Metropolis-Hastings sampling of paths

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Outline

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Introduction

- a paths is a connected sequence of nodes in a network
- objective: efficient path sampling from general distributions
- applications
  - estimation of route choice models
  - dynamic traffic assignment
  - generation of route guidance alternatives

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How to sample from large (path) choice sets?

- approach
  - give every path $i \in \mathcal{C}$ a weight $b(i) > 0$
  - sampling probability $q(i)$ shall be $\propto b(i)$
- direct sampling from $q(i)$ requires path enumeration

$$q(i) = \frac{b(i)}{\sum_{j \in \mathcal{C}} b(j)}$$

- but pair-wise comparison of paths is easily done

$$\frac{q(i)}{q(j)} = \frac{b(i)}{b(j)}$$

Metropolis-Hastings (MH) algorithm

1. set iteration counter $k = 0$
2. select arbitrary initial state i^k
3. repeat beyond stationarity
   - 3.1 draw candidate state j from **proposal distribution** $q(i^k, j)$
   - 3.2 compute **acceptance probability**

$$\alpha(i^k, j) = \min\left(\frac{b(j)q(j, i^k)}{b(i^k)q(i^k, j)}, 1\right)$$

   - 3.3 with probability $\alpha(i^k, j)$, let $i^{k+1} = j$; else, let $i^{k+1} = i^k$
   - 3.4 increase k by one

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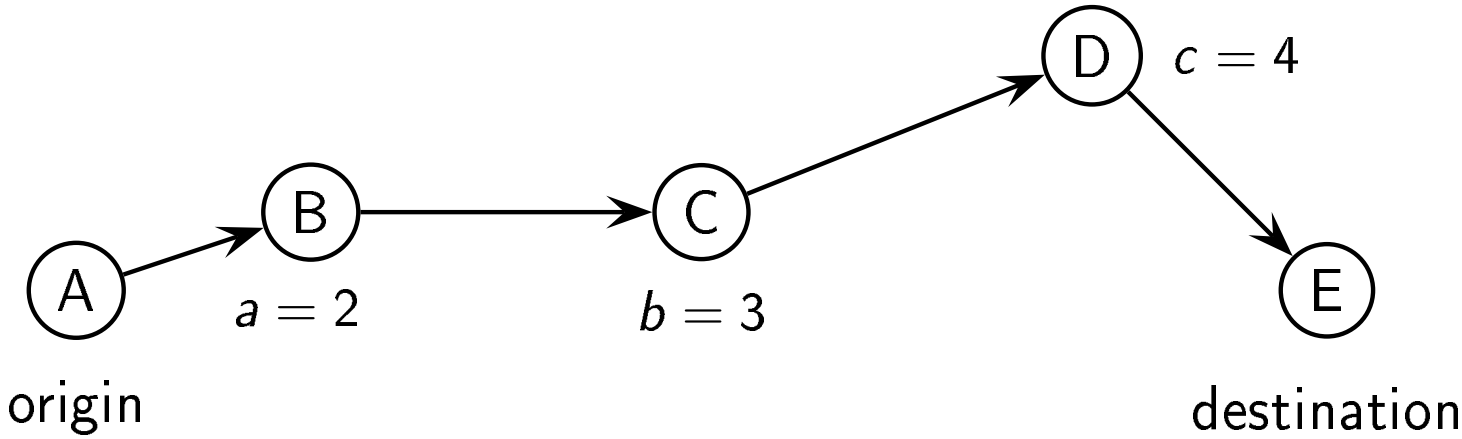
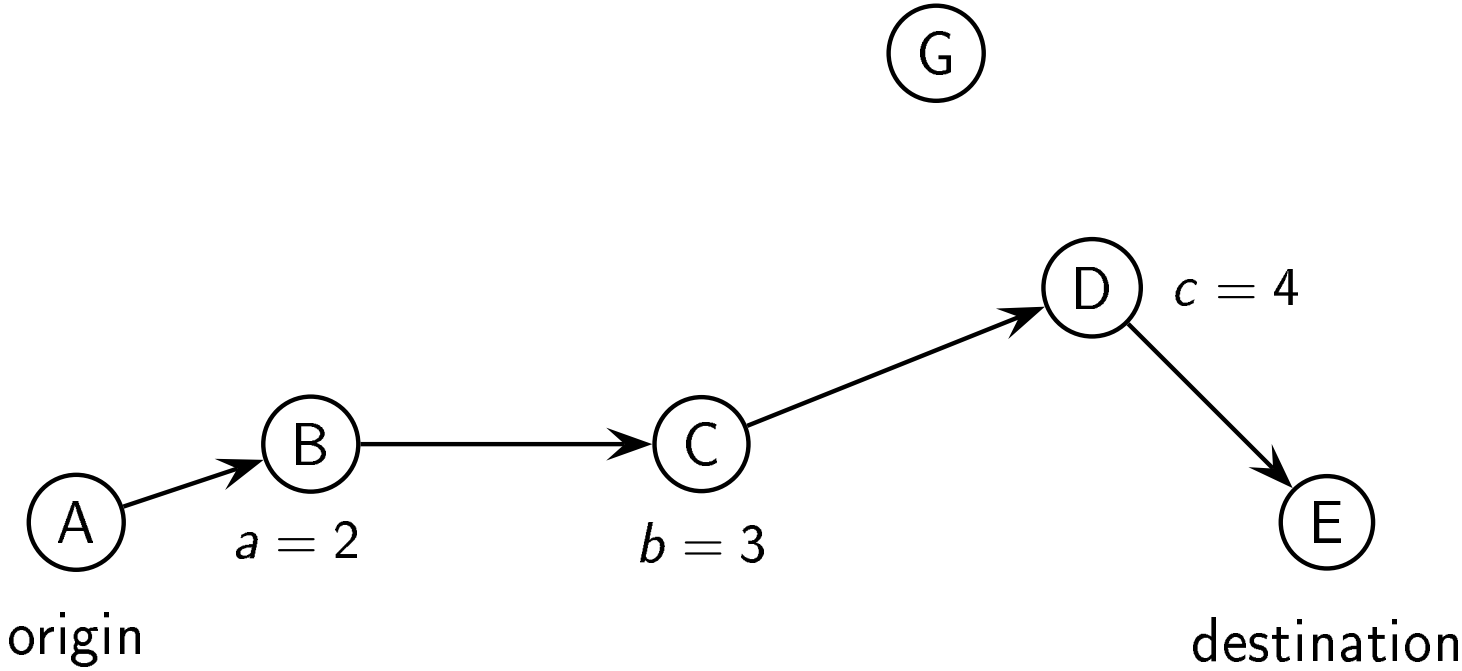
Building blocks

