Experimental parametric study for hydraulic design of PKWs

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ABSTRACT: The implementation of a new type of labyrinth spillway, called PKW (Piano Key Weir) reveals as a performing alternative for increasing the overflow capacity of existing dams. The optimal hydraulic design of such structure is however not obvious due to the large number of involved parameters. The present work explores experimentally the most relevant geometrical parameters, such as width, length, height and slope of keys and also the upstream and downstream flow conditions. The rating curve of a unit structure is then analysed considering the following dimensionless parameters: (*i*) the total developed crest length over PKW width (L/W), (*ii*) the inlet over outlet key widths (W_i/W_0), (*iii*) the vertical over the horizontal shapes (P/W_i), and (*iv*) the vertical height of the dam over the PKW height (P_d/P). A non-linear global stepwise regression approach is then applied to fit the most influent parameters in order to provide a mathematical formulation for the hydraulic design of a PKW. The discussion reveals that there is not only one optimal solution from the hydraulic point of view. The ideal solution can then only be selected when considering local and economical constraints related to excavation, materials, construction techniques and particularly to the downstream energy dissipation system.

1 INTRODUCTION

With the increase of hydrological data records and the development of new methodologies for flood discharge estimation, as well as higher requirements of the communities on safety issues, a large number of existing dams require spillway rehabilitation in order to improve their hydraulic capacity. For such projects, the new shape of labyrinth spillways, called Piano Key Weir (PKW) is an interesting alternative (Lempérière and Ouamane, 2003; Leite Ribeiro et al. 2009). As for labyrinth shapes, this structure provides a longer total effective crest length for a given spillway width, with the advantage that it can be placed in the upper part of most existing dams, due to its reduced base surface.

Over the last years, many efforts have been made in order to understand the hydraulic behavior of PKWs but for the moment, only few systematic laboratory tests as well as design basis can be found (Ouamane and Lempérière, 2006). Nowadays, the hydraulic behavior of PKWs is still not complete understood. All the current projects under development to assess the hydraulic capacity are based on physical modeling (Laugier 2007, Laugier et al. 2009¸ Bieri et al, 2009).

In the present research, an experimental study is conducted in order to evaluate the influence of selected geometrical parameters on the hydraulic efficiency of a PKW. The objective is (*i*) to analyze the influence of the main geometric parameters on the PKW discharge capacity, using non dimensional ratios and (*ii*) to propose an empirical formulation for the preliminary design of new PKWs.

2 PARAMETRIC STUDY

The total discharge over a PKW is a function of several parameters which can be summarized as follows:

$$Q=f(\rho, g, \nu, H, L, P, P_d, W, W_i, W_o, B, T_s, R, \alpha) \quad (1)$$

In Equation 1, the fluid is characterized by its density ρ and the kinematic viscosity ν, g is the acceleration of gravity, and H is the total upstream hydraulic head. The other parameters are related to the geometry of the PKW. L is the total developed crest length, P is the total height of the PKW, P_d is the dam height, W is the total width of the PKW, W_i and W_o are the widths of the inlet and outlet keys, B is the length of the side weir, α is the angle between the inlet/outlet key crest and the side weir of the PKW, T_s is the sidewall thickness and R is the radius of crest curvature. With exception of α, P_d and R, the other geometric parameters are shown in Figure 1.

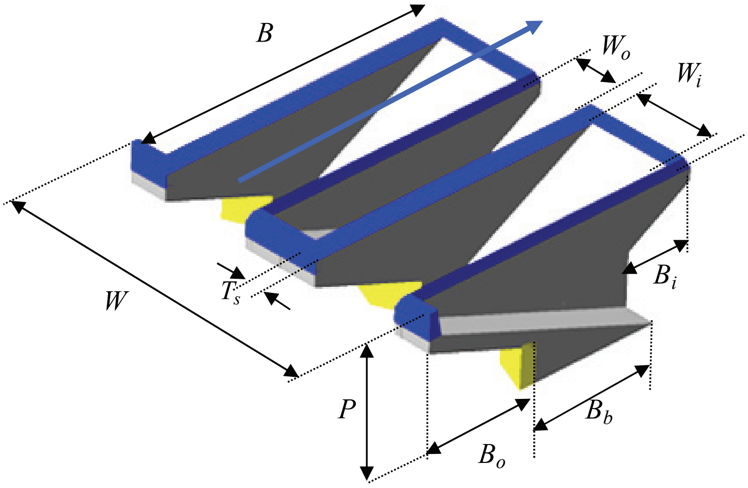


Figure 1. Fundamental parameters on an entire PKW – 3D-view (Pralong et al. 2011).

