

# **Optimization of evacuation instructions as a fixedpoint problem**

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**STRC 2011** 

May 2011



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May 2011

# Abstract

In this paper, a new framework is presented to optimize evacuation instructions. By giving optimized instructions to the evacuees (for example about which route to take), traffic conditions and, therefore, the evacuation efficiency are optimized. To solve the problem in an efficient way, the problem is decomposed into three simpler problems, namely the optimization of turning fractions, the optimization of instructions given the turning fractions, and the approximation of compliance behavior. Mutual consistency of these sub-problems is enforced through a fixed-point formulation.

# Keywords

Evacuation, optimization, instructions, fixed-point problem

## **1** Introduction

When a region is threatened by a disaster, like a flood or a fire, people have to be evacuated to avoid as many casualties as possible. To be able to evacuate these people in an efficient way, instructions can be given to guide them out of the endangered area. Many methods exist to optimize traffic assignments for (in particular car-based) evacuations, see for example Afshar and Haghani (2008); Liu *et al.* (2006); Miller-Hooks and Sorrel (2008); Saadatseresht *et al.* (2009); Sbayti and Mahmassani (2006); Stepanov and Smith (2009). The results of these methods yield bounds on the system performance since the results are valid for situations in which people behave exactly like in the optimized assignment. In reality, people will most probably deviate from the system-optimal assignment for two major reasons: (1) people have a lack of information about, and a lack of experience with, the extreme situation, and (2) people act out of a user- instead of system-optimal thinking. Giving optimized instructions to the people can solve the lack of information and can steer the evacues in the direction of a system-optimal traffic assignment. In this paper, a computationally efficient method for the generation of optimal evacuation instructions is proposed that takes compliance behavior into account.

In previous work (Huibregtse *et al.* (in press, 2011)), we developed a metaheuristic for the optimization of evacuation guidance strategies under (behavioral) uncertainty. The method is able to develop optimized instructions, but is computationally quite heavy for the development of robust instructions. In this paper, we present a new computational framework, where the problem to optimize evacuation instructions is decomposed into several simpler optimization problems. Mutual consistency of these sub-problems is enforced through a fixed-point formulation. The idea for this formulation is adopted from Bottom *et al.* (1999), where the problem to generate anticipatory route guidance is formulated as a fixed-point problem, and from Bierlaire and Crittin (2006) where an efficient approach is presented to solve such a fixed-point problem.

This paper contributes to the field by presenting a new and efficient method to optimize evacuation instructions that can be used under uncertain conditions, taking preferences of the people and compliance behavior into account, and is flexible with respect to the concrete modeling assumptions of both the behavioral model and the traffic flow model.

## 2 Overview fixed-point approach

The approach is illustrated in Figure 1. The problem of generating optimal guidance for a population of evacuees is decomposed into three sub-problems:

- 1. Optimization of turning fractions.
- 2. Optimization of instructions.

3. Approximation of compliance behavior.



Figure 1: Fixed-point optimization approach

Essentially, the interplay of these components is as follows: the optimization of turning fractions adopts a mathematically convenient yet simplistic perspective on the problem, in that it assumes a single-commodity flow, for which it computes optimal turning fractions and the resulting travel times. The turning flows resulting from these turning fractions are then optimally reproduced through commodity-specific guidance in the second building block, where a complex behavioral model (e.g., based on discrete choice theory) is applied. Since the behavioral model may exhibit limited compliance, the third component then aggregates the resulting compliance behavior into bounds on the turning fractions, which are then accounted for in a repeated computation of optimal turning fractions. The loop iterates until a fixed point is attained.

The remainder of this section elaborates on the individual components of this system.

## 2.1 Optimization of turning fractions

In the first block, the turning fractions are optimized. This block takes as input the following:

- the evacuation network;
- the evacuation demand in terms of a time-dependent origin-single destination matrix (where the artificial single destination represents all safe destinations);
- the upper and lower bounds on the turning fractions (from block 3).

The upper and lower bounds on the turning fractions represent behavior that could possibly be obtained through an appropriate guidance. A computationally efficient optimization routine that exploits differentiability is used to compute turning fractions that minimize, e.g., the total time needed to evacuate all people. A macroscopic dynamic traffic flow model is embedded as a constraint in this optimization. The block produces as output the following:

- the optimized turning flows;
- time-dependent travel times on all links in the network;
- the efficiency of the evacuation.

### 2.2 Optimization of instructions

To optimize the instructions, the building block takes as input:

- the optimized turning flows (from block 1);
- the time-dependent travel times (from block 1).

The purpose of the block is to identify disaggregate (at least by origin) guidance messages (e.g., in terms or route instructions) that come as close as possible to the given turning flows, assuming fixed time-dependent travel times. There are various ways of how this problem can be specified, where a concrete instance depends on the behavioral model at hand. The output of this building block consists of:

- the route instructions;
- the resulting turning fractions;
- the instructed turning fractions.

## 2.3 Approximation of compliance behavior

In this block, the bounds on the turning fractions are updated. The input of the block consists of:

- the resulting turning fractions (from block 2);
- the instructed turning fractions (from block 2).

