{wf}\left(T_f - T_{w,r}\right), \tag{4}$$

$$C_f \dot{T}_f = UA_{wf}\left(T_{w,r} - T_f\right) + UA_{fz}\left(T_z - T_f\right),. \tag{5}$$

$$C_z \dot{T}_z = UA_{fz}\left(T_f - T_z\right) + UA_z\left(T_{amb} - T_z\right) \tag{6}$$

- Periodic boundary conditions
$$\overline{T}(0\text{h}) = \overline{T}(24\text{h}) \tag{7}$$

- State constraints
$$\overline{T}_{\min} \le \overline{T}(t) \le \overline{T}_{\max} \tag{8}$$

- Input constraints
$$0 \le \dot{Q}_{hp}(t) \le COP(T_{amb}(t), T_{w,s}(t)).P_{\max} \tag{9}$$

The objective is to minimize the total electricity consumption for the time interval of one day as given by Equation 1. The profile for $\dot{Q}^*_{hp}(t)$ is discretized with a given control time step, yielding a finite number of optimization variables $\dot{Q}^*_{hp}(k)$. In this study, the control time step was chosen half an hour, yielding 48 optimization variables $\dot{Q}^*_{hp}(k)$.

The COP is approximated by a linear fit through the catalogue data of a modulating air-to-water heat pump (Daikin, 2006). Increasing the order of the COP regression was found to have almost no effect on the solution for the optimal $\dot{Q}^*_{hp}$ profile. The linear fit, given by Equation 2, and the catalogue data are represented in Figure 1.

The controller model used to represent the building dynamics, given by a set of differential equations, Equations 3 to 6, is based on the controller model identified for the MPC of a heavy-weight residential building with floor heating (Wimmer 2004). This model, with the return water temperature $T_{w,r}$, the floor temperature T_f and zone temperature T_z as states, has been extended with the supply water temperature $T_{w,s}$, as extra state, in order to be able to calculate the COP.

The state vector $\bar{T} = [T_{w,s}, T_{w,r}, T_f, T_z]$ is constrained by the lower and upper bounds $\bar{T}_{min}$ and $\bar{T}_{max}$ as given by Equation 8. Periodic boundary conditions, given by Equation 7, are put forward to eliminate boundary effects at the start and end of the control horizon. The last inequality constraint, Equation 9, guarantees that the calculated thermal power $\dot{Q}_{hp}^{*}$ is feasible given the maximum heat pump compressor power P_{max}. The parameters for the heat pump model and the building model are tabulated in Table 1. $\bar{T}_{min}$ and $\bar{T}_{max}$ are set to respectively [10°C, 10°C, 15°C, 18°C] and [65°C, 50°C, 30°C, 22°C]. Note that the thermal comfort requirement is translated into a lower and upper bound on T_z, specifying a comfort band between 18°C and 22°C. The ambient air temperature profile is shown in Figure 2, which represents a multisine with a daily mean temperature $T_{amb,m}$ of 0°C. The periodic boundary condition for the state vector $\bar{T}$ corresponds to the steady state values at the given $T_{amb,m}$ and for a zone temperature of 20°C. This way the solution for the $\dot{Q}_{hp}^{*}$ profile matches the steady state solution for the given ambient air temperature profile.

Table 1: Model parameters

Parameter	**Symbol**	**Value**	**Unit**
Constant term in linear fit for COP	COP_0	5.593	-
Coefficient for COP source temperature dependency	c_1	0.0569	1/K
Coefficient for COP supply temperature dependency	c_2	-0.0661	1/K
Maximal compressor power	P_{max}	2500	W
Water mass flow rate	$\dot{m}_w$	0.2660	kg/s
Capacity of supply water	$C_{w,s}$	1.193e5	J/K
Capacity of return water	$C_{w,r}$	5.357e6	J/K
Capacity of floor	C_f	4.550e7	J/K
Capacity of building zone	C_z	2.246e8	J/K
Heat exchange coefficient between water and floor	UA_{wf}	1.160e3	W/K
Heat exchange coefficient between floor and zone	UA_{fz}	6.155e3	W/K
Heat loss coefficient from the zone to the surroundings	κ_b	0.260e3	W/K

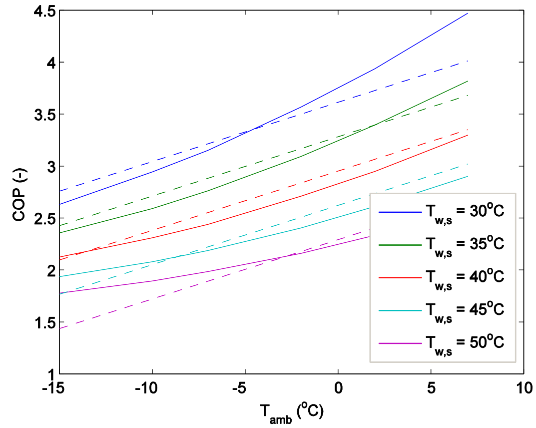


Figure 1: Catalogue data for the COP of a modulating air-to-water heat pump as a function of the source temperature T_{amb} and the water supply temperature $T_{w,s}$ at full load conditions (Daikin, 2006). The linear fit is presented in dotted lines.

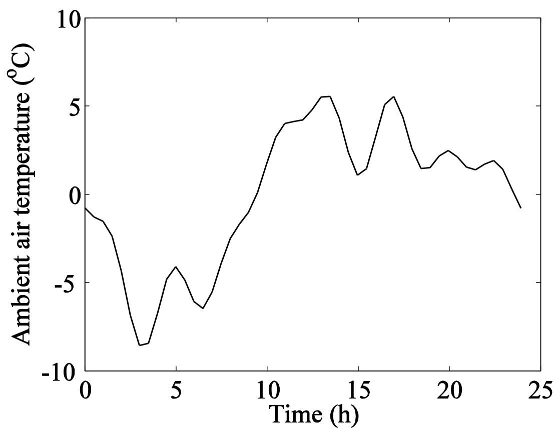


Figure 2: Ambient temperature profile for both optimization and simulation.