In order to analyze the PKW efficiency, a comparison to a sharp-crested weir with crest length W is made. As discussed by Leite Ribeiro et al. (2007), a PKW tends to behave like a linear weir as the upstream head increases. Consequently, a discharge enhancement ratio r between the PKW discharge (Q_{PKW}) and the corresponding rectangular sharp-crested weir discharge (Q_W) has been adopted for the analysis.

$$r=\frac{Q_{PKW}}{Q_W}=\frac{C_d L_{eff}\sqrt{2g}H^{\frac{3}{2}}}{C_d W\sqrt{2g}H^{\frac{3}{2}}} \quad (2)$$

In Equation 2, the discharge coefficient C_d is assumed as constant to 0.42, characteristic average value of sharp-crested weirs (Hager and Schleiss 2009), and L_{eff} is the effective crest length of the PKW that theoretically contributes to the overflow. L_{eff} decreases with increase of head, due to the interference of the overflow layers (Falvey 2003). For the analysis, the discharge capacity of the PKW was measured in the laboratory, while the theoretical discharge of a sharp-crested weir was calculated with reference to the measured hydraulic head.

The present investigation is only based on geometrical parameters of the PKW, thus the terms ρ, g and ν are not considered. The tested PKWs are all rectangular shaped (α=90°) and therefore α is nor included in the analysis. The length of the sidewall B can be omitted in the analysis because it is a function of L/W. Therefore, Equation 1 can be replaced by the following dimensionless relation:

$$r = \frac{L_{eff}}{W} = f\left(\frac{L}{W}, \frac{W_i}{W_o}, \frac{P}{W_i}, \frac{P_d}{P}, \frac{T_s}{P}, \frac{T_s}{R}, \frac{H}{P}\right) \tag{3}$$

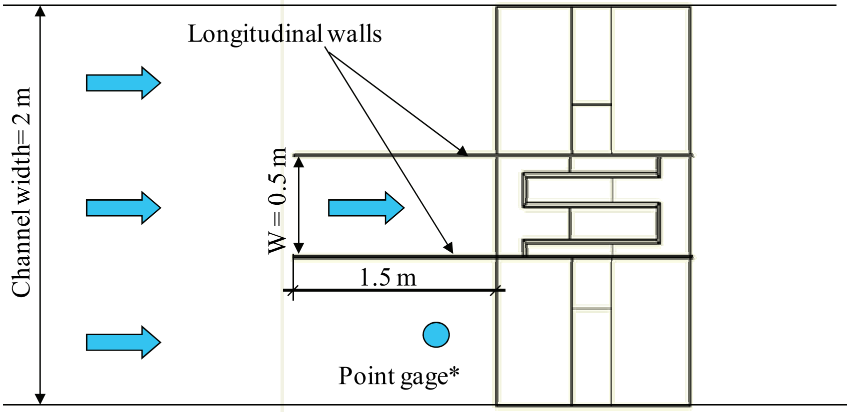
In the present study, T_s and R were maintained constant and therefore, only the parameters L/W, W_i/W_o, P_d/P, P/W_i and H/P will be discussed.

3 EXPERIMENTAL SET-UP

The experimental set-up is installed in a 2 m wide straight flume, on a platform placing the bottom of the PKW 0.5 m over the ground of the channel (P_d=0.5 m). A one and half unit configuration (1.5 inlet key + 1.5 outlet key) is constructed over a constant width (W= 0.5 m). Two longitudinal guide walls (1.5 m long) allow the water supply to be uniform when approaching the weir. A "reference PKW configuration" is defined, corresponding to the following geometrical values:

- ratio W_i/W_o = 1.25, with W_i = 0.163 m and W_o = 0.130 m ;
- ratio L/W = 5, with a total developed crest L = 2.5 m;
- height of the sidewall P = 0.217 m;

Sidewalls are 0.02 m thick (Ts=0.02 m) and have semi-circular crests. Moreover, semi-circular noses are installed under the outlet keys of the PKW. A schematic view of the experimental set-up is shown in Figure 2.



*Total head measurement $(H=h+v^2/2g)$

Figure 2. Schematic overview of the experimental set-up.

The experimental program was organized in order to allow a first analysis of the individual influence of the main dimensionless parameters. All the tested PKWs are described in Table 1. The analysis is divided into four parts.

