# Deployment of a Pentagonal "Hollow-Rope" Tensegrity Module

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# Summary

Tensegrity structures are spatial reticulated structures composed of cables and struts. Tensegrity systems are good candidates for adaptive and deployable structures and thus have applications in various engineering fields. A "hollow-rope" tensegrity system composed of tensegrity-ring modules has been demonstrated by the authors to be a viable system for a pedestrian bridge. This paper focuses on the deployment of pentagonal ring modules. A geometric study is performed to identify the deployment-path space allowing deployment without strut contact. Two actuation schemes are explored for deployment: the first scheme employs only actuated cables, while the second combines actuated cables with spring elements. In both schemes, continuous cables are used to reduce the number of actuators required. Finally, the structural response of the module during deployment is studied numerically using a modified dynamic relaxation algorithm.

**Keywords:** *tensegrity structures, deployable structures, adaptive structures, pedestrian bridges, dynamic relaxation.* 

# 1. Introduction

Tensegrity structures are spatial reticulated structures composed of cables and struts. Their stability

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René Motro, born 1946, graduated in Civil Engineering 1968, Professor Emeritus at Montpellier University, Vice President of the IASS, Chief Editor of the International Journal for Space Structures. His main research topics are tensegrity systems, structural morphology and conceptual design. is based on the equilibrium among cables and struts. The tensegrity concept exists for almost 60 years now and has received significant interest from scientist and engineers [1-3]. In structural engineering, tensegrity offers lightweight structures with a high strength to mass ratio. However, there are few examples of tensegrity structures for civil engineering applications [4-5]. Moreover, few large scale tensegrity structures have been constructed for experimental studies [6-7].

Tensegrity systems are attractive systems for adaptive architecture. In adaptive architecture, structures use embedded actuated members to interact with complex environments. In tensegrity structures, actuators can be integrated in both struts and cables. Telescopic struts were used to control a 15m<sup>2</sup> active tensegrity structure for service performance, self-diagnosis and self-repair [8-10]. Moreover, actuated members in tensegrity structures can be used to apply shape changes linking adaptive structures with deployable structures.

Deployable tensegrity systems were first studied in the 1990s [11]. Tibert and Pellegrino [12] compared deployable tensegrity masts to conventional ones in order to identify lack of stiffness during deployment and low bending stiffness as major drawbacks of the system. Lack of stiffness during deployment can be avoided using cable actuation, while bending stiffness can be increased if tensegrity systems with strut-to-strut connections are used. Proper cable actuation can keep the system close to an equilibrium manifold thus maintaining its stiffness [13]. Smaili and Motro [14] used actuated cables for the deployment of a double tensegrity layer grid. Deployment is thus based on the creation of finite mechanisms. Furthermore, equilibrium over finite displacements can be obtained with the integration of zero-free length springs in a tensegrity system [15].

This paper focuses on the deployment of a pentagonal tensegrity-ring module using embedded actuated cables. Pentagonal ring modules can be used as structural units for a deployable "hollow-rope" system. A geometric study is performed to identify the deployment-path space allowing deployment of a pentagonal ring module without strut contact. Two actuation schemes are explored for deployment. The first scheme is based on finite mechanism creation, while the second is inspired from statically balanced mechanisms combining actuated cables with spring elements. Finally, a modified dynamic relaxation algorithm is used to study the structural response of the module during deployment for both actuation schemes.

# 2. Tensegrity-ring module topology

Tensegrity-ring modules are class II tensegrity systems. Their topology is based on straight prism geometry with a single strut circuit around the empty space in the middle of the prism. Therefore, they are called "ring" modules after the form of their strut circuit. Ring modules can be assembled together by their basis to form a "hollow-rope" structural system. Motro [16] studied ring-module topology and its deployability through allowing changes in cable rest lengths. However, no actuation scheme for deployment was proposed. Furthermore, Nguyen [17] studied the deployed configuration of a pentagonal ring module and its folding using FEM. Folding was simulated using nodal displacements and allowing slack cables. Therefore, this method is not suitable for studying deployment actuation.