4. METHODOLOGY

The question addressed in this study is how the profile found for $\dot{Q}_{hp}^{*}$ and the resulting actual electricity consumption are affected by the OCP formulation. A first choice is the way the COP is incorporated in the cost function. Three approaches are compared:

- COP-formulation *A* takes both the ambient air temperature and the actual supply water temperature into account, as given by equation 2:

  $$COP(T_{amb}(t), T_{w,s}(t)) = COP_0 + c_1 T_{amb}(t) + c_2 T_{w,s}(t).$$

  As $T_{w,s}$ is an optimization variable, depending on the control input, the COP is also an optimization variable.

- COP-formulation B neglects the $T_{w,s}$ dependency of the COP. The COP is estimated a priori based on the predicted ambient air temperature profile and a fixed value for $T_{w,s}$, namely the steady state supply water temperature $T_{w,s,SS}$ for the given daily mean ambient air temperature $T_{amb,m}$ which corresponds to the setpoint supply water temperature which would be determined by a heating curve:

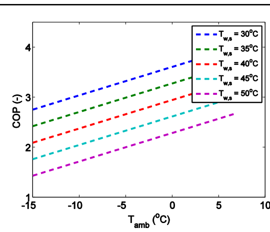
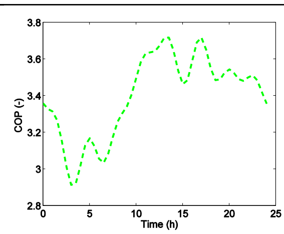
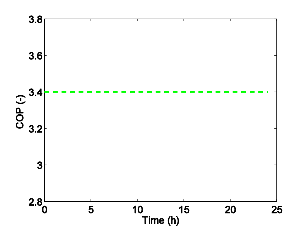
$$COP(T_{amb}(t)) = COP_0 + c_1 T_{amb}(t) + c_2 T_{w,s,SS}. \qquad (10)$$

- COP-formulation C goes a step further in the approximation, and also neglects the hourly variations in ambient air temperature. The COP is approximated by a predefined constant value, calculated as follows:

$$COP_{cte} = COP_0 + c_1 T_{amb,m} + c_2 T_{w,s,SS}. \qquad (11)$$

The choice for the COP-formulation determines whether the optimization problem is convex or not, as summarized in Table 2. A second choice is whether to penalize the predicted electricity consumption by a linear or by a quadratic term.

Table 2: Overview of COP-formulations compared

COP formulation A	COP formulation B	COP formulation C
$COP(T_{amb}, T_{w,s})$	$COP(T_{amb})$	COP_{cte}
 Optimization variable (Equation 2)	 Predefined profile based on T_{amb}-profile (Figure 2)	 Predefined constant
Non-convex problem	Convex problem	Convex problem

Hereby, two important questions have to be addressed. The first one is which discretization time step is allowed when solving the ordinary differential equations representing the building dynamics, i.e. Equations 3 to 6. For the convex approximations, the discretization time step can be chosen equal to the control time step without loss of accuracy. However, for the non-convex problem this does not necessarily hold. If the discretization time step is equal to the control time step, i.e. 0.5 hour, the supply water temperature, and by consequence the COP, are assumed constant during this time interval, which is not the case.

To quantify the impact of the time discretization on the optimization result, the OCP problems are first solved with the Automatic Control and Dynamic Optimization (ACADO)-solver (Houska et al., 2010) with a variable time step such that the discretization error remains below a given tolerance value. This allows a correct comparison of the three COP-formulations, which will be discussed in Section 5.1. In Section 5.2 then, the discretization time step is taken equal to the control time step. This allows us to quantify the impact of the discretization error for the non-convex formulation. It is also verified that the results for the convex formulations are not affected.

The second question to be addressed is whether the solution for the non-convex formulation has converged to the global optimum. To this end, a branch-and-bound strategy implemented in the BMIBNB-solver (Lawler et al., 2006) is used to find a global optimal solution of the non-convex problem and the result is compared to the (local) solution obtained by ACADO. As the BMIBNB-solver requires a discrete-time representation of the dynamics, the comparison is only done for the case with the discretization time step of 0.5 hour, discussed in Section 5.2. If the comparison shows that the ACADO solution lies close to the global one for the discrete-time model, it is reasonable to assume that the local solution found by ACADO for the continuous-time model (implicitly using a variable time step for numerical solution) is close to the global optimum as well. More details about the solvers used are given in Section 4.3.

As will be discussed in Section 5.1, the original non-convex problem gives rise to a smooth profile for $\dot{Q}_{hp}^{*}$ whereas the convex approximations tend to concentrate heat pump operation at certain time points, which results in higher power peaks. As this is found to increase the electricity consumption, Section 5.3 investigates the impact of penalizing power peaks by adding a quadratic term to the linear term in the cost function. In section 5.4 only the quadratic term is retained in the cost function, which corresponds to the approach put forward by Wimmer (2004) and Bianchi (2006). Also here the effect of the discretization time step Δt_s is investigated.

The different OCP formulations studied are schematically represented in Figure 3. They differ with respect to (1) the discretization scheme used for solving the differential equations representing the building dynamics, namely a variable time step in order to remain below a user-defined discretization error or Integration Tolerance *Int.Tol* of 10^{-9} versus a fixed discretization time step Δt_s of 0.5h, (2) the COP-formulation, namely formulation A versus B or C and (3) the objective function, namely a linear term versus a linear-quadratic term or a quadratic term only.

