Spatiotemporal Gradient Analysis of Differential Microphone Arrays*

MIHAILO KOLUNDŽIJA, CHRISTOF FALLER, AND MARTIN VETTERLI**
(mihailo.kolundzija@epfl.ch) (christof.faller@illusonic.com) (martin.vetterli@epfl.ch)

School of Computer and Communications Sciences, Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

The literature on gradient and differential microphone arrays makes a distinction between the two types, but it nevertheless shows how both types can be used to obtain the same directional responses. A more theoretically sound rationale for using delays in differential microphone arrays has not yet been given. A gradient analysis of the sound field viewed as a spatiotemporal phenomenon is presented, giving a theoretical interpretation of the working principles of gradient and differential microphone arrays. It is shown that both types of microphone arrays can be viewed as devices for approximately measuring spatiotemporal derivatives of the sound field. Furthermore the design of high-order differential microphone arrays using the aforementioned spatiotemporal gradient analysis is discussed.

0 INTRODUCTION

The concept of gradient microphones dates back to the middle of the last century and the works of Olson [1], [2], who described gradient microphones as arrays of pressure-sensing elements whose signals are combined in a similar way as gradients are approximated with finite differences. Gradient microphones have been used in a variety of applications, such as surround and spot recording (for example, see [3], [4]), and as particle velocity and sound intensity measurements called p-p probes [5]–[8].

The works of Olson [1], [2] also showed how the combination of responses from gradient microphones of different order can be equivalently obtained by combining signals from multiple pressure microphones with appropriately chosen delays. This more flexible family of microphone arrays found applications in hands-free communication [9] and hearing-aid devices [10], [11].

Later microphone arrays of the latter type, which use delay elements, have been termed differential microphone arrays, and they were more extensively analyzed in the works of Elko at the start of the last decade [12], [13]. However, to the best of our knowledge the relation between these two microphone array types, and the rationale behind the idea of combining the delayed microphone signals in differential microphone arrays, has not been presented yet.

This paper presents a slightly different analysis of the sound pressure field, which is viewed as both a spatial and a temporal phenomenon, that is, as a multivariate function of spatial location and time. This analysis then exposes the operations of taking gradients and directional derivatives of the sound pressure field as combinations of its spatial and temporal derivatives. This new perspective gives a clear interpretation of gradient and differential microphones and microphone arrays—the former as devices used for approximately measuring only spatial derivatives, and the latter as devices used for approximately measuring spatiotemporal derivatives of the sound pressure field. In other words, it shows their equivalence.

This paper is organized as follows. Section 1 gives a theoretical analysis of spatiotemporal derivatives of the sound pressure field when the latter is viewed as both a spatial and a temporal phenomenon, that is, as a multivariate function of space and time. Section 2 shows a number of practical differential microphone arrays which follow from the theoretical analysis in Section 1. Conclusions are given in Section 3.

1 THEORETICAL ANALYSIS OF SPATIOTEMPORAL DERIVATIVES OF THE SOUND FIELD

1.1 Spatial Derivatives of Sound Pressure Field

The sound pressure at a position defined by vector $\boldsymbol{r}$ of a plane wave propagating with wave vector $\boldsymbol{k}$ is given

*Presented at the 126th Convention of the Audio Engineering Society, Munich, Germany, 2009 May 7–10; revised 2010 November 2.

**Also with the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley, Berkeley, CA.

by[1]

$$p(\boldsymbol{r}, t) = A e^{j(\omega t + \boldsymbol{k} \cdot \boldsymbol{r})} \quad (1)$$

where ω is the temporal frequency of the plane wave.

The spatial derivative of the sound pressure field along the direction defined by vector $\boldsymbol{u}$ quantifies the rate of change of the sound pressure in that direction. It is given by the projection of the spatial gradient of the sound pressure field onto the vector $\boldsymbol{u}$,

$$\begin{aligned} D_u(\boldsymbol{r}, t) &= \nabla p(\boldsymbol{r}, t) \cdot \boldsymbol{u} \\ &= j p(\boldsymbol{r}, t)(\boldsymbol{k} \cdot \boldsymbol{u}) \\ &= jk \cos\theta\, p(\boldsymbol{r}, t) \end{aligned} \quad (2)$$

where θ is the angle between vectors $\boldsymbol{k}$ and $\boldsymbol{u}$.

Generalizing to the *n*th-order spatial derivative along the direction defined by vector $\boldsymbol{u}$ gives

$$D^n_{\boldsymbol{u}} p(\boldsymbol{r}, t) = (jk)^n (\cos\theta)^n p(\boldsymbol{r}, t). \quad (3)$$

The *n*th-order spatial derivative of the sound pressure field composed of a plane wave has a bidirectional characteristic whose shape has the form

$$d^n(\theta) = A(\omega)(\cos\theta)^n \quad (4)$$

where $A(\omega) = (jk)^n$ is a complex frequency-dependent gain, and θ is the angle between the direction along which the spatial derivative is taken and the direction of propagation of a plane wave. The directional characteristics of a plane wave's spatial derivatives of different orders *n* along the positive direction *x* are shown in Fig. 1.[2]

Also the plane wave's *n*th-order spatial derivative has at all angles a high-pass magnitude frequency characteristic proportional to $(jk)^n$ or, equivalently, $(j\omega/c)^n$, as shown in Fig. 2.