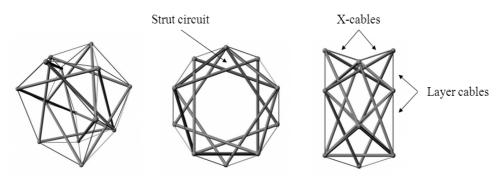


Fig. 1: The pentagonal ring module and its elements: struts, x-cables and layer cables

The authors studied ring-shaped tensegrities and showed that the pentagonal "hollow-rope" system is a viable system for a pedestrian bridge [18-19]. The pentagonal ring module includes 15 struts and 30 cables. Fig. 1 illustrates a pentagonal tensegrity-ring module. There are two sets of cables:

layer cables and x-cables. Layer cables form a pentagon on the front and back side of the module, while x-cables form an "x" on each lateral side of the module. In this paper, a module with a total length of 75cm with an external radius of 57cm is explored. Struts have a length of 100cm. Cable length is set at approximately 68cm and 51cm for layer cables and x-cables respectively.

# 3. Geometric study of deployment and actuation schemes

The deployment motion of a pentagonal tensegrity-ring module can be decomposed in three elementary motions: a translation, a rotation and a dilation of the pentagonal faces of the module (front and back face). The translation defines the deployment length *L* of the module (Fig.2). Therefore, *L* has a maximum value for the unfolded configuration and decreases with folding. Pentagonal faces rotate around the longitudinal axis of the module (axis x) during folding starting from zero for the unfolded configuration. The transverse rotation is described by the angle  $\theta$  between the two faces (Fig. 2). Finally, a dilation of the pentagonal faces of the module occurs during folding. The dilation is measured by the radius R of circumscribed circles on the nodes of the pentagonal faces (Fig. 2). Transverse rotation and dilation of the faces allow the strut circuit to adjust under a new module length. Therefore, all three motions are required for deployment. However, there is no explicit relationship between these parameters since many paths lead to the same module length *L*.

Deployment-path space depends only on L and  $\theta$  (Fig. 2), if a path is predefined for a known topology. Fig. 2 shows the deployment-path space for the pentagonal ring module described in Section 2. The space in white describes the space where deployment is conducted without strut contact. Isometric curves in the allowable space correspond to the closest distance between struts from 1 (inner curve) to 9 cm (outer curve). Curves are used to define the path with minimum contact risk. If this path is followed, deployment is smooth avoiding abrupt shape-changes. However, length-changes in all 30 cables of the module are required. A path requiring cable-length changes in only 10 layer cables and 10 x-cables is found close to this path. This path corresponds to the path with the minimum number of actuators (Fig.2). The two paths are similar with a small difference in the transverse rotation and extreme module-length values.

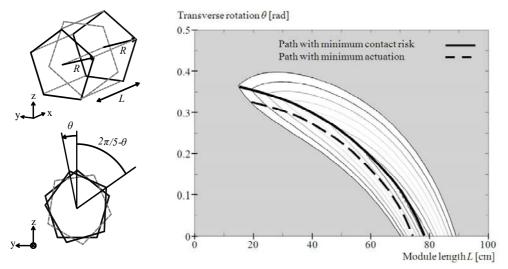


Fig. 2: Geometric deployment parameters and deployment-path space of the pentagonal ring module

Conventional tensegrity-ring modules are stable systems and therefore require cable-lengthening to become deployable. Consequently, cable actuation is critical for deployment of pentagonal ring modules as all paths require changes in cable-lengths. Although the number of actuators may differ for each path, their action is similar as it is directly related with ring-module topology. Layer-cable length decreases for unfolding while x-cable length increases (inversed scenario for folding). Consequently, actuated layer cables control unfolding while actuated x-cables control folding. However, actions in both sets of cables are required to obtain stable configurations with a new length without abrupt changes in the module geometry.