Each formulation gives rise to an optimal profile for $\dot{Q}_{hp}^{*}$(t). To compare the performance achieved by the different OCP formulations, the calculated profiles for

$\dot{Q}_{hp}{}^{*}$(t) are applied to a simulator model and the actual electricity power P_{hp} is calculated. As the focus of this study is to investigate the impact of the choice of the cost function, model mismatch on building level has been avoided by choosing the simulator model equal to the model used for optimization. This way, the temperature profiles will match the ones predicted by optimization and are therefore all feasible. For the same reason, the COP in the simulator model is represented by the linear fit used in the reference formulation A, i.e. Equation 2, and a perfect ambient air temperature prediction is assumed.

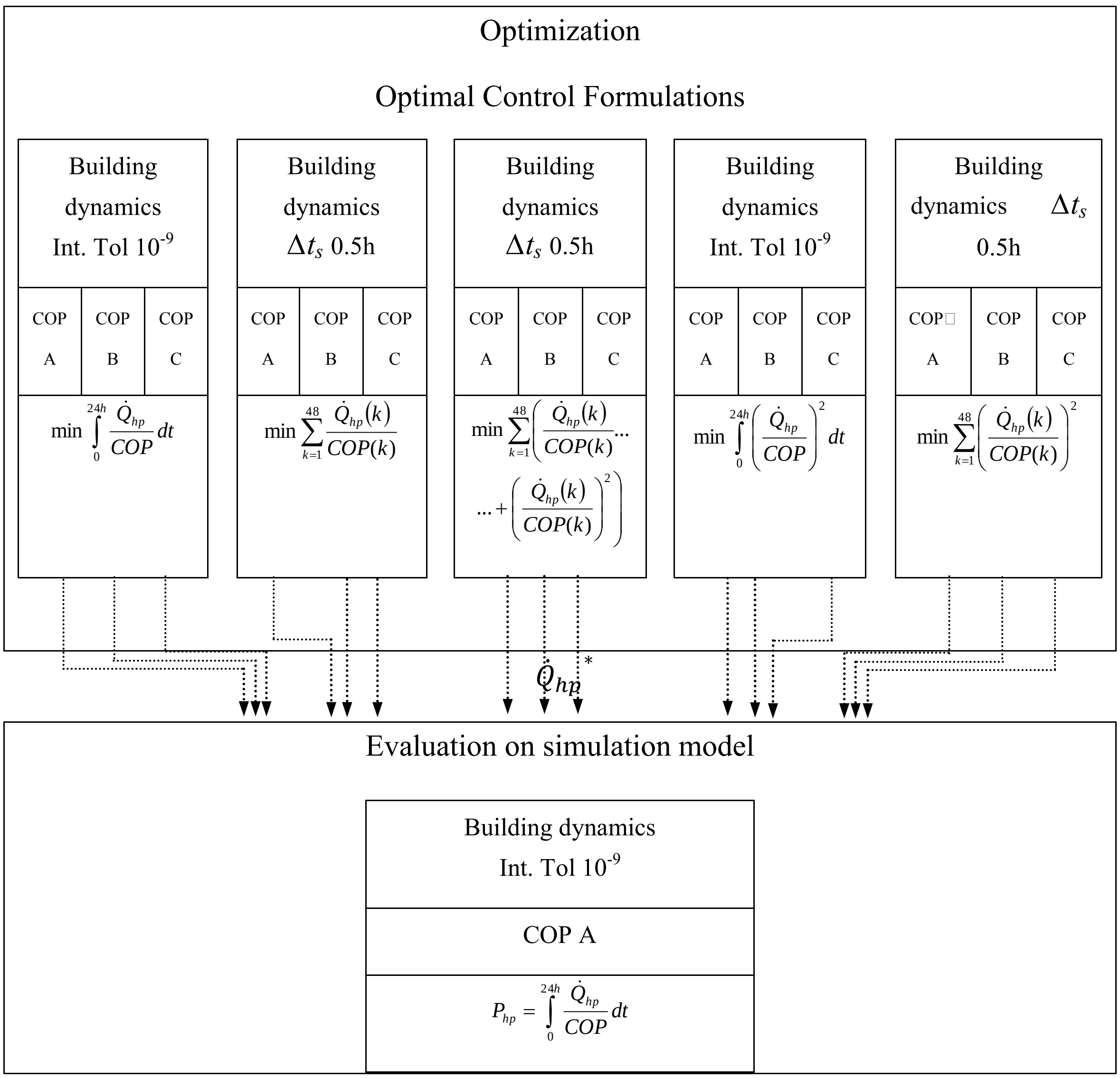


Figure 3: Schematic representation of the optimization problem formulations investigated (above) and the evaluation of the actual electricity consumption P_{hp} achieved with the calculated control profiles $\dot{\boldsymbol{Q}}_{\boldsymbol{hp}}^{*}$. Int. Tol stands for the integrator tolerance which represents the discretization error.

4.3 Optimization software

Two different optimization tools have been used, namely the C++ toolbox ACADO (Houska et al., 2010) and the Yalmip toolbox (Löfberg, 2004). The specifications of these tools are listed in Table 3.

Table 3: Overview of the optimization software used

	Problem type			Δt_s	Strengths
	Linear system + linear objective (LP)	Linear system + quadratic objective (QP)	Bilinear system + linear/quadratic objective (NLP)		
ACADO	SQP	SQP	SQP (local optimum)	variable or fixed	NLP & ODE
Yalmip	CPLEX	CPLEX	BMIBNB (*) (global optimum)	fixed	MILP, MIQP

(*) BMIBNB with CPLEX as lower solver and fmincon as upper solver.

Both optimization toolboxes enable to solve linear problems (LP), quadratic problems (QP) and nonlinear problems (NLP) of the bilinear type, i.e. the nonlinearity arises from the multiplication of two optimization variables. Within ACADO, they are solved using a Sequential Quadratic Programming solver (SQP), whereas in Yalmip different solvers can be called, depending on the problem type. In this study, the CPLEX solver (ILOG, 2010) was called when solving LP's and QP's. The BMIBNB-solver (Lawler et al., 2006) uses CPLEX and fmincon (MathWorks, 2010) to compute lower, respectively upper bounds for solving NLP's. The main difference between the solution obtained with ACADO and Yalmip for the bilinear case, is that ACADO gives a local optimum whereas Yalmip gives a global optimum.