1.2 Spatiotemporal Derivatives of Sound Pressure Field

Without loss of generality, the analysis will be given in a two-dimensional plane, such that the pressure field can be written as a function $p(x, y, t)$ of two spatial coordinates *x* and *y*, and one temporal coordinate *t*. Since such a pressure field is a function of three independent coordinates, its gradient is given by

$$\nabla p(x, y, t) = \left[\frac{\partial p}{\partial x}\ \frac{\partial p}{\partial y}\ \frac{\partial p}{\partial t}\right]^T. \quad (5)$$

The gradient of the sound pressure field, as defined in Eq. (5), shows how it—and subsequently all its projections—can be measured in a point. In particular one needs to measure the spatial derivatives of the sound field along directions *x* and *y*,[3] and its temporal derivative in the given point. This is the principle of p-p sound intensity probes (see, for example, [5], [6], [8]), which can measure the sound intensity vector in two and three dimensions, and also some sound field microphones [7].

The following analysis for a sound field composed of a plane wave with temporal frequency ω and wave vector $\boldsymbol{k} = [k\cos\theta \quad k\sin\theta]^T$ is given by the expression

$$p(x, y, t) = A e^{j(\omega t + kx\cos\theta - ky\sin\theta)} \quad (6)$$

where the angle θ defines the direction of propagation of a plane wave.

1.2.1 First-Order Spatiotemporal Derivatives of Sound Pressure Field

The gradient of the sound pressure field given in Eq. (6) has the form

$$\nabla p(x, y, t) = jkp(x, y, t)[\cos\theta \quad \sin\theta \quad c]^T \quad (7)$$

where *c* is the speed of sound propagation ($k = \omega/c$).

Let the unit vector $\boldsymbol{u}$, onto which the pressure gradient is projected, be defined as

$$\boldsymbol{u} = [\rho_u \cos\phi_u \quad \rho_u \sin\phi_u \quad u_t]^T \quad (8)$$

where ρ_u ($\rho_u \in [0,1]$) and ϕ_u ($\phi_u \in [0,2\pi]$) define the spatial coordinates, and u_t ($u_t \in [0,1]$) is the temporal coordinate of the vector $\boldsymbol{u}$. Note that since the vector $\boldsymbol{u}$ has a unit norm, $\rho_u^2 + u_t^2 = 1$, the ratio ρ_u/u_t gives the relation between its spatial part and its temporal part.

The derivative of the sound pressure field along the spatiotemporal direction defined by the vector $\boldsymbol{u}$ is given by

$$\begin{aligned} D_u p(x, y, t) &= \nabla p(x, y, t) \cdot \boldsymbol{u} \\ &= jkp(x, y, t)[\rho_u \cos(\theta - \phi_u) + cu_t]. \end{aligned} \quad (9)$$

The directional characteristic of a spatiotemporal derivative of a plane wave with wave vector $\boldsymbol{k}$ and temporal frequency ω is a combination of a first-order (bidirectional) directional characteristic of its spatial gradient, given by the term $\rho_n \cos(\theta - \phi_u)$, and a zero-order (omnidirectional) directional characteristic of a temporal differentiator, given by the term cu_t.

The relative contributions of the two derivatives—spatial and temporal—given by the ratio ρ_u/u_t determine the shape of the directional characteristic of a spatiotemporal derivative of a plane wave. In the two extreme cases when $\rho_u = 0$ and $\rho_u = 1$, the directional responses are omnidirectional and bidirectional, respectively. When $\rho_u = cu_t$, the directional response has a well-known cardioid polar pattern, and when the value of ρ_u is smaller or larger than cu_t, the directional response is a variation of a subcardioid or a "tailed cardioid." Some well-known first-order polar patterns, resulting from different ρ_u/u_t ratios, are given in Table 1 and shown in Fig. 3.

1.2.2 Higher Order Spatiotemporal Derivatives of Sound Pressure Field

The expression for a general *n*th-order spatiotemporal derivative of the sound pressure field along a single

[1] In this paper the wave vector $\boldsymbol{k}$ is chosen to point to the direction from which waves emanate, as opposed to the direction of wave propagation used in standard texts, such as [14].

[2] The directional characteristics are plotted in the plane $z = 0$.

