Optimal Inventory Policies with Non-stationary Supply Disruptions and Advance Supply Information

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July 11, 2011

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Introduction and Motivation

- A manufacturer with supplier uncertainty
- Supply is either fully available or unavailable
- Supply availability is time-dependent
  - seasonality
  - scarcity of the resource
  - supplier's contract with other manufacturers
- Supplier provides information on future availability (ASI)
  - periods of unavailability
  - ASI is not modified

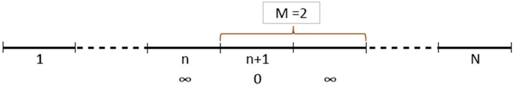
Introduction and Motivation

- Manufacturer faces deterministic non-stationary demand
  - firm production quantities from an upstream stage (MPS)
- Decision of timing and quantity of orders
  - as a function of ASI
  - considering expected holding, backorder and fixed costs
- A supply system with
  - non-stationary inter-delivery times
  - the supplier keeps track of the manufacturer's inventory level
  - partial knowledge of delivery times is revealed

Relevant Literature

- Uncertain supply/capacity models
  - Henig and Gerchak (1990)
  - Yano and Lee (1995)
  - Ciarallo et al. (1994)
  - Iida (2002)
  - Güllü et al. (1996), (1997)
- Markovian Supply Uncertainty
  - Parlar, Wang and Gerchak (1995)
  - Song and Zipkin (1996)
- Recent papers on ASI
  - Altug and Muharremoglu (2008)
  - Jaksic et al. (2009)

Description of the Model



$q_n = \infty \quad w_n = (0, \infty) \quad z_n = (q_n, w_n)$

- Supply State:
  - $z_n = (q_n, w_n)$
  - A vector of size $M + 1$: 0 and ∞'s
  - $w_n = (o_l, r_{M-l})$, $l = 0, 1, \ldots, M$

- Single period cost

$$L_n(y) = h \ \max(0, y - D_n) + b \ \max(0, D_n - y)$$

- DP recursion

$$\begin{aligned} C_n(I, z_n) &= \min_{I \leq y \leq I + q_n} \{A \ \delta(y - I) + L_n(y) \\ &+ E[C_{n+1}(y - D_n, w_n, Q_{n+M+1})]\} \end{aligned}$$

Optimal Policy, $A > 0$

- auxiliary function

$$G_n(y, w_n) := L_n(y) + E[C_{n+1}(y - D_n, w_n, Q_{n+M+1})]$$

$$C_n(I, z_n) = \min_{I \leq y \leq I + q_n} \{A\ \delta(y - I) + G_n(y, w_n)\}$$

Theorem

(a) $G_n(y, w_n)$ is A-convex in y for all w_n
(b) the optimal ordering policy is a state dependent $(s_n(w_n), S_n(w_n))$ policy where $S_n(w_n)$ minimizes $G_n(y, w_n)$ and $s_n(w_n)$ is the smallest value of y for which $G_n(y, w_n) = A + G_n(S_n(w_n), w_n)$
(c) $C_n(I, z_n)$ is A-convex in I for all z_n and it is minimized at $S_n(w_n)$

Optimal Policy, $A = 0$

Theorem

(a) $G_n(y, w_n)$ is convex in y. Let the minimum of $G_n(y, w_n)$ be at $y_n(w_n)$
(b) $C_n(I, z_n) = C_n(I, q_n, w_n)$ is convex in I and it is minimized at $I = y_n(w_n)$
(c) the optimal ordering policy is of order-up-to type. The ordering quantity at the beginning of the period is $u_n(w_n) = \max\{y_n(w_n) - I, 0\}$

Characterization for $A = 0$

- First thing to notice

$$y_n(w_n) \geq D_n \text{ for all } n = 1, 2, ..., N$$

- Things get a little simpler for $A = 0$:

Characterization for $A = 0$

When there is at least one supply period in the ASI horizon...