One of the strengths of ACADO is the ability to solve general nonlinear optimal control problems with single or multiple shooting techniques. This enables one to solve the differential state equations with high integrator tolerance. This is done by internally varying the integration time step Δt_s. In Yalmip, as with most optimization software, a discrete-time representation of the dynamics is required. The user determines the discretization time step Δt_s. The strength of Yalmip is the ability to compute global solutions of certain non-convex optimization problems like mixed integer linear problems (MILP), mixed integer quadratic problems MIQP and bilinear problems.

For the continuous-time representation of the building dynamics, the differential state equations, Equations 3 to 6, are integrated with a relative error of less than 10^{-9}. For the discrete-time representation, a zero-order hold discretization scheme with a time step Δt_s equal to the control time step, i.e., half an hour, is applied. The formulations with continuous-time representation can only be solved with ACADO, whereas the ones with discrete-time representation can be handled by both ACADO and Yalmip. As previously mentioned, the motivation for solving the discrete-time formulations both in ACADO and Yalmip is to verify the local solution obtained by ACADO using the global BMIBNB solver in Yalmip.

5. RESULTS AND DISCUSSION

Section 5.1 discusses the impact of the COP formulation using a continuous-time representation of the system dynamics. Section 5.2 quantifies the impact of discretization errors when using a discretization time step for the building dynamics equal to the control time step. Section 5.3 discusses the impact of adding an extra term (the square of the estimated electricity consumption) to the cost function. Finally, Section 5.4 deals with the case where the cost function consists of the quadratic term only.

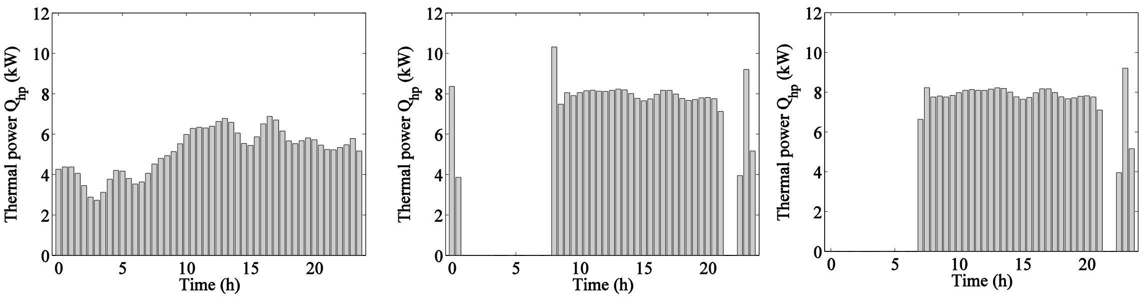
5.1 Effect of COP-formulation with continuous-time system model

The optimal heat pump power profiles $\dot{Q}_{hp}^{*}$ calculated by the three OCP formulations are depicted in Figure 4. The corresponding temperature profiles are shown in Figure 5. The actual COP, calculated based on the profiles for $T_{w,s}$ and T_{amb}, is compared to the COP assumed by the optimization in Figure 6. The values for the predicted and actual electricity consumption are tabulated in the first column of Table 4. Following observations are made:

- **COP-formulation *A*** The reference OCP with the COP($T_{w,s}$, T_{amb})-formulation yields an almost continuous $\dot{Q}_{hp}^{*}$ profile, see Figure 4.a. Comparison of this $\dot{Q}_{hp}^{*}$ -profile with the T_{amb} -profile depicted in Figure 2 shows a strong correlation between both. The continuous heat pump operation results in a smooth profile for both the supply water temperature and the COP, as can be seen in respectively Figure 5.a and 6.a. As the same model for the COP and the building was used for both evaluation and optimization, the actual and predicted COP profiles coincide. This way, the actual electricity consumption is correctly predicted and effectively minimized. The optimum found, further used as reference value, is 36.17 kWh.

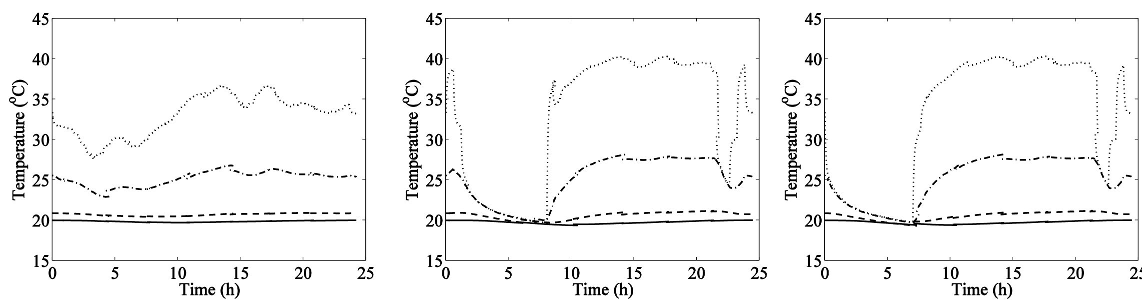
- **COP-formulation *B*** The OCP with the COP(T_{amb})-formulation concentrates the total heat production during the day, see Figure 4.b, as at that time the ambient temperature and thus the predicted COP are higher. Consequently, $T_{w,s}$ rises in the afternoon, causing a drop in the actual COP. The effect of $T_{w,s}$ on the COP is not taken into account during optimization, as the COP profile is predicted a priori and does not depend on the actual value of $T_{w,s}$. This way the assumed and actual COP differ substantially, as shown in Figure 6.b, and the resulting electricity consumption is higher than predicted. Compared to the reference solution, the consumption is increased by 7%.