[3] In three dimensions, one needs to add the *z* direction.

direction given by the vector $\boldsymbol{u}$ is obtained by iterating the operations of taking its gradient and projecting it along the vector $\boldsymbol{u}$. For the sound field of a plane wave given by Eq. (6) the nth-order spatiotemporal derivative along the direction given by Eq. (8) has the form

$$D_u^n p(x, y, t) = (\mathrm{j}k)^n [\rho_u \cos(\theta - \phi_u) + c u_t]^n p(x, y, t). \quad (10)$$

Also, instead of along a single direction, higher order spatiotemporal derivatives can be taken along multiple directions. Given an n-tuple of vectors $\boldsymbol{U} = (\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n)$, with each vector of the form

$$u_i = [\rho_{u_i} \cos \phi_{u_i} \quad \rho_{u_i} \sin \phi_{u_i} \quad u_{t_i}]^{\mathrm{T}},$$

a mixed derivative of the sound pressure field along directions given by $\boldsymbol{U}$ has the form

$$D_U^n p(x, y, t) = (\mathrm{j}k)^n p(x, y, t) \prod_{i=1}^{n} [\rho_{u_i} \cos(\theta - \phi_{u_i}) + c u_{t_i}]. \quad (11)$$

As with the spatial derivative, the spatiotemporal derivative of the sound pressure field composed of a single plane wave has a high-pass frequency characteristic at all angles which is proportional to $(\mathrm{j}k)^n$, as shown in Fig. 2.

The directional characteristic is, however, proportional to a linear combination of spatial gradients of different orders, resulting from expanding the product $\Pi_{i=1}^{n}[\rho_{u_i}\cos(\theta - \phi_{u_i}) + c u_{t_i}]$ in Eq. (11), or the term $[\rho_u \cos(\theta - \phi_u) + c u_t]^n$ in Eq. (10), which is a special case of Eq. (11).

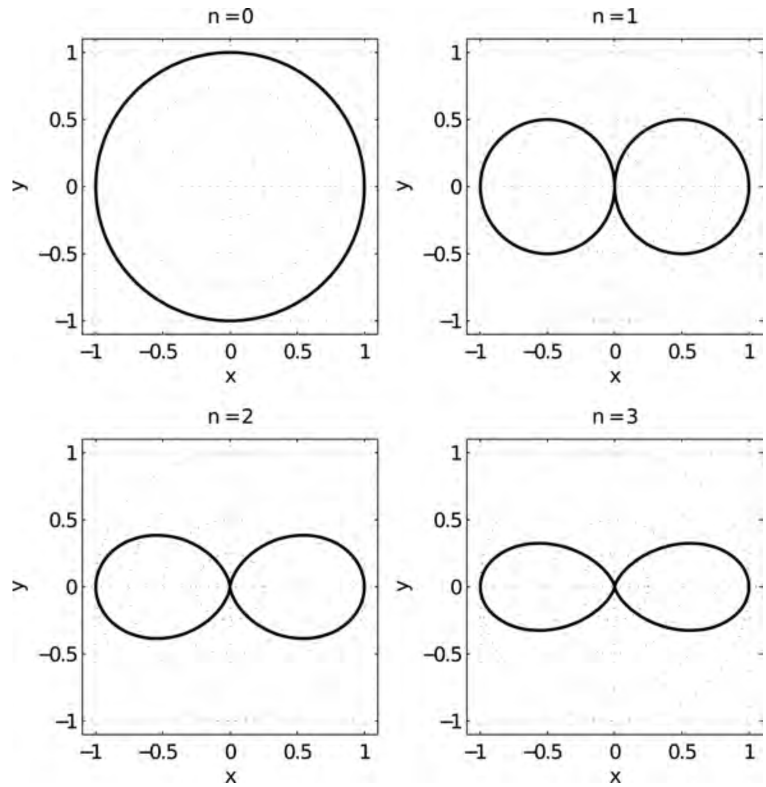

Fig. 1. Directional characteristics of plane-wave spatial derivatives for different derivative orders n.

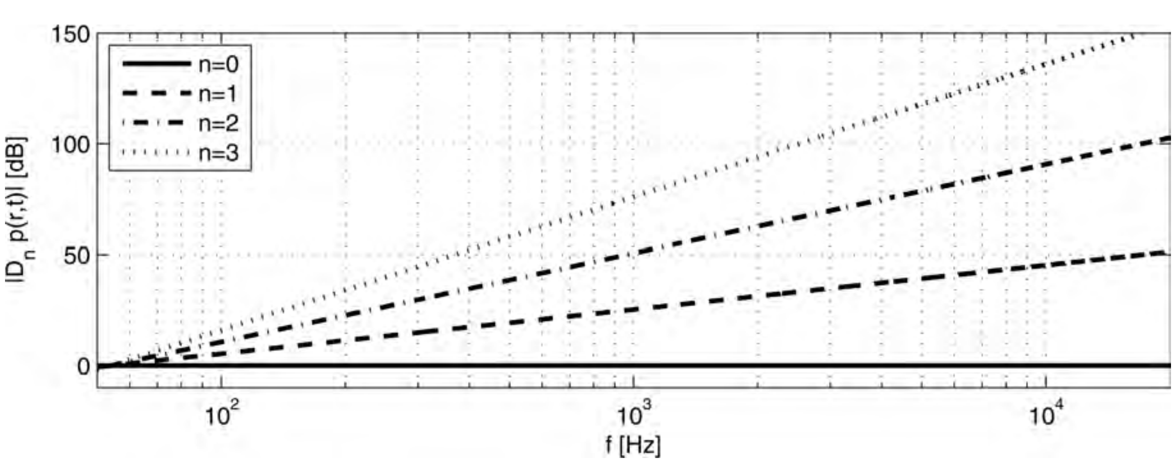

Fig. 2. Magnitude responses of plane-wave spatial derivatives of different orders n.