Lemma

(a) $y_n(r_M) = D_n$ (where the next period's supply is known to be ∞)
(b) Let $w = (o_l, r_{M-l})$ for some $l \in 1, 2, ..., M-1$. Then there exists $K(w) \in 1, 2, ..., l+1$ such that $y_n(w) = D(n, n+K(w)-1)$. Moreover,

$$K(w) = \begin{cases} 1 & \text{if } h \geq lb, \\ j & \text{if } h \in \left\{\frac{l-j+1}{j}b, \frac{l-j+2}{j-1}b\right\} j = 2, ..., l, \\ l+1 & \text{if } h < \frac{1}{l}b. \end{cases} \quad (1)$$

Remark: Does not depend on n and the demand values.

Characterization for $A = 0$

When there is no supply period in the ASI horizon, $y_n(o_M)$:

- Easy: $y_N(o_M) = D_N$
- Using Lemma: $y_n(o_M) \leq D_n + y_{n+1}(o_M)$
- Suppose that $y_{n+1}(o_M) = D(n+1, n+K)$ (K-period demand)
- For fixed $j \in \{1, 2, \ldots, K\}$ set $y = D(n, n+j)$
- For a small $\eta > 0$
- $G_n(y-\eta, o_M) - G_n(y, o_M) \leq 0$ iff

$$\frac{\sum_{i=j}^{N-n} \mathcal{P}_n(i)}{1 + \sum_{i=1}^{N-n} \mathcal{Q}_n(i,j)} \geq \frac{h}{h+b}$$

Characterization for $A = 0$

- $\mathcal{P}_n(i)$ is the probability that supply does not become available for the periods $n+1, ..., n+i$:

$$\mathcal{P}_n(i) = \begin{cases} 1 & \text{if } i < M+1 \\ \prod_{k=M+1}^{i} (1 - p_{n+k}) & \text{if } i \geq M+1 \end{cases}.$$

- $\mathcal{Q}_n(i,j)$ is the probability that the inventory level can not be raised to the optimal order-up-to level in periods $n+1, ..., n+i$ whenever the starting inventory at the beginning of period n is $D(n, n+j)$.
- $\mathcal{Q}_n(i,j)$ can be obtained efficiently in a recursive manner using the first hitting time probabilities of an appropriately constructed non-stationary Markov chain.

Characterization for $A = 0$

Theorem

The optimal order-up-to level $y_n(o_M)$ is equal to K_n period demand $D(n, n + K_n - 1)$ for some $1 \leq K_n \leq N - n + 1$ with $K_N = 1$. If $y_{n+1}(o_M) = D(n + 1, n + K)$, then $y_n(o_M) = D(n, n + K')$ where

$$K' = \max\{j = 1, 2, \ldots, K : \frac{\sum_{i=j}^{N-n} \mathcal{P}_n(i)}{1 + \sum_{i=1}^{N-n} \mathcal{Q}_n(i, j)} \geq \frac{h}{h + b}\}$$

If no such K' exists, then $y_n(o_M) = D_n$.

Algorithm

Step 0. $K = 1 \;\; (y_N(o_M) = D_N)$

Step 1. For $n = N - 1$ to 1, find $K^{'}$ satisfying the Theorem.

Set $y_n(o_M) = D(n, n + K^{'})$ and $K = K^{'} + 1$.

If no such $K^{'}$ exists, set $y_n(o_M) = D_n$ and $K = 1$.

Numerical Findings: Parameters

- Demand is generated by discretizing Gamma distribution:
  - $\mu \in \{5, 10, 15\}$
  - $cv \in \{0.1, 0.5, 1\}$
- 12 periods
- Stationary availability probabilities: $\{0.1, 0.5, 0.9\}$
- $h = 1$, $b \in \{5, 10\}$, $A \in \{5b, 10b\}$.