If the turning flows realized in the second building block coincide with those computed in the first building block, mutual consistency is attained. Otherwise, it can be concluded that the

optimization of the turning fractions was too optimistic with respect to the evacuees' compliance, which needs to be reflected through tighter bounds on the feasible turning fractions. In this block, the bounds on the turning fractions are updated based on the commodity-specific compliance to the disaggregate guidance. The output of this block is the following:

• the upper and lower bounds on the turning fractions.

## **3** Specification fixed-point approach

In this section, the different blocks of the approach are specified.

#### **3.1** Optimization of turning fractions

The optimal turning fractions  $\beta_{ij}^{0}(\hat{t})$  are determined that minimize the objective function J, where  $\beta_{ij}(t)$  is the turning fraction from link i to link j at time t. The objective represents the efficiency of the evacuation, for example the time needed to evacuate all people, or the number of arrivals (in this case, the problem is a maximization problem). The turning fractions are constrained by lower and upper bounds, respectively  $\beta_{ij}^{\ell}(t)$  and  $\beta_{ij}^{u}(t)$ :

$$\beta_{ij}^{\ell}(t) \le \beta_{ij}(t) \le \beta_{ij}^{u}(t). \tag{1}$$

The bounds follow from the third building block, and have to be initialized in the first iteration. The macroscopic dynamic traffic flow model  $f_1$  which is a constraint in the optimization problem determines the value of J, the outflow of the links  $y_i(t)$ , and the entering time-dependent travel times  $\tau_i(t)$ , as function of the network N, the demand D, and the turning fractions  $\beta_{ij}(t)$ :

$$(J, y_i(t), \tau_i(t)) = f_1(N, D, \beta_{ij}(t)).$$
(2)

Each dynamic traffic flow model that can produce the mentioned output given the input can be applied. Theoretically, specifying the optimal turning fractions for each time step t would lead to the best solution (the solution with the highest efficiency). However, in order to create route instructions that are feasible from a practical point of view, and to limit the computational complexity of the optimization both in block 1 and 2, it is preferable to make the turning fractions and the route instructions constant for sets of time steps t. This means that the upper and lower bounds should be averaged over the time steps in the set.

#### **3.2** Optimization of instructions

The route instructions are optimized such that the deviation between the optimal and the resulting turning flows, respectively  $y_{ij}^{o}(t)$  and  $y_{ij}^{r}(t)$ , is minimized. The deviation is determined as follows:

$$\sum_{t \in T, ij \in IJ} |y_{ij}^{\mathbf{o}}(t) - y_{ij}^{\mathbf{r}}(t)|, \tag{3}$$

where T is the set of all time steps t and IJ is the set of all turns ij. The optimal turning flows follow from the optimal turning fractions and outflow of the links:

$$y_{ij}^{\mathbf{o}}(t) = \beta_{ij}^{\mathbf{o}}(t)y_{i}^{\mathbf{o}}(t).$$

$$\tag{4}$$

A behavioral model produces the turning fractions as a function of the disaggregated route instructions, assuming fixed time-dependent travel times:

$$\beta_{ij}(t) = f_2(\chi^g(t), \tau_i(t)) \tag{5}$$

where  $\chi^g(t)$  is the fraction of people leaving at time t instructed to follow route g. The optimal route instructions are the instructions that minimize the deviation given in 3. The instructed turning fractions  $\beta_{ij}^{i}(t)$  follow directly from these route instructions, and the resulting turning fractions  $\beta_{ij}^{\mathbf{r}}(t)$  are the turning fractions resulting from equation 5 with the optimized route instructions as input. The resulting turning flows follow from the resulting turning fractions and outflow, computed in the same way as the optimal turning flows in equation 4. The behavioral model can be any model that produces the mentioned output as function of the input.

#### 3.3 Approximation of compliance behavior

The bounds on the turning fractions are updated based on the commodity-specific compliance to the disaggregate guidance. The lower and upper bounds on the feasible turning fractions are determined as follows:

$$\beta_{ij}^{\ell}(t) = (1 - c_i(t))\beta_{ij}^{\mathbf{p}}(t) \tag{6}$$

$$\beta_{ij}^{u}(t) = c_i(t) + (1 - c_i(t))\beta_{ij}^{\mathbf{p}}(t)$$
(7)

where  $c_i(t)$  is the aggregated compliance on upstream link level, and  $\beta_{ij}^{\mathbf{p}}(t)$  is the preferred turning fraction from link *i* to link *j* at time *t*. The turn-specific compliance is determined as

follows:

$$c_{ij}(t) = \frac{\beta_{ij}^{\mathbf{r}}(t) - \beta_{ij}^{\mathbf{p}}(t)}{\beta_{ij}^{\mathbf{i}}(t) - \beta_{ij}^{\mathbf{p}}(t)}$$
(8)

The time-specific compliance values of all turns with the same upstream link are averaged to obtain the aggregated compliance on upstream link level  $c_i(t)$ . The preferred turning fractions follow from the behavioral model given in equation 5, without route instructions.

#### 3.4 Mutual consistency

The loop iterates until a fixed-point is reached. This fixed-point can be recognized in different ways, for example by comparing the realized and the optimized turning flows: if these flows coincide, mutual consistency is attained.

#### 4 Conclusions

A fixed-point approach is presented to optimize evacuation instructions. The decomposition of the problem in simpler problems, separating the traffic flow model and the behavioral model, makes the approach efficient. While writing this paper, the approach is applied to a case study to analyze the efficiency of the approach.

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