Table 1. Some well-known first-order polar patterns expressed through the ratio ρ_u/u_t of the spatiotemporal derivative.

Response Type	ρ_u/u_t
Cardioid	c
Subcardioid	$(0, c)$
Hypercardioid	$3c$
Supercardioid	$\frac{3-\sqrt{3}}{\sqrt{3}-1}c$

As with the first order, the shape of the directional characteristic of a higher order spatiotemporal derivative of a plane-wave sound field is determined by the choice of the vectors $\boldsymbol{u}_i$, that is, the parameters ρ_{u_i}, ϕ_{u_i}, and u_{t_i}. Some well-known second-order polar patterns, resulting from different choices of ratios ρ_{u_1}/u_{t_1} and ρ_{u_2}/u_{t_2} and angle differences $\Delta\phi = \phi_{u_1} - \phi_{u_2}$, are given in Table 2, and shown in Fig. 4.

2 PRACTICAL SPATIOTEMPORAL DIFFERENTIAL MICROPHONE ARRAYS

Section 1 presented a theoretical analysis of the spatiotemporal derivatives of a plane-wave sound field, which serves as a basis for designing gradient and differential microphone arrays with desired directional responses.

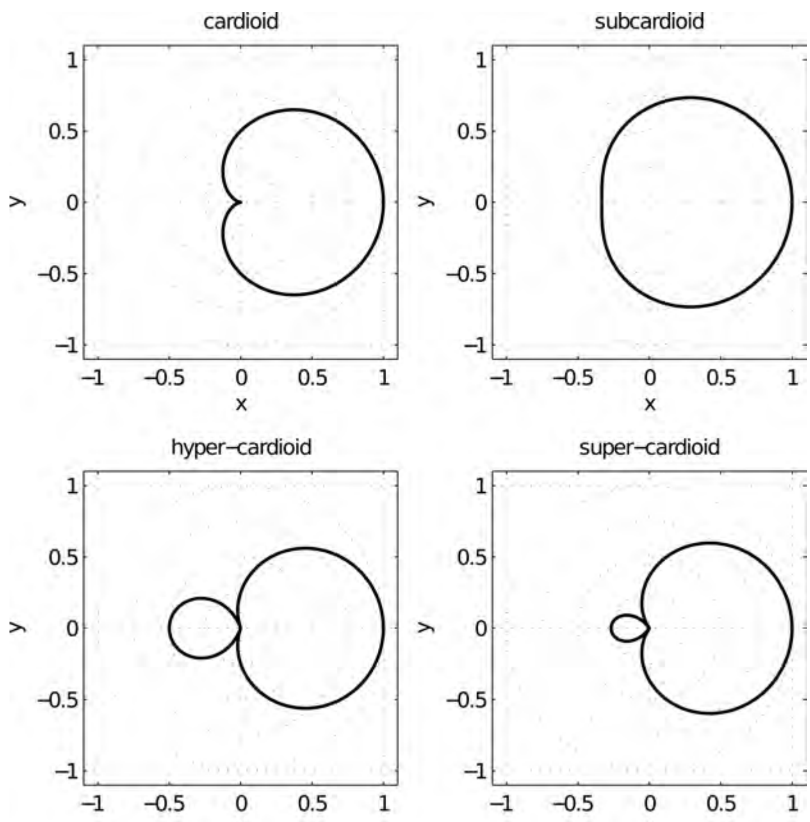

Fig. 3. Directional characteristics of plane-wave first-order spatiotemporal derivatives for different ratios ρ_u/u_t as given in Table 1.

Table 2. Some well-known second-order polar patterns expressed through the ratios ρ_u/u_t and angle differences $\Delta\phi$ of the spatiotemporal gradient.

Response Type	ρ_{u_1}/u_{t_1}	ρ_{u_2}/u_{t_2}	$\Delta\phi = \phi_{u_1} - \phi_{u_2}$
Cardioid	c	c	0
Hypercardioid	$(\sqrt{6}-1)c$	$(\sqrt{6}+1)c$	π
Supercardioid	$\frac{4-\sqrt{7}+\sqrt{8-3\sqrt{7}}}{\sqrt{7}-2-\sqrt{8-3\sqrt{7}}}c$	$\frac{4-\sqrt{7}-\sqrt{8-3\sqrt{7}}}{\sqrt{7}-2+\sqrt{8-3\sqrt{7}}}c$	0

Practical differential microphone arrays are based on the principle of the finite-difference approximation of the spatiotemporal derivatives of a sound pressure field. They combine values of the sound pressure field in multiple closely spaced points in space and time,[4] either acoustically (pressure at two faces of a diaphragm, and different-length acoustic paths to the two faces of a diaphragm) or electronically (pressure at different microphones of a microphone array combined with delay elements).