- **COP-formulation *C*** The OCP formulation with a constant COP yields similar results as with formulation B. Despite the difference in the COP profile used for optimization (compare Figure 6.b with 6.c), the calculated $\dot{Q}_{hp}{}^{*}$- profiles (see Figures 5.b and 5.c) and resulting electricity cost are almost the same.



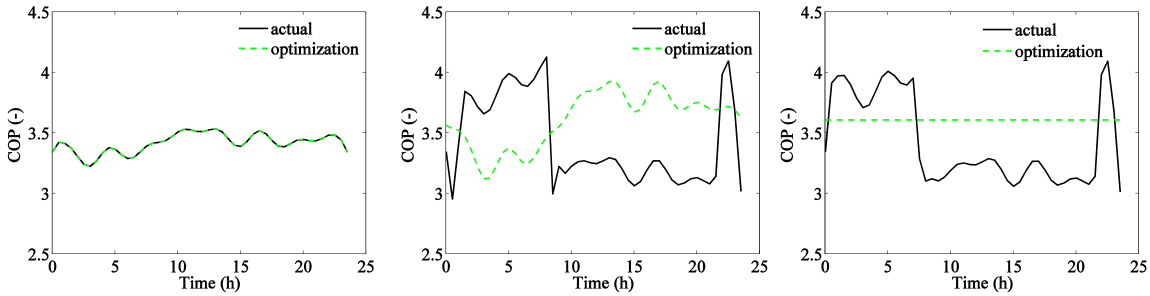
(a) COP-formulation *A* (b) COP-formulation *B* (c) COP-formulation *C*

*Figure **4**: Optimal thermal power profiles $\dot{Q}_{hp}{}^{*}$ for the COP-formulations A, B and C with a continuous-time representation of the system dynamics.*



(a) COP-formulation *A* (b) COP-formulation *B* (c) COP-formulation *C*

*Figure **5:** Temperature profiles $T_{w,s}{}^{*}$ (.), $T_{w,r}{}^{*}$(.-) ,$T_f{}^{*}$ (--) and $T_z{}^{*}$ (-) for the COP-formulations A, B and C with a continuous-time representation of the system dynamics.*



(a) COP-formulation *A* (b) COP-formulation *B* (c) COP-formulation *C*

Figure 6: Comparison between the actual COP (-) and the COP found/assumed by the OCP (--) for the COP-formulations A, B and C with a continuous-time representation of the system dynamics.

Table 4: Overview of the predicted (grey) and the actual electricity consumption (black) for the studied cases (kWh) and comparison to the reference solution (bold).

Section	5.1	5.2		5.3	5.4	
Objective function	$\min \int_0^{24h} \frac{\dot{Q}_{hp}}{COP} dt$	$\min \sum_{k=1}^{48} \frac{\dot{Q}_{hp}(k)}{COP(k)}$		$\min \sum_{k=1}^{48} \left(\frac{\dot{Q}_{hp}(k)}{COP(k)} \ldots + \left(\frac{\dot{Q}_{hp}(k)}{COP(k)} \right)^2 \right)$	$\min \int_0^{24h} \left(\frac{\dot{Q}_{hp}}{COP} \right)^2 d$	$\min \sum_{k=1}^{48} \left(\frac{\dot{Q}_{hp}(k)}{COP(k)} \right)^2$
System dynamics	Int.tol. 10^{-9}	$\Delta t_s = 0.5h$		$\Delta t_s = 0.5h$	Int.tol. 10^{-9}	$\Delta t_s = 0.5h$
Solver	ACADO	ACADO	Yalmip	Yalmip	ACADO	Yalmip
Formulation A *COP(T_{ws},T_{amb})*	36.16 36.17 **REF**	33.92 40.00 **+10%**	32.54 40.42 **+12%**	36.27 36.27 **+0.3%**	36.30 36.30 **+0.4%**	36.27 36.27 **+0.3%**
Formulation B *COP(T_{amb})*	34.95 38.72 **+7.1%**	34.95 38.73 **+7.1%**	34.95 38.73 **+7.1%**	36.22 36.19 **+0.1%**	36.30 36.20 **+0.1%**	36.29 36.21 **+0.1%**
Formulation C *COP_{cte}*	36.22 38.76 **+7.2%**	36.22 38.76 **+7.2%**	36.22 38.76 **+7.2%**	36.53 36.52 **+1.0%**	36.53 36.54 **+1.0%**	36.53 36.53 **+1.0%**

The results for the considered case show that the actual electricity consumption obtained with the convex approximations is more than 7% higher than with the original formulation. Figure 7, depicting the zone temperature profiles resulting from the different formulations, shows that this energy reduction is not achieved at the expense of thermal comfort. On the contrary, the reference formulation not only yields the lowest electricity consumption but also the highest thermal comfort.

5.2 Effect of the COP-formulation with discrete-time system model

In this section it is investigated what happens to the calculated optimal profile if the differential equations representing the building dynamics, Equations 3 to 6, are discretized with a time step Δt_s equal to the control time step, i.e. half an hour.

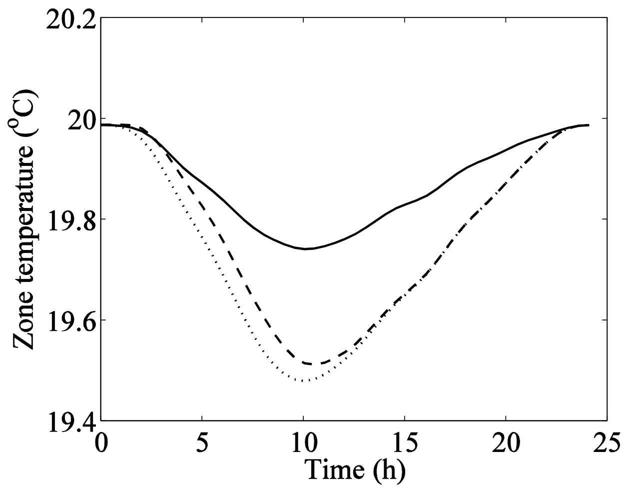


Figure 7: Comparison of the zone temperature profiles T_z^* for the different COP formulations: A (-),B (--) and C(.)

