Exact Solution Approaches to the Berth Allocation Problem in Bulk Sea Port Terminals

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Outline

- **Motivation**
- **Introduction to bulk ports**
  - Bulk port operations and equipment
  - Case Study of SAQR
- **Research Challenges and Objectives**
- **Berth Allocation Model**
  - Mixed Integer Linear Model
  - Set Partitioning Approach
- **Preliminary Results and Analysis**
- **Conclusions and Future Work**
Motivation

- Bulk port terminals have received significantly less attention than container terminals in the field of large scale optimization.

- High level of uncertainty in bulk port operations due to weather conditions, mechanical problems etc.
  - Disrupt the normal functioning of the port
  - Require quick real time action.

- The major objective of planning robust port operations is to minimize operational costs while maximizing system reliability.
Bulk terminal operations

- Vessel and Berth Activities

- Ship Loading or Discharge
- **Apron to Storage Transfer**
  - Pipelines
  - Loading Shovels
  - Wheel loaders

- **Storage**

- **Intermodal transfer and inland distribution**
Case Study: SAQR Port, Ras Al Khaimah, UAE

- Biggest bulk port in the entire middle east, handling 30 million tons of bulk and assorted cargo annually

- Deals with a wide variety of imported and exported commodities - aggregates, cement, coal, clinker, iron ore, feldspar, clay, soda ash, petroleum products etc.

- Wide range of equipment facilities including MHC’s, load shovels, mini, wheel and ship loaders etc.
Port Layout

- 12 berths, with alongside depth of 12.2 meters at mean low water spring tide
- 8 x 200 meters bulk handling berths, 3 x 200 meters container handling berths and 1 general purpose roll-on/roll-off berth
- Conveyors at berths 5 and 7; pipelines at berths 6, 7 and 11 (variable demand across different berths)
Research challenges

- **Key issues and sources of disruption at SAQR**: High waiting times and delays at berths owing to
  
  - Congestion at berths
  
  - Unavailability of required number and type of equipment when needed
  
  - *Uncertainty in arrivals* of vessels and cargo trucks
Research Objectives

- *Integration of the two crucial problems of berth allocation and yard allocation* for better coordination between berthing and yard activities

- Include *robustness* in planning process to account for uncertainties in arrival times of vessels and cargo trucks which lead to unforeseen disruptions and delays in operations.

- Develop methodologies and algorithms that can be extended to other domains such as container ports, railways and airlines.
The Berth Allocation Problem
Problem Definition

- **Find**
  - Berthing assignment and schedule of vessels along the quay

- **Given**
  - Time windows on arrivals of vessels
  - Handling times dependent on berthing position and cargo type

- **Objective**
  - Minimize total service times of vessels berthing at the port
Discretization

Discrete Layout

Continuous Layout

Hybrid Layout

Our Model
BAP Model

Objective Function

\[
\min \sum_{i \in N} \left( m_i - a_i + h_i \right)
\]

Decision variables:

- \( m_i \) starting time of handling of vessel \( i \in N \);
- \( a_i \) arrival time of vessel \( i \in N \);
- \( h_i \) total handling time of vessel \( i \in N \);
BAP Model

Dynamic vessel arrival constraints

\[ m_i - a_i \geq 0 \quad \forall i \in N, \]
\[ a_i = A_i r_i + U_i (1 - r_i) \quad \forall i \in N, \]

\[ A_i \quad \text{expected arrival time of vessel } i \in N; \]
\[ U_i \quad \text{upper bound to the arrival time of vessel } i \in N; \]
BAP Model

Non overlapping constraints

\[
\sum_{k \in M} (b_k s_k^j) + B(1 - y_{ij}) \geq \sum_{k \in M} (b_k s_k^i) + L_i \quad \forall i, j \in N, i \neq j,
\]

\[
m_j + B(1 - z_{ij}) \geq m_i + C_i \quad \forall i, j \in N, i \neq j,
\]

\[
y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1 \quad \forall i, j \in N, i \neq j,
\]

Section covering constraints

\[
\sum_{k \in M} s_k^i \geq 1 \quad \forall i \in N,
\]

\[
\sum_{k \in M} (b_k s_k^i) + L_i \leq L \quad \forall i \in N,
\]

\[
\sum_{p \in M} x_{ipk} s_p^i = x_{ik} \quad \forall i \in N, \forall k \in M,
\]
Draft Restrictions

\[(d_k - D_i) x_{ik} \geq 0 \quad \forall i \in N, \forall k \in M,\]

Determination of Handling Times

\[h_i \geq h_{ik}^w p_{ilk} Q_i s_l^i \quad \forall i \in N, \forall k \in M, \forall l \in M, \forall w \in W_i\]

- \(Q_i\) quantity of cargo to be loaded on or discharged from vessel \(i\)
- \(h_{ik}^w\) handling time for unit quantity of cargo \(w\) when vessel \(i \in N\) is berthed in section \(k \in M\);
Generation of Instances