- a state consists of
  - a path
  - three anchor points within the path
- weights $b(i) = \exp[-\mu\delta(i)]$ decrease with path length $\delta(i)$
- proposal distribution "splices" path segments together

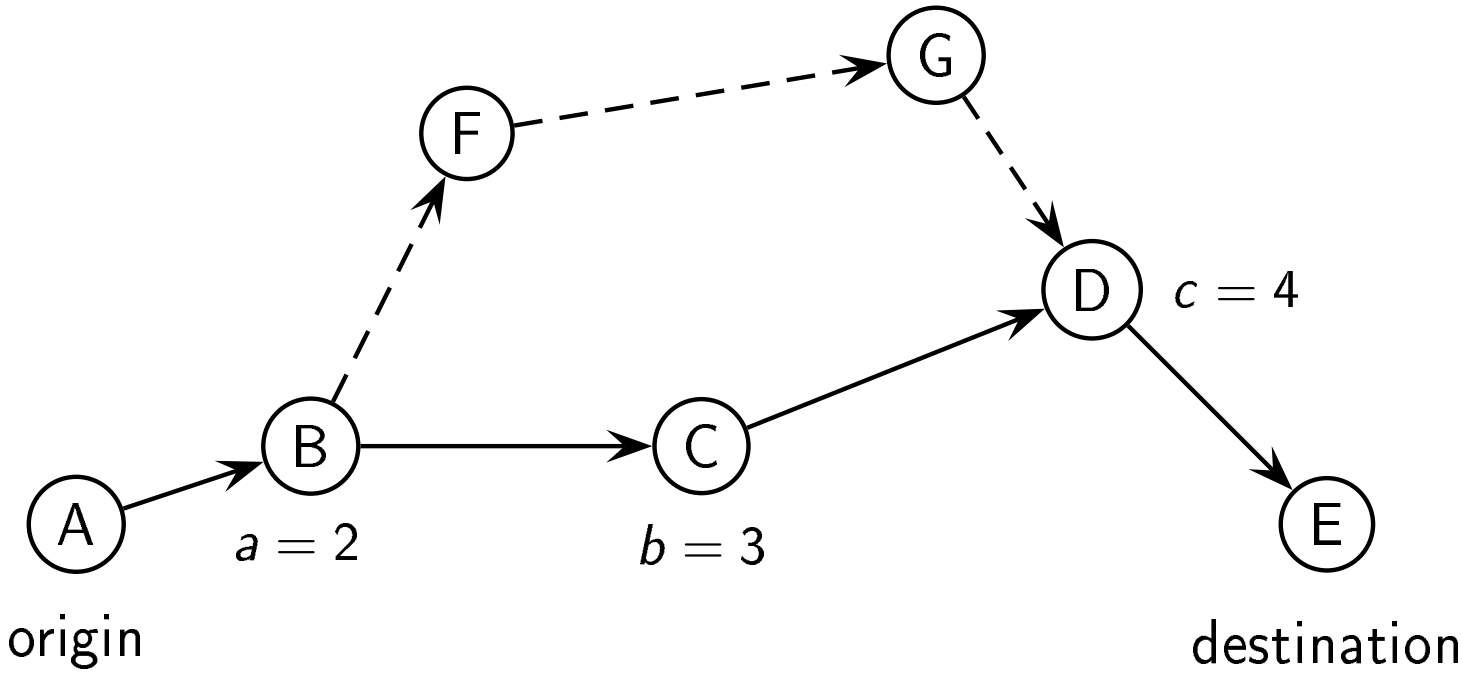
Example



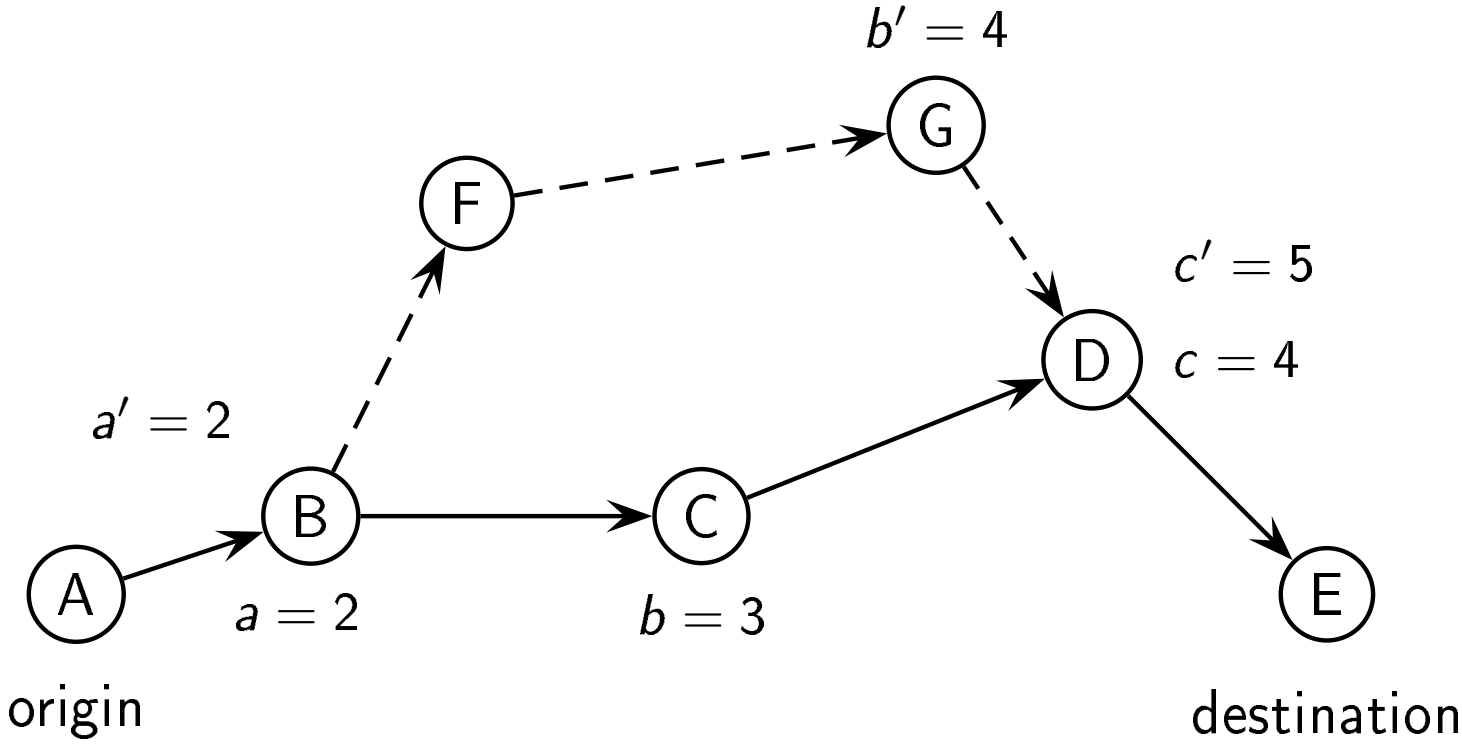
Example



Example



Example