- Influence of the dam height (P_d) on the discharge capacity of the reference PKW: For these experiments, a movable bottom was installed inside the longitudinal walls and different ratios P_d/P were tested.
- Influence of P/W_i: Different values of P were tested for the configurations with L/W=3 and 5.
- Influence of W_i/W_o: Different W_i/W_o ratios were tested with constant values of L/W=5, P=0.217 m and P_d=0.50 m. For all tests, the sum W_i+W_o was maintained constant and equal to 0.293 m.
- Influence of L/W: Three values of L/W=3, 5 and 7 were tested for constant values of W_i/W_o = 1.25, P = 0.217 m and P_d=0.50 m.

For each PKW configuration, rating curves (Q versus H) are established. Measurement of the total hydraulic head is performed outside of the guide walls by a point-gage whereas the discharge is measured by an electromagnetic flowmeter.

Table 1. Summary of the tested PKW configurations with main geometric and dimensionless parameters.

[°]	Configuration	*L*	*W*	*B*	B_i	B_o	W_i	W_o	*P*	P_d	*L/W*	W_i/W_o	P/W_i
		[m]	[m]	[m]	[m]	[m]	[m]	[m]	[m]	[m]	[-]	[-]	[-]
1	L/W=5; Wi/Wo=1.25; P/Wi=1.33	2.5	0.5	0.67	0.23	0.23	0.163	0.130	0.217	0.50	5	1.25	1.33
2	L/W=5; Wi/Wo=1.25; P/Wi=0.96	2.5	0.5	0.67	0.23	0.23	0.163	0.130	0.157	0.50	5	1.25	0.96
3	L/W=5; Wi/Wo=0.80; P/Wi=1.67	2.5	0.5	0.67	0.23	0.23	0.130	0.163	0.217	0.50	5	0.80	1.67
4	L/W=5; Wi/Wo=0.80; P/Wi=1.21	2.5	0.5	0.67	0.23	0.23	0.130	0.163	0.157	0.50	5	0.80	1.21
5	L/W=5; Wi/Wo=1.60; P/Wi=1.20	2.5	0.5	0.67	0.23	0.23	0.181	0.113	0.217	0.50	5	1.60	1.20
6	L/W=5; Wi/Wo=1.60; P/Wi=1.87	2.5	0.5	0.67	0.23	0.23	0.181	0.113	0.157	0.50	5	1.60	0.87
7	L/W=5; Wi/Wo=0.63; P/Wi=1.92	2.5	0.5	0.67	0.23	0.23	0.113	0.181	0.217	0.50	5	0.63	1.92
8	L/W=5; Wi/Wo=0.63; P/Wi=1.39	2.5	0.5	0.67	0.23	0.23	0.113	0.181	0.157	0.50	5	0.63	1.39
9	L/W=5; Wi/Wo=2.00; P/Wi=1.11	2.5	0.5	0.67	0.23	0.23	0.195	0.098	0.217	0.50	5	2.00	1.11
10	L/W=5; Wi/Wo=2.00; P/Wi=0.81	2.5	0.5	0.67	0.23	0.23	0.195	0.098	0.157	0.50	5	2.00	0.81
11	L/W=5; Wi/Wo=0.50; P/Wi=2.21	2.5	0.5	0.67	0.23	0.23	0.098	0.195	0.217	0.50	5	0.50	2.21
12	L/W=5; Wi/Wo=0.50; P/Wi=1.60	2.5	0.5	0.67	0.23	0.23	0.098	0.195	0.157	0.50	5	0.50	1.60
13	L/W=7; Wi/Wo=1.25; P/Wi=1.33	3.5	0.5	1.00	0.40	0.40	0.163	0.130	0.217	0.50	7	1.25	1.33
14	L/W=7; Wi/Wo=0.80; P/Wi=1.67	3.5	0.5	1.00	0.40	0.40	0.130	0.163	0.217	0.50	7	0.80	1.67
15	L/W=7; Wi/Wo=2.00; P/Wi=1.11	3.5	0.5	1.00	0.40	0.40	0.195	0.098	0.217	0.50	7	2.00	1.11
16	L/W=3; Wi/Wo=1.25; P/Wi=1.33	1.5	0.5	0.33	0.07	0.07	0.163	0.130	0.217	0.50	3	1.25	1.33
17	L/W=3; Wi/Wo=1.25; P/Wi=0.82	1.5	0.5	0.33	0.07	0.07	0.163	0.130	0.134	0.50	3	1.25	0.82
18	L/W=3; Wi/Wo=1.25; P/Wi=0.59	1.5	0.5	0.33	0.07	0.07	0.163	0.130	0.096	0.50	3	1.25	0.59
19	L/W=3; Wi/Wo=0.80; P/Wi=1.67	1.5	0.5	0.33	0.07	0.07	0.130	0.163	0.217	0.50	3	0.80	1.67
20	L/W=3; Wi/Wo=0.80; P/Wi=1.03	1.5	0.5	0.33	0.07	0.07	0.130	0.163	0.134	0.50	3	0.80	1.03
21	L/W=3; Wi/Wo=0.80; P/Wi=0.74	1.5	0.5	0.33	0.07	0.07	0.130	0.163	0.096	0.50	3	0.80	0.74