Assuming individually actuated cables, deployment requires a minimum of 20 actuators (Fig. 2). The number of actuators can be reduced using continuous cables. However, integrating continuous cables in the tensegrity system affects its mechanics [20]. The number of independent states of self-stress in the module decreases, while the number of infinitesimal mechanisms increases. For the pentagonal ring module, the use of continuous cables in both sets of actuated cables (layer cables and x-cables) results in an unstable configuration. Therefore, continuous cables are used only for actuated x-cables. The total number of actuators decreases thus to 15 including 10 actuators in layer cables and 5 actuators in continuous x-cables. The deployment of the module with continuous actuated x-cables follows a similar path with the deployment of the module with discontinuous actuated cables. Therefore, continuous x-cables do not affect the deployment path of the module.

Spring elements can be used to further reduce the number of actuators required for deployment as they allow length changes without requiring an individual actuation mechanism. However, actions in spring elements are driven by actuated cables. For the pentagonal ring module, spring elements are used to replace the actuated layer cables. In this case, both unfolding and folding are controlled by x-cables. Spring elements can be combined with both discontinuous and continuous actuated cables. The module configuration employing continuous cables in actuated x-cables and spring elements instead of layer cables requires only 5 actuators for deployment. The deployment path followed remains similar with the deployment of the module with discontinuous actuated cables. Consequently, the use of continuous cables in actuated x-cables and spring elements in layer cables does not affect the deployment of the module.

# 4. Structural analysis and deployment

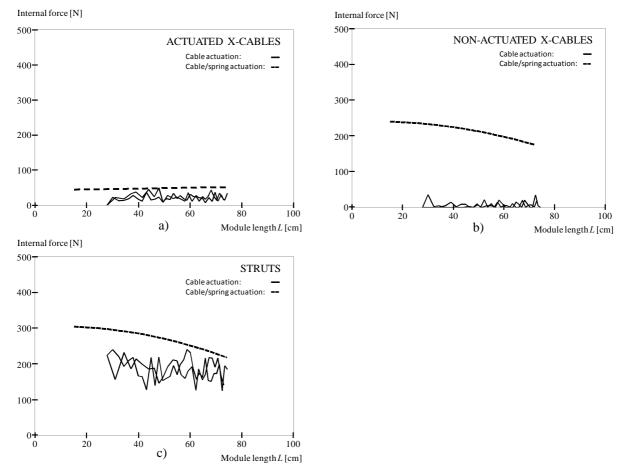
The pentagonal ring module is analyzed using a modified version of the dynamic relaxation method. Dynamic relaxation is a static analysis method suitable for highly non-linear structures such as tensegrity systems. The method was extended to accommodate continuous cables and spring elements. Continuous cable action requires the formulation of a *clustering matrix* relating continuous cables with the traditional module topology [20]. Furthermore, continuous cables and spring elements affect the calculation of internal forces in the system. Continuous cables carry the same internal force in all their segments as frictionless pulleys are considered. Spring force depends on spring characteristics: spring constant k and initial spring length  $l_s$ . Details about the modified dynamic relaxation method are given in [21].

Deployment simulation is conducted under dead load using a deployment-analysis algorithm that integrates actuation schemes with the modified version of the dynamic relaxation method. Actuation is implemented as changes in the length of actuated members and depends on the deployment path followed. In continuous actuated cables, actuation is applied equally to all segments of the cable. Simulation can start from any known topology applying cable-length changes that allow the module to change shape. Therefore, with appropriate actuation the module can unfold or fold.

For deployment, actions taken on actuated elements depend on the deployment path chosen and the actuation scheme applied. If a deployment path similar to paths of Section 3 is followed and cable actuation is applied, then x-cable length increases while layer-cable length decreases with unfolding (inversed scenario for folding). Increasing x-cable length creates a finite mechanism that is next stabilized by decreasing layer-cable length. Deployment is thus based on a series of finite mechanisms and on a series of equilibrium configurations with different module lengths. Moreover, equilibrium finding can be controlled with the implementation of topology constraints such as planarity among pentagonal faces. If the actuation scheme combines cables with spring elements, then deployment is based exclusively on a series of equilibrium configurations. Spring elements allow the system to move from one equilibrium configuration to another without the creation of a finite mechanism. However, due to spring action, the energy stored in the system varies for each equilibrium configuration.