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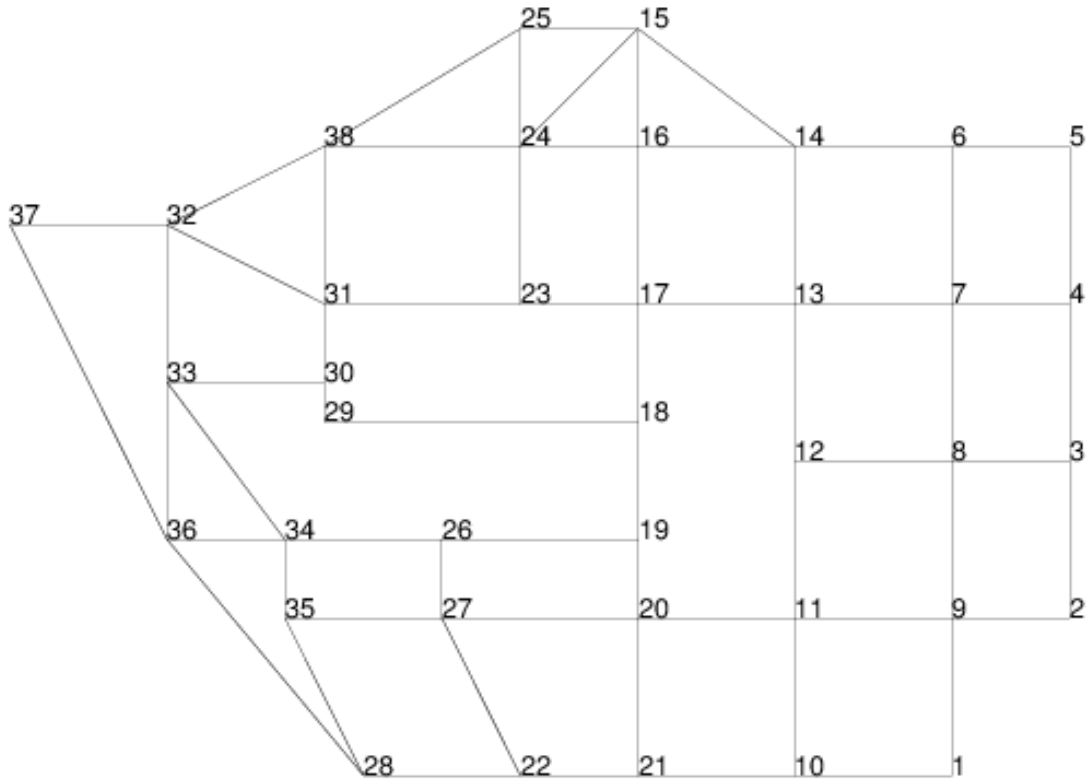
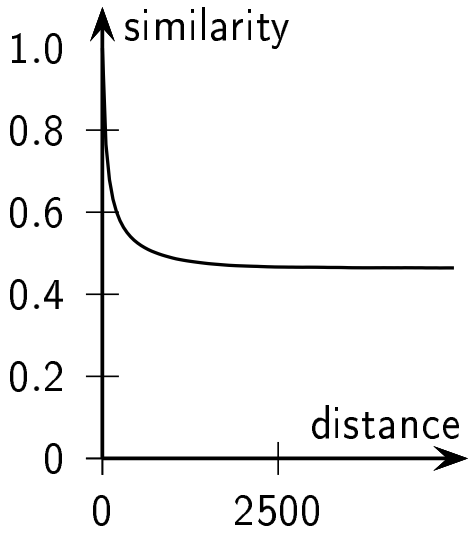
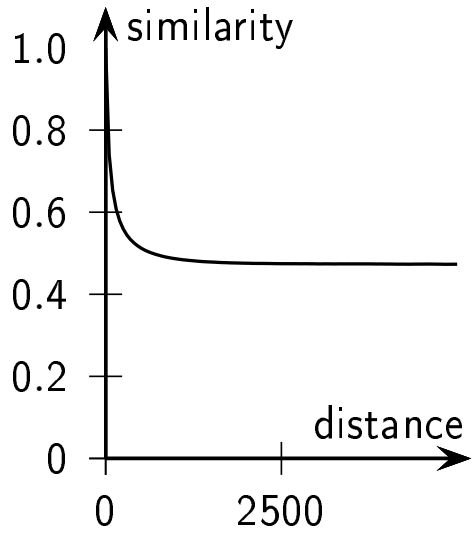
Simple example