The time constants in the building model range from 3 minutes to 10 days, with the fastest dynamics being related to the supply water temperature and the slowest dynamics to the building thermal mass. The discretization error on $T_{w,s}$ will thus be significant. It is expected that this will affect the results found with the COP($T_{w,s}$,T_{amb}) formulation as $T_{w,s}$, and by consequence also the COP, are assumed constant during one time step instead of continuously varying. It is expected that the optimal control profiles found with the predicted and constant COP will not be affected as their cost function does not depend on $T_{w,s}$. As mentioned in Section 3, the discrete-time representation enables also to find out whether the local solution obtained for the non-convex problem by the SQP-solver in ACADO lies close to the global solution found by the BMIBNB –solver called by Yalmip. The results for the predicted and actual cost with the discrete- time formulation obtained by the solvers in ACADO and Yalmip are tabulated in respectively the second and third column of Table 4.

From these values, following conclusions are drawn:

- **COP-formulation *A*** A first observation is that ACADO and Yalmip yield slightly different results for this bilinear problem. Yalmip yields a solution with a predicted cost of 32.54 kWh, which is 4% lower than the predicted cost of 33.92 kWh found by ACADO. Second, the actual electricity consumption is 10% higher than the reference solution, which is even higher than the solutions found using the approximated COP-formulations A and B. Third, the actual electricity consumption significantly differs from the one predicted by the optimization. These results are further analyzed below.

- **COP-formulation *B* and C** The conclusions are opposed to the former ones. First, ACADO and Yalmip do yield identical results. This was expected as both problems are Linear Programming problems which have one single optimal value. Second, the values for the actual electricity consumption are almost identical to the values found with the continuous-time representation. This was also expected, as the COP in this formulation does not depend on the supply water temperature $T_{w,s}$ and is therefore not affected by discretization errors on this state.

To give more insights in the unsatisfactory results obtained with the $T_{w,s}$ -dependent COP model combined with the discrete-time model using a time step of 0.5 h, the control strategy from this formulation is discussed more in detail. Figure 8.a shows the proposed $\dot{Q}_{hp}^{*}$ -profile, which cycles between maximum and minimum power. This on-off - switching causes the supply water temperature to fluctuate with an amplitude of almost 15°C in one time step, as shown in Figure 8.b. Consequently, the COP assumed by the optimization, shown by the upper bars in Figure 9.a, also fluctuates significantly. To understand why this is effectively the optimal solution for the discrete-time optimization problem, Figure 9.a plots $\dot{Q}_{hp}^{*}$ and the predicted COP together. It is observed that the production of thermal power is concentrated in the time steps with a high predicted COP. Concentrating power, however, results in an increase of the supply temperature during the control time step considered. This negative feedback on the COP is not captured in this case, as the supply temperature – and thus the COP - is assumed constant during the control time step due to the discretization method used. Figure 8.b shows the predicted piece-wise constant temperature profiles and 8.c shows the actual, continuously varying ones. Figure 9.b shows how the discretization error on the supply water temperature causes a significant discrepancy between on the one hand the piecewise varying COP assumed by the optimization and on the other hand the actual, continuously varying one.

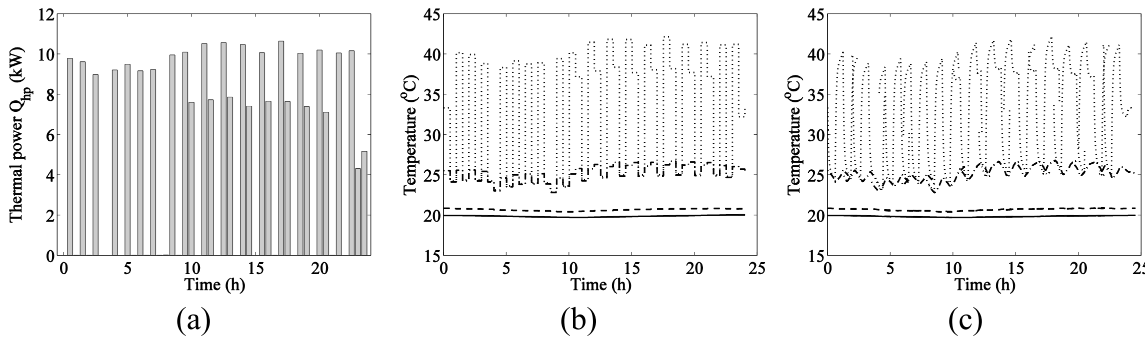


Figure 8: (a) Optimal thermal power profile $\dot{\boldsymbol{Q}}_{hp}{}^{*}$ found with the OCP with COP-formulation A and the discrete-time system model (b) Temperature profile predicted by optimization, (c) Actual resulting temperature profile.