The deployment of the pentagonal ring module described in Section 2 is analyzed using both actuation schemes. Aluminium struts and steel cables with cross-sections of 2.54cm<sup>2</sup> and 0.03cm<sup>2</sup> respectively are used. Spring elements have a constant *k* of 1N/cm with initial length  $l_s$  of 66cm. A contact-free deployment is successfully conducted using both actuation schemes. However, starting from a similar deployed configuration, cable/spring actuation results in a more folded configuration. The folded length obtained with cable/spring actuation corresponds to 20% of the deployed length,

while it is only 40% when only actuated cables are used. Fig. 3a and 3b show the evolution of internal forces in actuated and non-actuated x-cables. Fig. 3c shows the evolution of compressive internal forces in struts during deployment for both actuation schemes. Cable actuation maintains internal forces approximately at the same level for all elements. Furthermore, internal forces with cable actuation are lower compared with forces under cable/spring actuation. This is due to finite mechanism creation that results with cable actuation (cable-length changes). Mechanism creation maintains a low level of internal forces by stabilizing intermediate mechanisms during deployment. Deployment is thus composed of a series of mechanisms and equilibrium configurations. However, intermediate equilibrium configurations for folding and unfolding are not the same as internal forces vary. If cable/spring actuation is applied, the structural behaviour of the module is governed by spring action. Spring elements can undergo length changes based on actuated x-cables allowing thus the deployment of the ring module. Spring elements replace layer cables that increase with folding and decrease with unfolding. Therefore, spring-force increases with folding and decreases with unfolding. Internal forces in non-actuated x-cables and struts increase with folding and decrease with unfolding. Forces in actuated cables remain almost constant. For both actuation schemes, highest internal forces during deployment are found in non-actuated cables. Moreover, cables and struts remain in tension and in compression respectively throughout the entire process respecting the tensegrity principle.



*Fig. 3: Internal forces in actuated x-cables a), non-actuated x-cables b) and struts c) during deployment for both actuation schemes* 

The total energy in the pentagonal tensegrity-ring module during deployment is estimated according to [13]:

$$W = U + K - T_d \sum \int M^T \omega^r d\tau$$
 (1)

where U is the potential energy including gravitational potential energy  $U_{G}$ , elastic potential energy

 $U_{el}$  and spring potential energy  $U_s$ . K corresponds to the kinetic energy and is negligible due to quasi-static actuation. The last term of Eq. 1 represents energy loss due to friction on strut-to-strut connections based on deployment time  $T_d$ , friction torque  $M^T$  and angular velocity  $\omega^r$ . Cable friction is not considered in this study. Energy in the tensegrity-ring module during deployment depends on the actuation scheme applied. If only actuated cables are used for deployment, then most of the energy in the system is stored as elastic potential energy in cables and is given by:

$$U_{el} = 0.5 \cdot N \cdot \Delta l \tag{2}$$

where *N* corresponds to internal forces in cables and  $\Delta l$  to corresponding length changes. If spring elements are included in the actuation scheme, then spring potential energy is the highest energy component during deployment. Furthermore, the total energy is much higher compared with the energy of cable-actuation scheme. Fig. 4 shows the evolution of elastic potential energy in cables during deployment for both actuation schemes. Cable actuation maintains energy approximately at the same level during deployment. Energy varies during folding and unfolding as different self-stress are found. When cable-spring actuation is used, elastic potential energy in cables increases with unfolding. However, potential spring energy is much higher then the elastic potential energy. Moreover, it was found that energy stored in spring elements during folding can be used for unfolding the module.