This section will present a few practical differential microphone array realizations based on the analysis from the previous section.

2.1 First-Order Differential Microphone Arrays: Cardioid, Hypercardioid, and Supercardioid

The first-order directional responses of the spatiotemporal derivatives of a sound field, presented in Section 1 and shown in Fig. 3 can be obtained by a finite-difference approximation of the spoatiotemporal derivative of a sound field, which involves taking differences of the sound pressure, both in space and in time. Two closely spaced microphones, spaced at a distance d, together with a delay element, as shown in Fig. 5, can be used for a practical realization of any first-order differential microphone array.

The response of a practical first-order differential microphone array, as shown in Fig. 5, in a sound field of a plane wave, expressed in Eq. (1), is given by

$$p_d(t) = 2\mathrm{j}\sin\left[\frac{k}{2}(d\cos\theta + ct_d)\right]p\left(\boldsymbol{r}, t - \frac{t_d}{2}\right) \qquad (12)$$

where k is the wavenumber, d the intermicrophone distance, t_d the delay used, and $\boldsymbol{r}$ the position of the microphone array center (midpoint between the two microphones). At low frequencies, Eq. (12) can be approximated by

$$p_d(t) \approx \mathrm{j}k(d\cos\theta + ct_d)p\left(\boldsymbol{r}, t - \frac{t_d}{2}\right) \qquad (13)$$

from which it can be seen that the ratio d/t_d determines the directional response of a practical differential microphone array in the same way the ratio ρ_u/u_t determines the directional response of the spatiotemporal derivative in Eq. (9) of the plane-wave sound field.

Fig. 6 shows directional responses of the practical cardioid, supercardioid, and hypercardioid microphones realized with the microphone combination shown in Fig. 5, with $d = 20$ mm.

From Fig. 6 it can be seen that the shape of the directional responses of practical first-order microphone arrays is frequency dependent, and that it corresponds to the desired responses, shown in Fig. 3, only at low frequencies. Above the aliasing frequency[5] the directional characteristics

[4] Points are spaced at a distance much shorter than the wavelength, and at a time much shorter than the period.

[5] The aliasing frequency of a first-order gradient microphone array is dependent on the intermicrophone distance d and the delay t_d used.

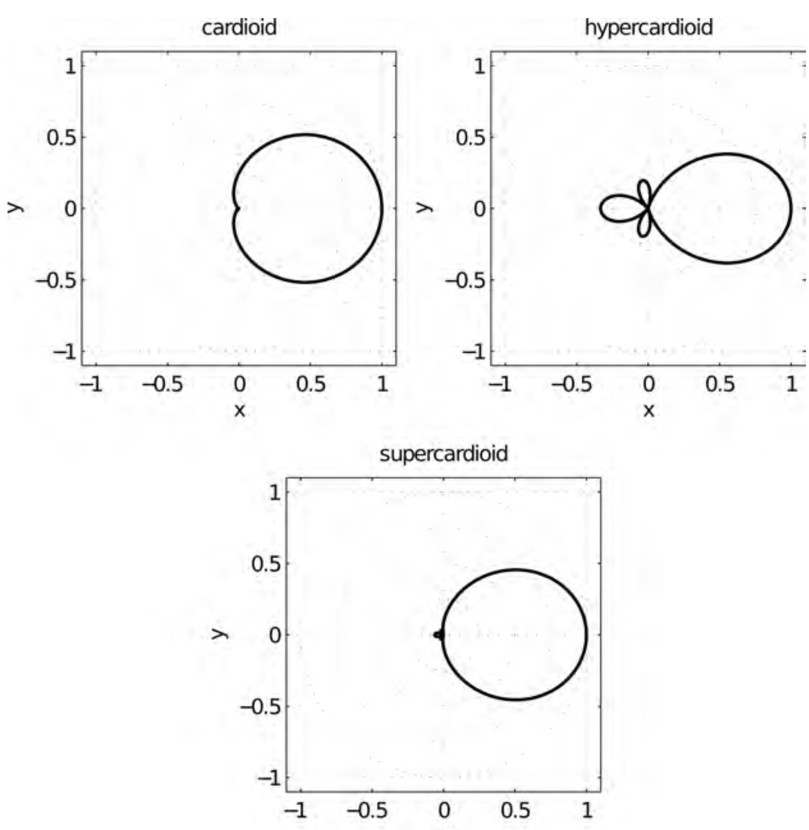

Fig. 4. Directional characteristics of plane-wave second-order spatiotemporal derivatives for different ratios ρ_u/u_t and angle differences $\Delta\phi$ as given in Table 2.

deviate from the desired ones, as can be observed in Fig. 6 for frequency $f = 7000$ Hz.

2.2 Second-Order Differential Microphone Arrays

In this part it is shown how clover-leaf directional responses sin 2θ and cos 2θ can be realized in two different ways based on the analysis from Section 1.