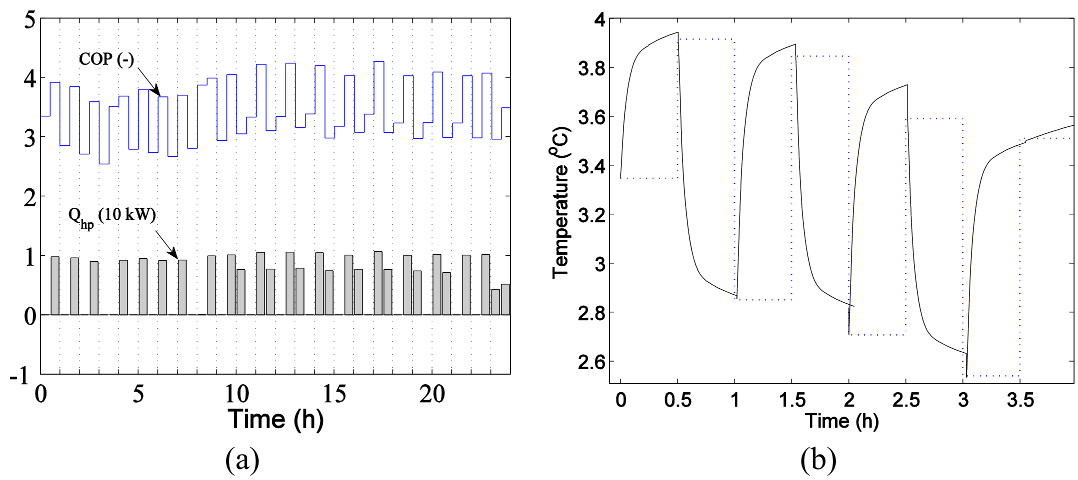


Figure 9: (a) Optimal thermal power profile $\dot{\boldsymbol{Q}}_{hp}{}^{*}$ (lower bars, scaled) and predicted COP (upper bars) found with the OCP with COP-formulation A and the discrete-time system model. (b) COP predicted by the optimization using a discrete-time model (--) versus the actual COP when applying the $\dot{\boldsymbol{Q}}_{hp}{}^{*}$ on the simulation model (-).

On-off-switching heat pump operation is suboptimal when applied to the actual system, which is clear from the values for the actual electricity consumption, tabulated in the third column of Table 4. The actual electricity consumption with Formulation *A*, namely 40 kWh, is 3% higher than the values obtained with Formulation *B* and *C*.

The above mentioned observations suggest that power peaks, or concentration of heat production in general, has a negative impact on the electricity consumption. This hypothesis is confirmed by the fact that the reference optimal profile, depicted in Figure 5.a, is indeed relatively smooth. Therefore it is investigated how the optimal profile and the resulting actual electricity consumption change if power peaks are penalized.

5.3 Effect of adding a quadratic term in P_{hp} to the cost function

To reduce the power peaks in the solution of the discrete-time formulation with COP formulation *A*, a quadratic term is added to the cost function:

$$\min \sum_{k=0}^{48} \left(\frac{\dot{Q}_{hp}(\mathrm{k})}{COP(\mathrm{k})} + K \left(\frac{\dot{Q}_{hp}(\mathrm{k})}{COP(\mathrm{k})} \right)^2 \right) \tag{12}$$

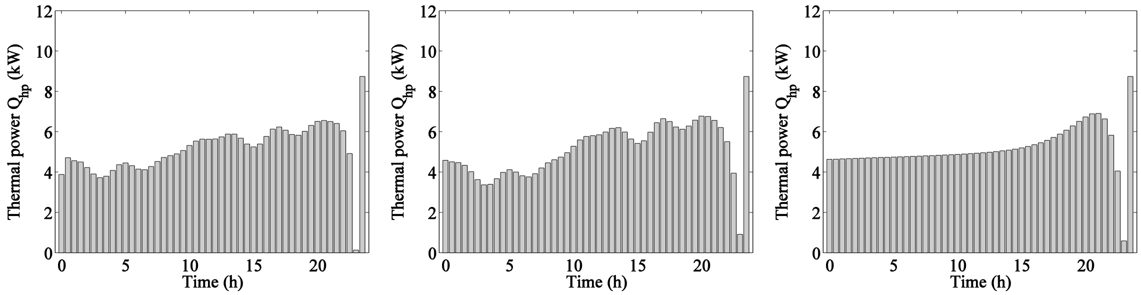
The scaling factor *K* is chosen such that the linear and the quadratic term have the same order of magnitude. Although the incentive for investigating the impact of this cost function is triggered by the unsatisfactory results obtained with COP-formulation *A* in the discrete-time formulation, this cost function will also be studied in combination of COP-formulations *B* and *C*.

The optimal control profiles found with this cost function are shown in Figure 10. They are characterized by an almost continuous heat pump operation resembling the optimal profile found for the reference OCP discussed in Section 5.1. The values for the actual electricity consumption, obtained when applying these input profiles to the simulation model, are presented in the fourth column of Table 4. Comparison with the results discussed in section 5.2 reveals that the actual costs are reduced by almost 10%, just by adding the quadratic term in the cost function. What is even more striking is that the actual cost resulting from the formulation with a constant COP, formulation C, is only 1% higher than the reference solution discussed in section 5.1. With the predefined COP-profile, formulation B, the actual cost is even only 0.1% higher, which is quasi identical to the reference solution.

- **COP-formulation *A*** The actual cost obtained with the discrete-time formulation (discretization time step of 0.5 h) is significantly reduced by adding a quadratic term to the cost function. The cycling behaviour is eliminated and the electricity consumption decreases from 40.0 kWh for the solution discussed in Section 5.2, to 36.30 kWh. This value is only 0.4% higher than for the reference solution.

- **COP-formulation *B*** The actual cost obtained with the COP(T_{amb})-formulation is only 0.1% higher than for the reference solution. Unlike the power profile depicted in Figure 4.b, the heat pump now has a continuous operation and lower peak power. Just as for the reference solution, there is a strong correlation between $\dot{Q}_{hp}^{\,*}$ and the ambient air temperature profile.

- **COP-formulation *C*** Contrary to the power profile resulting from formulation B, the formulation with the constant COP, depicted in Figure 10.c, yields an almost flat profile for $\dot{Q}_{hp}^{\,*}$. There is no incentive to shift heat production to the afternoon as the COP is assumed constant. The actual cost is in this case 1.0% higher than the reference solution.



(a) COP-formulation A *(b) COP-formulation B* *(c) COP-formulation C*

Figure 10: Optimal thermal power profiles $\dot{\boldsymbol{Q}}_{\boldsymbol{hp}}^{*}$ found with the OCP for the three COP-formulations with a linear-quadratic cost and the discrete-time system model.