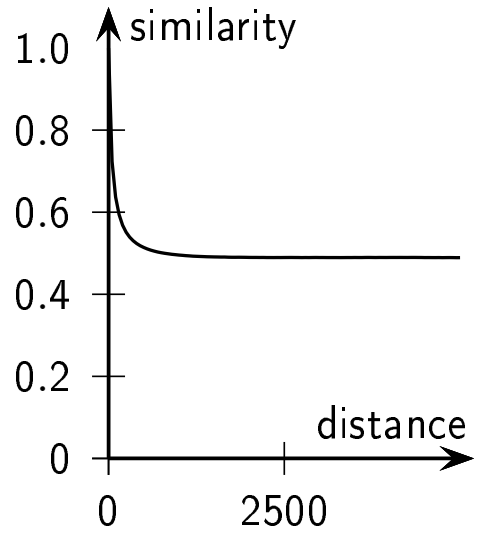
Simple example: correlation within the chain



(a) $\mu = 0.0$



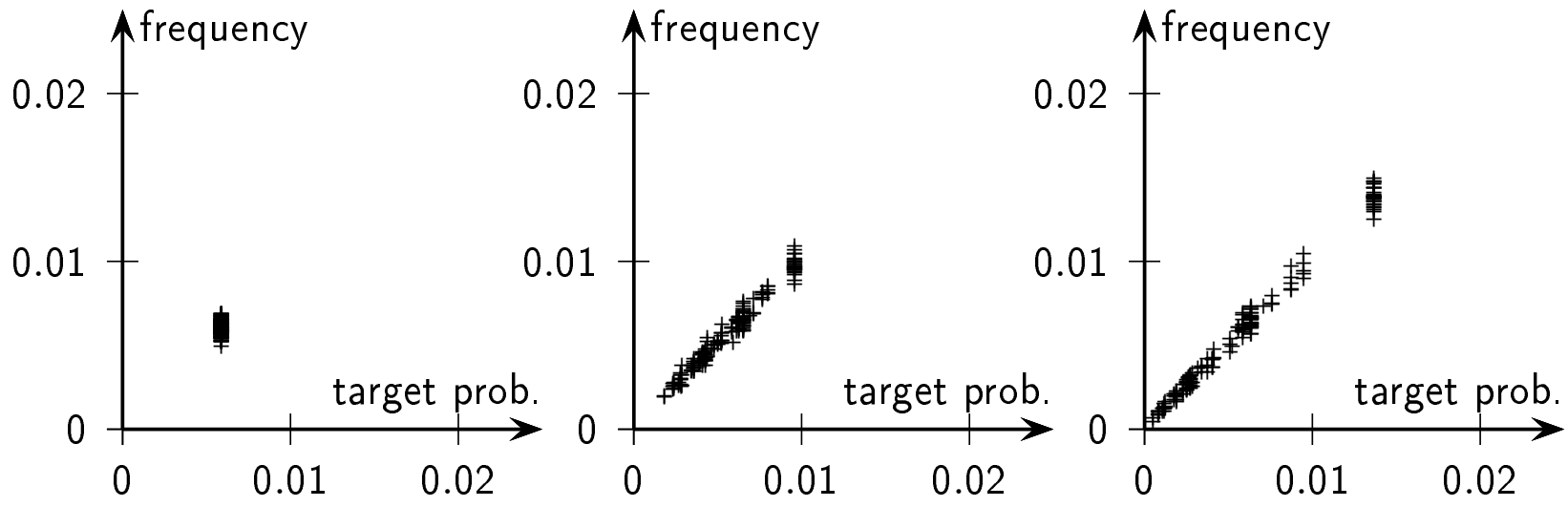
(b) $\mu = 2.0$



(c) $\mu = 4.0$

- for independent draws, extract every 2500th path

Simple example: scatterplots



(a) $\mu = 0.0$ (b) $\mu = 2.0$ (c) $\mu = 4.0$

- χ^2 statistics indicate proper functioning
  - test statistics: 156.74, 188.28, 164.42 for $\mu = 0, 2, 4$
  - 0.5, 0.9, 0.95 quantiles: 168.33, 192.95, and 200.33

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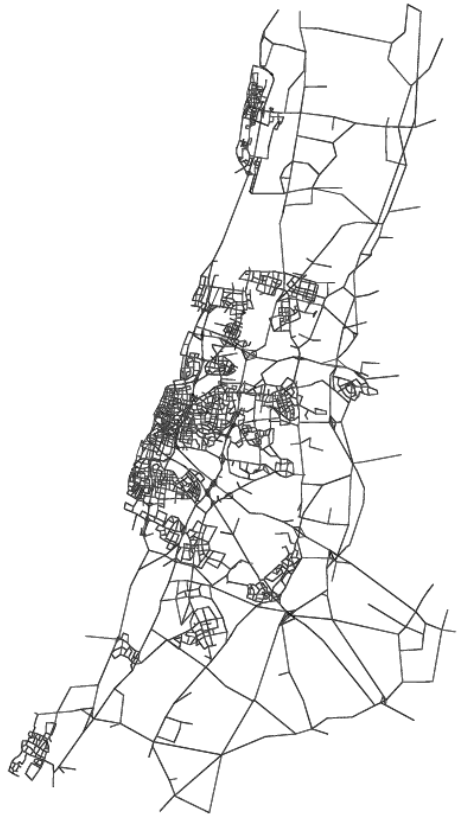
Tel-Aviv example: network



Tel-Aviv example: computational performance

μ	nodes in subnetwork	link in subnetwork	comp. time for 10^5 iterations	comp. share for shortest paths
0.04	802	395	47 seconds	77 %
0.02	1627	774	109 seconds	80 %
0.01	2987	1362	196 seconds	91 %

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Summary

- sampling of paths in general networks from general distributions
- implemented in new BIOROUTE
- most recent version of paper:
  `http://transp-or.epfl.ch/documents/technicalReports/FloeBier11.pdf`