2.2.1 Clover-Leaf Response sin 2θ: Quadrupole Microphone Array

The clover-leaf directional response sin 2θ can be represented as the product of the directional responses of two spatial derivatives of a plane-wave sound field, the spatial derivative along the axis x, which has a directional response cos θ, and the spatial derivative along the axis y, which has a directional response sin θ [or cos (θ − π/2)]. As such the directional response sin 2θ can be realized as a cascade of two spatial derivative approximations—first along the axis x and then along the axis y, or vice versa.

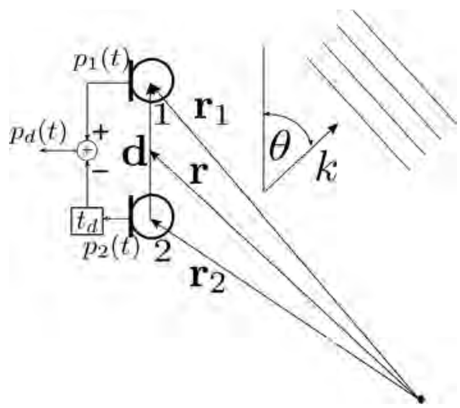

Fig. 5. First-order differential microphone realization using two pressure microphones and a delay element.

Fig. 7 illustrates a configuration of four pressure microphones used as an approximation of the previously described cascade of spatial derivatives of the sound field. Fig. 8 shows the directional responses at various frequencies of the quadrupole microphone array shown in Fig. 7 when the intermicrophone distance $d = 20$ mm is used.

2.2.2 Clover-Leaf Response cos 2θ: Three-Microphone Line Array

The clover-leaf directional response of the form cos 2θ can be represented as

$$\cos 2\theta = 2\cos^2\theta - 1 \tag{14}$$

or, equivalently, as

$$\cos 2\theta = (\sqrt{2}\cos\theta - 1)(\sqrt{2}\cos\theta + 1) \tag{15}$$

which is a product of the directional responses of two first-order spatiotemporal derivatives of a plane-wave

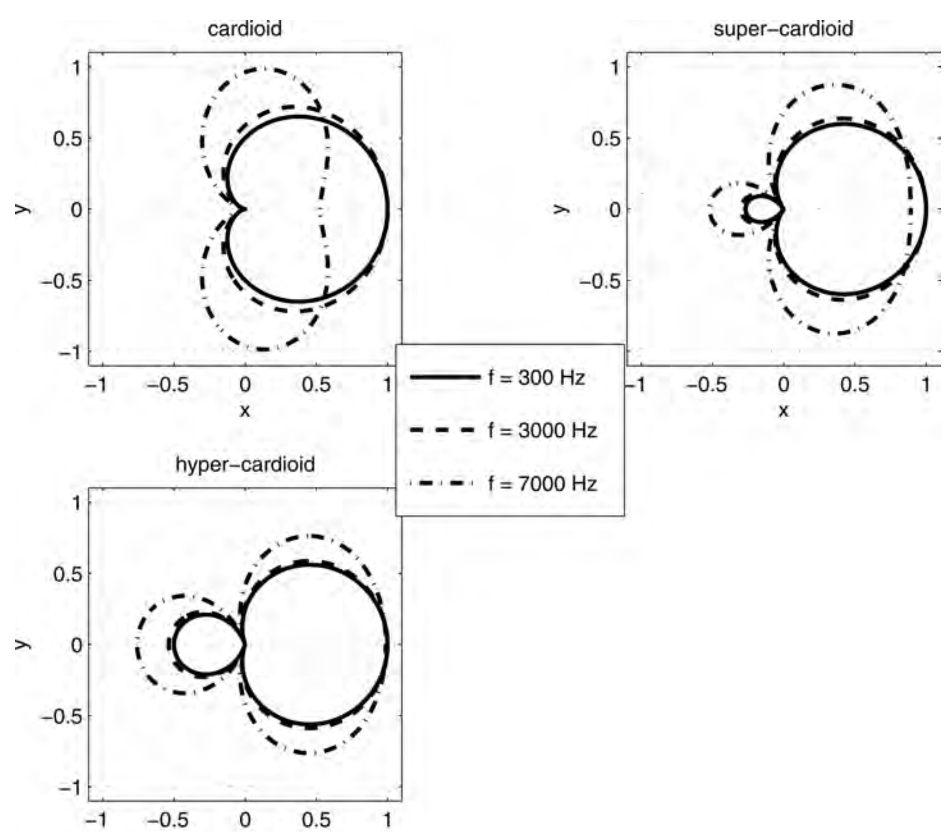

Fig. 6. Directional responses at various frequencies of first-order differential microphones realized as shown in Fig. 5, with $d =$ 20 mm and $t_d = d/c$ (cardioid), $t_d = d(\sqrt{3}-1)/c(3-\sqrt{3})$ (supercardioid), and $t_d = d/3c$ (hypercardioid).

sound pressure field. Consequently the response cos 2θ can be obtained by cascading two spatiotemporal derivative operations: one with $\rho_u/u_t = -\sqrt{2}$ and the other with $\rho_u/u_t = \sqrt{2}$ or, equivalently, two spatiotemporal finite differences: the first with $d/t_d = -\sqrt{2}c$ and the second with $d/t_d = \sqrt{2}c$, or vice versa, as shown in Fig. 9.