Numerical Findings: Effect of ASI horizon

Stationary Probabilities and for $b = 5$ and $A = 0$

p=0.1	cv_1			cv_2			cv_3		
	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3
%VOI$_{0-1}$	0.240	0.240	0.240	0.237	0.238	0.237	0.244	0.241	0.230
%VOI$_{1-2}$	0.396	0.396	0.396	0.391	0.393	0.390	0.404	0.395	0.378
%VOI$_{2-3}$	0.444	0.444	0.444	0.438	0.439	0.438	0.457	0.439	0.422
p=0.5	cv_1			cv_2			cv_3		
	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3
%VOI$_{0-1}$	16.25	16.20	16.19	16.00	16.12	16.02	16.18	16.50	15.83
%VOI$_{1-2}$	15.86	15.83	15.79	15.57	15.67	15.53	15.74	16.07	15.20
%VOI$_{2-3}$	8.65	8.64	8.61	8.44	8.54	8.46	8.54	8.72	8.13
p=0.9	cv_1			cv_2			cv_3		
	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3
%VOI$_{0-1}$	67.56	67.42	67.37	66.85	67.28	67.22	67.42	67.92	66.56
%VOI$_{1-2}$	14.69	14.62	14.59	14.31	14.49	14.36	14.59	15.03	14.05
%VOI$_{2-3}$	1.10	1.10	1.09	1.07	1.08	1.08	1.09	1.12	1.03

- %VOI increases as the supply availability probability increases.
- There is a diminishing rate of return of ASI for medium and high availability of supply.

Numerical Findings: Effect of non-stationarity

For $h = 1$, $b = 5$, $A \in \{0, 20\}$

Demand Cases

Demand	Period 1	Period 2	Period 3	Period 4
1	5	15	25	35
2	35	25	15	5
3	20	20	20	20
4	10	10	10	50
5	30	10	30	10

Probability Scenarios

Scenarios	Period 1	Period 2	Period 3	Period 4
Scenario 1	0.9	0.9	0.9	0.9
Scenario 2	0.9	0.9	0.1	0.1
Scenario 3	0.9	0.1	0.9	0.1
Scenario 4	0.1	0.1	0.1	0.1
Scenario 5	0.1	0.1	0.9	0.9
Scenario 6	0.1	0.9	0.1	0.9

Numerical Findings: Effect of non-stationarity

For $h = 1$, $b = 5$, $A = 0$

Order-up-to levels (number of periods covered)

	A=0						
ASI	**M=2**				**M=1**		**M=0**
	$y(0,0)$	$y(0,\infty)$	$y(\infty,0)$	$y(\infty,\infty)$	$y(0)$	$y(\infty)$	y
Scenario 1	3	2	1	1	2	1	1
Scenario 2	4	2	1	1	4	1	2
Scenario 3	4	2	1	1	2	1	2
Scenario 4	4	2	1	1	4	1	4
Scenario 5	3	2	1	1	2	1	2
Scenario 6	3	2	1	1	3	1	1

- In the absence of fixed costs, the optimal policy is insensitive to the non-stationarity of demand.

Numerical Findings: Effect of non-stationarity ($A = 20$)

ASI		M=2				M=1		M=0	
Scenario 1	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	3	2	1	2	2	2	2	22.33
	2	4	2	1	1	2	1	1	11.31
	3	3	2	1	1	2	1	2	9.14
	4	3	2	1	2	3	2	3	19.86
	5	3	2	1	2	2	2	2	13.67
Scenario 2	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	4	2	1	2	4	1	2	22.64
	2	4	2	1	1	4	1	4	18.21
	3	4	2	1	1	4	1	2	23.03
	4	4	2	1	2	4	1	2	17.43
	5	4	2	1	2	4	1	4	19.93
Scenario 3	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	4	2	1	2	2	2	2	17.88
	2	4	2	1	1	2	1	2	8.97
	3	4	2	1	1	2	1	2	12.85
	4	4	2	1	2	2	2	2	12.40
	5	4	2	1	2	2	2	2	17.97
Scenario 4	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	4	2	1	2	4	1	4	0.43
	2	4	2	1	1	4	1	4	0.05
	3	4	2	1	1	4	1	4	0.17
	4	4	2	1	2	4	1	4	0.53
	5	4	2	1	2	4	1	4	0.13
Scenario 5	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	3	2	1	2	2	2	2	7.39
	2	4	2	1	1	2	1	2	0.14
	3	3	2	1	1	2	1	2	0.44
	4	3	2	1	2	3	2	3	8.83
	5	3	2	1	2	2	2	2	0.63
Scenario 6	**Demand**	$S(0,0)$	$S(0,\infty)$	$S(\infty,0)$	$S(\infty,\infty)$	$S(0)$	$S(\infty)$	S	$\%VOI_{0-2}$
	1	3	2	1	2	3	1	3	9.25
	2	4	2	1	1	4	1	4	0.86
	3	3	2	1	1	3	1	3	3.64
	4	3	2	1	2	3	3	3	10.06
	5	3	2	1	2	3	1	1	0.63

