Design of Acoustic Metamaterials based on the Concept of Dual Transmission Line

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Layout of the presentation

• Introduction & Objectives
• Transmission line concept
  • Conventional & Dual acoustic TL
• Lumped element models
  • Membrane, Stub & CRLHTL
• Finite element models
  • Membrane, Stub & CRLHTL
• Methodologies for the assessment
  • Lumped & Finite element models
• Results
  • Bloch, Scattering parameters & Unbalanced case
• Conclusion
Introduction

- **Acoustic metamaterials**: artificial structures using inclusions of elements, whose dimensions are smaller than the wavelengths of interest, so as to enact effective macroscopic behavior not readily available in nature.

- Growing interest for acoustic metamaterials

  - Capability to achieve new properties like negative refraction

- Lots of development in Electromagnetics

- Proposed structure based on transmission line concept: acoustic waveguide loaded with membranes and open radial stubs
Objectives

- Negative refraction: an illustration

![Diagram of waveguide with conventional and dual mediums]
Transmission line concept

- Conventional acoustic TL
  - Describe the propagation of waves
  - Positive index of refraction

- Dual acoustic TL
  - Dual topology
  - Negative index of refraction

- Unit cell of the composite right/left-handed (CRLH) TL
  - Dual acoustic TL has to load a conventional TL
  - Equivalent circuits assumed lossless

Conventional TL for high frequencies

Dual TL for low frequencies
Lumped element models

- Membrane
  - Implements a **series compliance**
  - Mechanical element

- Stub
  - Implements a **parallel mass**
  - Acoustic element

- Host waveguide
  - Conventional TL

\[\begin{align*}
  m_{as} &= m_{am} + m_a \\
  C_{ap} &= C_{at} + C_a \\
  C_{as} &= C_{am}
\end{align*}\]
Lumped element models

- CRLHTL
  - Balanced condition between RH & LH bands
    \[ m_{as}C_{as} = m_{ap}C_{ap} \]
  - Balanced condition for \( f_0 = 1 \text{kHz} \)
  - Lattice constant \( d \) small compared to the wavelength \( \lambda \)
    \[ d/\lambda = 0.1 \text{ at } f_0 \rightarrow d = 34 \text{ mm} \]
  - Dimensions and values of masses & compliances

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
<th>Value (kg/m(^4))</th>
<th>Value (x10(^{-12}) m(^3)/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Membrane radius</td>
<td>9.06 mm</td>
<td></td>
</tr>
<tr>
<td>( h )</td>
<td>Membrane thickness</td>
<td>125 ( \mu \text{m} )</td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>Stub thickness</td>
<td>1 mm</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>Stub length</td>
<td>43.5 mm</td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td>Lattice constant</td>
<td>34 mm</td>
<td></td>
</tr>
</tbody>
</table>
**Finite element models**

- **Membrane**
  - 2D axi-symmetric acoustic structure interaction model
  - Thin circular membrane clamped at its perimeter modeled as a thin plate: no tension is applied
  - \( \Delta p \): average pressure applied on the membrane
  - \( v \): average velocity of the membrane

\[
Z_{ac} = \frac{\Delta p}{S v} = \frac{p_2 - p_1}{q}
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>Pressure of 1 Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls</td>
<td>Sound Hard Walls</td>
</tr>
<tr>
<td>Output</td>
<td>Plane Wave Radiation</td>
</tr>
</tbody>
</table>
Finite element models

• **Stub**
  - 2D axi-symmetric pressure acoustic model
  - Radial open tube radiating in a surrounding medium
  - \( p \): average pressure at the entrance of the stub
  - \( v \): average velocity at the entrance of the stub

\[
Z_{at} = \frac{p}{S v} = \frac{p}{S(v_2 - v_1)} = \frac{p}{q_2 - q_1}
\]
Finite element models

- CRLH-TL
  - 2D axi-symmetric acoustic structure interaction model
  - Periodic structure with the same boundary conditions as before
  - Connection to an adapted host waveguide to avoid reflection at the interfaces
  - CRLH-TL with 10 cells

- 1 symmetric cell

\[
\begin{align*}
C_{as}/2 & \quad 2m_{as} & \quad C_{as} \\
2m_{sp} & \quad C_{as}/2 & \quad 2m_{sp}
\end{align*}
\]

\[d\]
Methodologies for the assessment

- Lumped element model
  - Computation of the Bloch parameters
  - Description of the propagation of waves in periodic structures
  - Pulsation of the two branches
    \[ \omega_R = \frac{1}{\sqrt{m_{as} C_{ap}}} \text{ and } \omega_L = \frac{1}{\sqrt{m_{ap} C_{as}}} \]
  - Bloch impedance \( Z_B \) and dispersion diagram \( \beta_B d \)
    \[
    \cos(\gamma_B d) = 1 - \frac{\left( \frac{\omega}{\omega_R} - \frac{\omega}{\omega_L} \right)^2}{4} \\
    Z_{B,\pi} = \sqrt{\frac{m_{as}}{C_{ap}}} \sqrt{1 - \frac{\left( \frac{\omega}{\omega_R} - \frac{\omega}{\omega_L} \right)^2}{4}}
    \]
Methodologies for the assessment

- Finite element model
  - Four probes sensing pressure
  
<table>
<thead>
<tr>
<th>Distance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = -x_4$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>$x_2 = -x_3$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>s</td>
<td>0.05 m</td>
</tr>
</tbody>
</table>

- Propagation of plane waves under $f < 0.586 \frac{c}{2a} = 11 k\text{Hz}$
- 2 & 4 microphones methods
- Scattering parameters: 1 & 10 cells

\[
R = \frac{H_{12} - e^{jks} e^{2jks}}{e^{jks} - H_{12}}
\]

\[
T' = \frac{C}{A} = \frac{P_3 e^{jks} - P_4 e^{jks}}{P_1 e^{jks} - P_2 e^{jks}}
\]

- Bloch parameters: 1 cell

\[
\cos(\gamma_B d) = \frac{1 - R^2 + T'^2}{2T'}
\]

\[
Z_B = \pm Z_0 \sqrt{\frac{(1 + R)^2 - T'^2}{(1 - R)^2 - T'^2}}
\]
Results

- Bloch parameters

![Dispersion diagram with negative and positive refraction](image)

**Negative refraction**

Pressure at $f = 950$ Hz

**Positive refraction**

Pressure at $f = 1050$ Hz
Results

- Scattering parameters
  - 2 & 4 microphones methods can be realized experimentally
  - 2 microphones \( \rightarrow \) reflection coefficient
  - 4 microphones \( \rightarrow \) transmission coefficient
  - LEM & FEM match well (previous results) so FEM results can be used as a reference for experimental results
Results

- Unbalanced case
  - Finite element model
  - $L = 80$ mm

Negative refraction
No propagation
Pressure at $f = 900$ Hz

Positive refraction
Conclusion

- Circuit theory concepts efficiently used to design a TL-based metamaterial
- Inclusion of mechanical elements to realize series compliance
- Proposed structure: negative refractive band of almost one octave with a balanced condition & unbalanced case
- LEM & FEM models confirm performances
- Future experimental measurement of scattering parameters to validate the results
- Further work in FEM to predict performances of a 2D version of this structure
Thank you for your attention

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