Fig. 10 shows the directional responses at various frequencies of the microphone array shown in Fig. 9, with the intermicrophone distance $d = 20$ mm and the delay $t_d = d/\sqrt{2}c$.

Like the first-order differential microphone arrays, the second-order differential microphone array has a directional response that is frequency dependent. At low frequencies it corresponds well to the desired response, and above the aliasing frequency it deviates from the desired response. This can be observed in Fig. 10, which shows how the shape of the directional response of the microphone array from Fig. 9 deforms at the frequency $f = 7000$ Hz.

Note that the directional response of the form sin 2θ can also be obtained by rotating by 45° the microphone array of Fig. 9.

3 CONCLUSIONS

This paper presented an analysis of the sound pressure field as a multivariate function of spatial location and time, which helps explaining the working principles of gradient microphones, differential microphones, and arrays as devices for approximately measuring the spatiotemporal derivatives of a sound pressure field and shows their equivalence.

The presented analysis framework enables not only analyzing the response of a given gradient or differential microphone or microphone array, but it can also be used for designing differential microphone arrays. The appropriate adjustment of the microphone array parameters—such as array orientation and shape, intermicrophone distances, and microphone signal delays—enables meeting the desired response requirements of the microphone array.

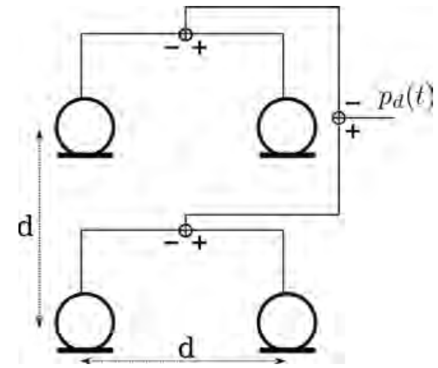

Fig. 7. Quadrupole microphone array used to obtain clover-leaf directional response sin 2θ.

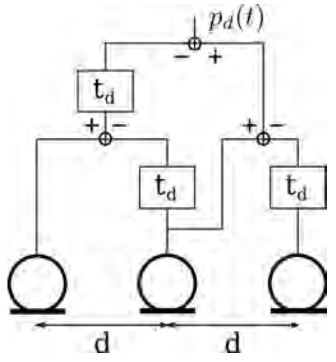

Fig. 9. Line array with three microphones used to obtain clover-leaf directional response cos 2θ.

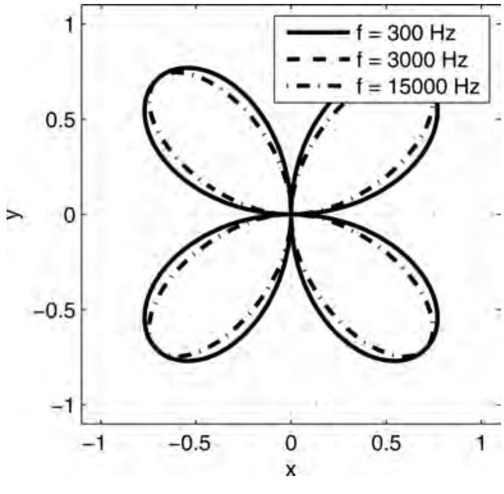

Fig. 8. Directional responses at various frequencies of quadrupole microphone array shown in Fig. 7, with intermicrophone distance $d = 20$ mm.

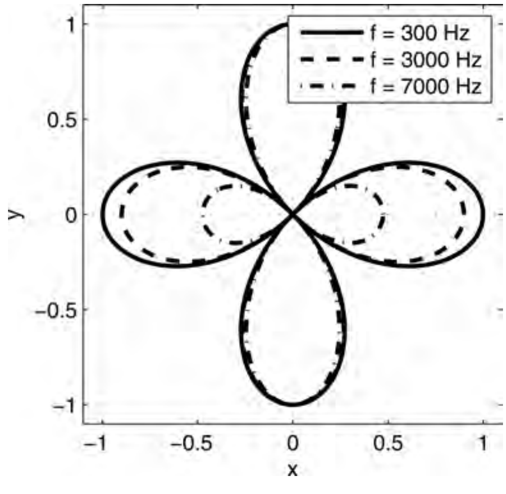

Fig. 10. Directional responses at various frequencies of microphone array shown in Fig. 9, with intermicrophone distance $d = 20$ mm and intermicrophone delay $t_d = d/\sqrt{2}c$.

4 REFERENCES

[1] H. F. Olson, “Gradient Microphones,” *J. Acoust. Soc. Am.,* vol. 17, pp. 192–198 (1946).

[2] H. F. Olson, “Directional Microphones,” *J. Audio Eng. Soc.,* vol. 15, pp. 420–430 (1967).