5.4 Effect of a purely quadratic term in P_{hp} in the cost function

Given the satisfactory results obtained when adding a quadratic term to the cost function, the question arises whether the linear term is actually needed. The OCP problems with the three COP-formulations are now solved with only the square of the predicted electricity consumption in the objective function:

$$\min \int_{0}^{24h} \left(\frac{\dot{Q}_{hp}(\mathrm{t})}{COP(\mathrm{t})} \right)^2 dt \qquad (13)$$

The results are tabulated in the fifth and sixth column of Table 4 for respectively the continuous-time and the discrete-time formulation. Again, it is observed that the results obtained with the OCP's with COP-formulations *B* and *C* are not affected by the time-discretization of the system dynamics, contrary to the OCP with COP-formulation *A*. Following observations are made based on the tabulated values in Table 4:

- **COP-formulation *A*** The optimal control profile obtained with the quadratic cost and the continuous-time model in ACADO, yields a slightly higher electricity consumption than with the linear cost function, discussed in Section 5.1. However, if the discrete-time model is used, the resulting consumption with this quadratic cost function, 36.27 kWh, is much lower than the 40.0 kWh obtained with the linear cost function discussed in Section 5.2 and equal to the linear-quadratic cost function discussed in Section 5.3.

- **COP-formulation *B* and C** The electricity consumption is approximately the same as with the linear-quadratic cost function discussed in Section 5.3.

Comparing the values for the electricity consumption resulting from these OCP's which minimize the square of the predicted electricity consumption, respectively 36.30, 36.20 and 36.54 kWh for COP-formulations A, B and C with the value found by the reference formulation, namely 36.17 kWh, shows that the results obtained are close to optimal. A second important observation is that the discrepancy between the predicted and the actual electricity consumption is very small. This reflects the fact that the optimal profile determined for the supply water temperature lies close to the steady state supply water temperature, $T_{ws,ss}$, used for the calculation of the predefined COP-profile.

Note that with COP-formulation C, where the COP is a constant value, the OCP boils down to minimizing the square of the thermal power $\dot{Q}_{hp}^{*}$. The actual electricity consumption resulting from this formulation is only 1% higher than for the reference formulation. The computational gains, however, are large as the problem is convex instead of nonlinear. Moreover, the discretization time step to simulate the building dynamics can be chosen larger than in the non-convex formulation with the T_{ws}-dependent COP as the latter is sensitive to discretization errors on this state. Knowledge about the heat pump characteristics is however still required in order to correctly constraint the thermal power in Equation 9.

6. SUMMARY AND CONCLUSION

In this study it was investigated to what extent the choice of the objective function in the OCP formulation for a modulating air-to-water heat pump, influences the calculated thermal power profile $\dot{Q}_{hp}^{*}$ and the related actual electricity consumption. More specifically, it was investigated to what extent convex approximations, often encountered in practice, approach the result of the original nonlinear problem resulting from the supply water temperature dependency of the COP.

A first comparison is performed using a continuous-time representation of the building dynamics and the predicted electricity consumption as cost criterion. The reference (non-convex) formulation yields a smooth profile for the heat pump thermal power which is strongly correlated to the ambient air temperature profile, whereas the formulations with predetermined COP concentrate the heat pump operation in the time periods with high ambient temperature, neglecting the negative impact of the corresponding supply temperature rise on the actual COP. The actual electricity consumption with the convex approximations is found to be 7% higher compared to the non-convex reference formulation.

A second comparison shows that these conclusions do not hold when the building dynamics are discretized using a time step larger than the time constant of the supply water heating process. The formulations with a predefined profile for the COP are not affected by the discretization errors on the supply water temperature. In the formulation with the reference COP-model, however, the discretization error on the supply water temperature is propagated in the prediction of the COP and gives rise to a cycling behaviour. This causes a significant increase in the actual electricity consumption compared to the case with the numerically accurate time discretization of the system dynamics.

This cycling problem was tackled in a third comparison, by adding a quadratic term to the cost function to penalize power peaks. The resulting power profile is found to be very similar to the reference solution. Moreover, the formulations with predefined COP are close to optimal, even the formulation with a constant COP.

Finally, the comparison was performed with the square of the predicted electricity consumption as minimization criterion. The quadratic cost function using a predefined COP profile based on the predicted ambient air temperature and a constant supply water temperature, yields an almost identical cost as the reference solution. The actual electricity cost is found to be only 0.1% higher than the optimal solution, which is negligible. The quadratic cost function with a constant COP, which is equivalent to minimizing the square of the heat demand, is found to be close to optimal as well, only 1% higher.

To summarize: the control profile obtained with the convex formulation minimizing the square of the heat demand is almost identical to the result of the original, non-convex formulation which minimizes the electricity consumption, taking both the ambient air temperature and supply water temperature dependency of the COP into account. The actual electricity consumption is only 1% higher than for the reference formulation. If ambient air temperature predictions are used to predict the COP profile, assuming a fixed value for the supply water temperature, the convex formulation minimizing the square of the predicted electricity demand yields even better results, up to 0.1% close to the optimal solution. These findings are beneficial for optimal control purposes, as with the convex approximations the discretization time step can be chosen equal to the control time step and convergence to the global optimum is guaranteed. Moreover, the considered convex problems can be solved much faster than considered non-convex ones.

Future work will comprise validation of these results with more detailed models for the modulating heat pump and the building in the evaluation step, to take the effect of model mismatch into account. This includes amongst others the study of the influence of the part load performance of the heat pump on the optimal operation profile. As part load performance of modulating air-source heat pumps is often superior to the full load performance, it is expected that the tendency will be to further reduce power peaks. Future work will also comprise verification of the generic character of the conclusions, more specifically for the case with time-varying electricity price and the monetary cost as minimization criterion, and for the case with backup system.

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