- Instances based on data from SAQR port
  - Quay length of 1600 meters and vessel lengths in the range 80-260 meters
- Test instances for $|N| = 5, 10$ and 15 vessels, and $|M| = 10, 20$ and 30 sections
- Rate of handling is 15 hours per $10^4$ tonnes per crane, and number of cranes dependent on length of each section
- Drafts of all vessels $D_i$ are less than the minimum draft along the quay.
- Instances solved using commercially available CPLEX 12.1 solver!
Preliminary Results and Analysis

$|N| = 5$ vessels, and $|M| = 10, 20$ and $30$ sections

<table>
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<th>5 x 10</th>
<th>5 x 20</th>
<th>5 x 30</th>
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<tr>
<td></td>
<td>obj</td>
<td>t(s)</td>
<td>obj</td>
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<tr>
<td>Large Time Horizon</td>
<td>52.50 0.34</td>
<td>66.59 0.53</td>
<td>66.59 0.61</td>
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<td>52.50 0.27</td>
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<td>60.00 0.50</td>
<td>66.59 0.43</td>
<td>66.59 0.64</td>
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</table>

- All instances solved in less than a second
\( |N| = 10 \) vessels, and \( |M| = 10, 20 \) and 30 sections

<table>
<thead>
<tr>
<th></th>
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<tr>
<td><strong>10 x 10</strong></td>
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<td><strong>10 x 30</strong></td>
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<tr>
<td>Large Time Horizon</td>
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</tr>
<tr>
<td>Small Time Horizon</td>
<td>123.60</td>
<td>17.56</td>
</tr>
</tbody>
</table>

- All (except one) instances solved within 1 hour of computation time
Very few instances solved within CPLEX time limit of 1 hour!

\( |N| = 15 \) vessels, and \( |M| = 10, 20 \) and 30 sections

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<td>136.66</td>
<td>17.67%</td>
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<tr>
<td>Small Time Horizon</td>
<td>166.82</td>
<td>30.09%</td>
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Sensitivity Analysis: Effect of discretization and congestion!

<table>
<thead>
<tr>
<th></th>
<th>Discretization I</th>
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<td>104.71</td>
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<td>115.74</td>
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<td>congested</td>
<td>118.39</td>
<td>537.48</td>
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<tr>
<td>congestion free</td>
<td>118.84</td>
<td>1.87</td>
<td>117.50</td>
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<tr>
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<tr>
<td>congested</td>
<td>127.60</td>
<td>19.78</td>
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</tr>
</tbody>
</table>

- Increase in objective function value and computation time with congestion!
- Discretization I better than II
Sensitivity Analysis: Effect of vessel length

- Test for 4 pairs of instances with everything same except vessel lengths
  - one instance with vessel lengths in the range 80 -120 meters with vessel arrivals close together
  - other instance with vessel lengths in the range 180-220 meters with vessel arrivals close together

5 vessels: % difference in OFV for long and small vessels

10 vessels: % difference in OFV for long and small vessels
Generalized Set Partitioning Model

- Used in context of container terminals by Christensen and Holst (2008)

- Generate set $P$ of columns, where each column $p \in P$ represents a feasible assignment of a single vessel in both space and time

- Generate two matrices

  - Matrix $A = \left( A_{ip} \right)$; equal to 1 if vessel $i \in N$ is the assigned vessel in the feasible assignment represented by column $p \in P$

  - Matrix $B = \left( b_{st}^{p} \right)$; equal to 1 if section $s \in M$ is occupied at time $t \in H$ in column $p \in P$

Note: Assume integer values for all time measurements
GSPP Model Formulation

Objective Function:

\[
\min \sum_{p \in P} \left( d_p \lambda_p + h_p \lambda_p \right)
\]

Constraints:

\[
\sum_{p \in P} \left( A_{ip} \lambda_p \right) = 1 \quad \forall \ i \in N
\]
\[
\sum_{p \in P} \left( b_{st}^{st} \lambda_p \right) \leq 1 \quad \forall \ s \in M, \forall \ t \in H
\]

\[d_p : \text{delay in service associated with assignment } p \in P\]

\[h_p : \text{handling time associated with assignment } p \in P\]

\[\lambda_p : \text{binary parameter, equal to 1 if assignment } p \in P \text{ is part of the optimal solution}\]
## Comparison between GSPP and MILP formulations

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- GSPP model is very fast, but memory issues due to large number of variables
- For small planning horizon, GSPP model can solve very big instances
Summary of Results

- Preliminary results inspired by port data show that the problem is complex and general purpose solvers fail to produce good solutions as soon as the problem size increases.

- Sensitivity Analysis
  - Choosing the most appropriate discretization is critical!
  - Complexity increases significantly when vessel arrivals are close together!
  - Optimal Objective Function Value is more sensitive to vessel lengths for larger number of sections
  - Limitations: It can only be performed for small instance sizes up to 10 vessels.

- GSPP approach is extremely fast, but memory issues for large instances
Ongoing and Future Work

- **Experimental Analysis**
  - Impact of parameters
  - Robust vs. Non-Robust: For given berthing schedule, test different scenarios of arrival delays and different distributions for arrivals times (such as triangular, uniform etc.)

- **For bigger sized instances**
  - Use dynamic column generation approach to resolve memory issues with the GSPP Model
  - Possibly explore heuristic approaches for faster results

- Integration of the berth allocation problem with yard allocation
Thank you!