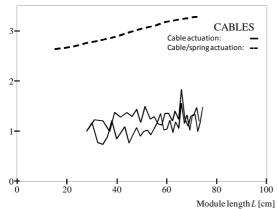


Fig. 4: Elastic potential energy in cables during deployment for both actuation

# 5. Experimental validation

A small scale physical model of the pentagonal ring module was constructed for experimental validation of the findings (Fig. 5). The module was designed according to the ring-module topology described in Sections 2 and 4. It is composed of aluminium struts and steel cables. Joints and supports allow the required movement for deployment. The actuation scheme with the minimum number of actuated elements was implemented in the model. Continuous cables are thus used in actuated x-cables and spring elements are used to replace layer cables. Cable actuation is conducted manually using hand cranks placed on supports of the model. Extreme values of the deployment-path space and the deployment path with minimum number of actuators were validated successfully.

Fig. 5 shows the deployment-path space of the module including the computed and the measured deployment path. The two paths are similar with same extreme values (folded and unfolded length). Consequently, continuous cables and spring elements do not affect the deployment path of the tensegrity-ring module. Furthermore, the measured path reveals a systematic error in the transverse rotation due to joint design. Both unfolding and folding were conducted successfully without strut contact. Finally, energy stored in spring elements during folding is found sufficient for unfolding the module.

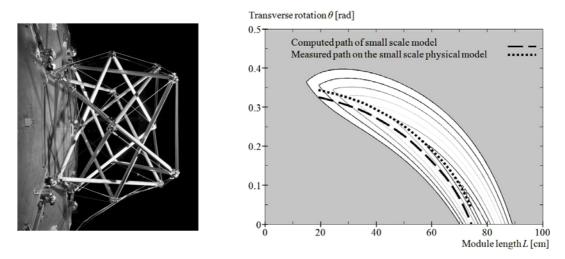


Fig. 5: The small scale physical model and its deployment-solution space and path

# 6. Discussion

Tensegrity-ring modules are stable systems. Therefore, their deployment requires cable actuation in order to apply cable-length changes. However, a smaller number of actuators results in a simpler design and control of the system. The number of actuators required is thus reduced using continuous cables and spring elements. Continuous cables may run in more than one module reducing significantly the number of actuators required for a hollow-rope configuration (cable/spring actuation requires only 5 actuators for the deployment of three interconnected ring modules). Deployment actuation schemes employed affect the structural behaviour of the module. Cable actuation maintains internal forces at the same level during deployment. Fig. 3 reveals that equilibrium configurations for a folding-unfolding cycle with cable actuation are not exactly the same. Intermediate equilibrium configurations may have the same deployment length with a different self-stress state. If cable/spring actuation is applied, internal forces increase significantly during folding and may become critical for the design. Finally, actuation characteristics, such as actuation-step size and spring stiffness, control the deployment motion. Large actuation steps in actuated cables may result in unstable configurations while small actuation steps are computationally expensive. Furthermore, low values of spring constant k combined with large values of initial spring length  $l_s$  do not provide sufficient stiffness to the system resulting in unstable configurations. On the contrary, large values of k combined with low values of  $l_s$  induce large internal forces that may exceed element strength during deployment. Therefore, actuation-step size and spring characteristics (when springs are used) are critical parameters for deployment. Moreover, applying individual actuation steps and different spring constants for each actuated element and spring element respectively may result in better control of the structure during deployment.

# 7. Conclusions

The conclusions from this study are as follows:

- Contact-free deployment of pentagonal tensegrity-ring modules is feasible with appropriate cable-spring actuation.
- Continuous cables and spring elements can be used to reduce the number of actuators.
- Although continuous cables and spring elements affect the internal forces and the energy in the tensegrity-ring module during deployment, they do not change the deployment path followed.
- Actuation steps and spring characteristics are critical parameters for deployment.
- Cable actuation maintains internal forces during deployment in lower values compared with cable/spring actuation.
- Energy stored in spring elements during folding can be used for unfolding.

Work in progress includes construction and study of a 1:4 scale physical model of the active

deployable pentagonal tensegrity-ring bridge.

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