[3] A. Blumlein, “Improvements in and Relating to Sound Transmission, Sound Recording and Sound Reproduction Systems,” British patent 394325 (1931), reprinted in *Stereophonic Techniques* (Audio Engineering Society, New York, 1986).

[4] R. Streicher and W. Dooley, “The Bidirectional Microphone: A Forgotten Patriarch,” *J. Audio Eng. Soc.,* vol. 51, pp. 211–225 (2003 Apr.).

[5] H. E. de Bree, M. Iwaki, K. Ono, T. Sugimoto, and W. Woszczyk, “Anechoic Measurements of Particle-Velocity Probes Compared to Pressure Gradient and Pressure Microphones,” *Convention of the Audio Engineering Society, (Abstracts)* www.aes.org/events/122/122ndWrapUp.pdf, (2007 May), convention paper 7107.

[6] R. Raangs, W. F. Druyvesteyn, and H. E. De Bree, “A Low-Cost Intensity Probe,” *J. Audio Eng. Soc.,* vol. 51, pp. 344–357 (2003 May).

[7] J. Merimaa, “Applications of a 3-D Microphone Array,” presented at the 112th Convention of the Audio Engineering Society, *J. Audio Eng. Soc., (Abstracts),* vol. 50, p. 496 (2002 June), convention paper 5501.

[8] F. J. Fahy, “Measurement of Acoustic Intensity Using the Cross-Spectral Density of Two Microphone Signals,” *J. Acoust. Soc. Am.,* vol. 62, pp. 1057–1059 (1977).

[9] G. W. Elko, “Steerable and Variable First-Order Differential Microphone Array,” U.S. patent 6,041,127 (2000).

[10] R. J. Geluk and L. de Klerk, “Microphone Exhibiting Frequency-Dependent Directivity,” E.U. patent application 01201501.2 (2001).

[11] D. Preves, T. Peterson, and M. Bren, “In-the-Ear Hearing Aid with Directional Microphone System,” U.S. patent 5,757,933 (1998).

[12] G. W. Elko, “Microphone Array Systems for Hands-Free Telecommunication,” *Speech Commun.,* vol. 20, pp. 229–240 (1996).

[13] G. W. Elko, “Superdirectional Microphone Arrays,” in *Acoustic Signal Processing for Telecommunication* (Kluwer Academic, Boston, MA, 2000), pp. 181–238.

[14] E. G. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography* (Academic Press, London, 1999).

THE AUTHORS

M. Kolundžija

C. Faller

M. Vetterli

Mihailo Kolundžija received a Dipl. Ing. degree in electrical engineering from the University of Novi Sad, Serbia, in 2004, an M.Sc. degree in communication systems from the Ecole Polytechnique Fédérale (EPFL), Lausanne, Switzerland, in 2007, and is currently a Ph.D. student there. His diploma thesis work, carried out at Micronas GmbH, Freiburg, Germany, involved developing video upscaling algorithms and upscaling units on FPGAs. His M.Sc. thesis work at EPFL was on techniques for spatial sound capture using compact microphone arrays. He is currently working on techniques for spatial sound reproduction using loudspeaker arrays and sound field analysis using arrays of microphones. His main research interests are spatial sound acquisition and reproduction, array signal processing, and room acoustics.

●

Christof Faller received an M.S. (Ing.) degree in electrical engineering from ETH Zurich, Switzerland, in 2000, and a Ph.D. degree for his work on parametric multichannel audio coding from Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland, in 2004.

From 2000 to 2004 he worked in the Speech and Acoustics Research Department at Bell Labs Lucent and its spin-off Agere Systems, where he worked on audio coding for satellite radio, MP3 Surround, and MPEG Surround. He is currently managing director at Illusonic, a company he founded in 2006, and part-time research associate at EPFL in Lausanne.

●

Martin Vetterli received an engineering degree from the Eidgenössiche Technische Hochschule (ETHZ), Zurich, Switzerland, an M.S. degree from Stanford University, Stanford, CA, and a Ph.D. degree from the Ecole Polytechnique Fédérale (EPFL), Lausanne, Switzerland.

In 1986 he joined Columbia University, New York, first with the Center for Telecommunications Research and then with the Department of Electrical Engineering, where he was an associate professor of electrical engineering. In 1993 he joined the University of California at Berkeley, where he was a full professor until 1997. Since 1995 he has been a professor at EPFL, where he headed the Communication Systems Division (1996–1997) and now is heading the Audiovisual Communications Laboratory. From 2001 to 2004 he directed the National Competence Center in Research on mobile information and communication systems. He is also vice president for international affairs at EPFL. He has held visiting positions at ETHZ (1990) and Stanford (1998). His research interests are in the areas of applied mathematics, signal processing, and communications.

Dr. Vetterli is the coauthor of *Wavelets and Subband Coding* with J. Kovacevic, *Signal Processing for Communications* with P. Prandoni, and *Fourier and Wavelet Signal Processing* with J. Kovacevic and V. Goyal. He has published about 150 journal papers.