Numerical Findings: Effect of non-stationarity ($A = 20$)

- ASI does not only make the system less costly to operate, but it also makes it more robust in terms of dependency on the demand pattern.
- ASI that signals an upcoming supply period decreases the system's desire to stock against supply scarcity.
- Similarly, ASI that signals an upcoming supply scarcity elevates the order-up-to levels protecting the system from shortage.
- Not only the content of ASI, but also the mere existence of ASI may change the optimal solution. This is because the overall uncertainty of the problem decreases as the length of ASI horizon increases.
- The existence of the fixed ordering cost triggers intricate cost interactions in the system, which makes it difficult to draw simple conclusions regarding the order-up-to level as a function of ASI.

A Heuristic for the non-zero fixed cost case

- A Silver-Meal based heuristic
- T: the number of periods of coverage
- In a period with supply availability
  - Decide on T that will minimize fixed, holding, and expected backorder costs per period, based on available ASI
  - Once a new supply period is encountered, check if a new order should be given.

Performance of the heuristic algorithm

p = 0.1		cv_1			cv_2			cv_3			**average %av**
		μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	
b=5, A=25	**%av**	0.71	0.83	0.83	0.67	0.75	0.86	2.11	1.69	1.39	**0.8**
b=5, A=50	**%av**	0.72	0.71	0.77	0.63	0.61	0.69	2.10	1.60	1.29	
b=10, A=50	**%av**	0.09	0.10	0.14	0.13	0.17	0.17	2.14	1.39	1.07	
b=10, A=100	**%av**	0.24	0.15	0.17	0.23	0.18	0.19	2.26	1.42	1.07	
p = 0.5		cv_1			cv_2			cv_3			**average %av**
		μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	
b=5, A=25	**%av**	0.47	2.12	1.25	2.41	5.24	5.95	4.09	5.15	5.47	**5.6**
b=5, A=50	**%av**	1.85	0.25	3.14	1.77	2.55	3.75	3.29	4.12	4.23	
b=10, A=50	**%av**	2.68	6.87	6.15	6.12	10.45	12.22	11.00	13.32	14.47	
b=10, A=100	**%av**	2.72	2.47	4.03	4.53	6.81	8.41	9.85	11.63	11.72	
p = 0.9		cv_1			cv_2			cv_3			**average %av**
		μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	μ_1	μ_2	μ_3	
b=5, A=25	**%av**	0.85	1.53	1.09	2.33	3.19	3.63	3.61	3.76	5.13	**3.2**
b=5, A=50	**%av**	5.23	1.52	0.79	2.86	2.21	2.75	3.33	2.78	4.29	
b=10, A=50	**%av**	3.22	2.51	0.51	3.16	3.93	5.72	4.47	6.04	7.96	
b=10, A=100	**%av**	0.67	3.10	0.22	2.88	2.91	2.58	3.62	4.39	5.36	

- Heuristic method performs best for $p = 0.1$.
- $p = 0.5$ case has the biggest variability in terms of the supply availability and therefore heuristic algorithm performs better in low and high availability cases.

Conclusions

- A stylized model
  - Deterministic demand
  - All-or-nothing type supply
- Analytical findings
  - non-stationary demand
  - non-stationary supply and supply availability information

Future Studies

- Extensions
  - Random demand
  - More than a single product/location (shared supply)
  - Variations of the supply process and ASI structure

Thank you for your attention !

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