4 EXPERIMENTAL RESULTS

The analysis is made by comparison of the discharge enhancement ratio (*r)* versus *H/P*. Only values of *H/P* higher than 0.2 are considered in order to avoid capillarity effects, responsible for aspiration due to negative pressure under the overflowing jet.

4.1 *Influence of the height of the dam (P_d)*

Experiments revealed that the flow approach conditions, characterized by the height of the dam (*Pd*), can have a significant influence on the discharge capacity. As illustrated in Figure 3, the influence of P_d tends to diminish with the increase of P_d/P. However, it can reduce the PKW efficiency to about 15% for P_d/P=0. This effect can be related to a head loss increase associated to the approach velocity in front of the outlet keys.

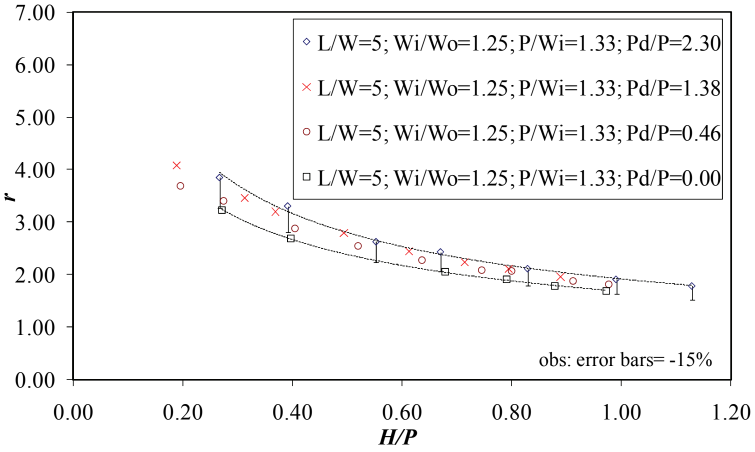


Figure 3. Discharge enhancement ratio *r* as function of *H/P* for different ratios of P_d/P.

4.2 *Influence of P/W_i*

The ratio of the vertical to horizontal dimensions of the inlet key (P/W_i) does not affect the discharge capacity of a PKW. As illustrated in Figure 4, for same values of *H/P*, the PKWs with identical ratios *L/W* and W_i/W_o have approximately the same efficiency for different P/W_i.

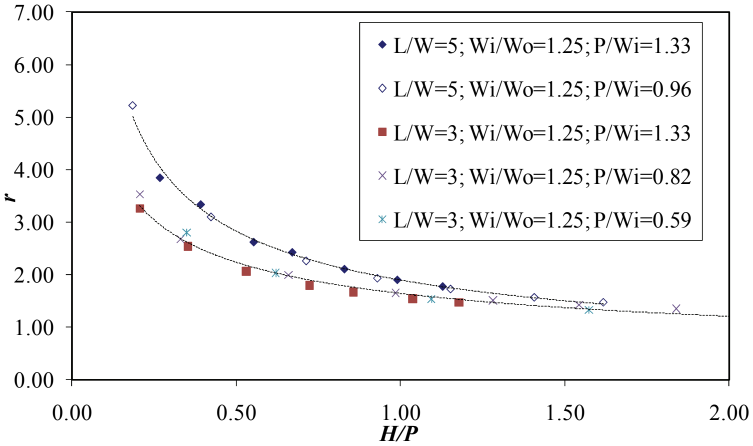


Figure 4. Discharge enhancement ratio as function of *H/P* for different values of P/W_i.

4.3 *Influence of W_i/W_o*

The ratio between the inlet (W_i) and outlet widths (W_o) reveals a higher efficiency when $W_i/W_o>1$ than for $W_i/W_o<1$ (Fig. 5). This suggests that the most efficient part of the PKW is the inlet key that combines lateral and frontal overflows. However, there are practically no differences between the measurements with $W_i/W_o=1.25$, 1.60 and 2.00.

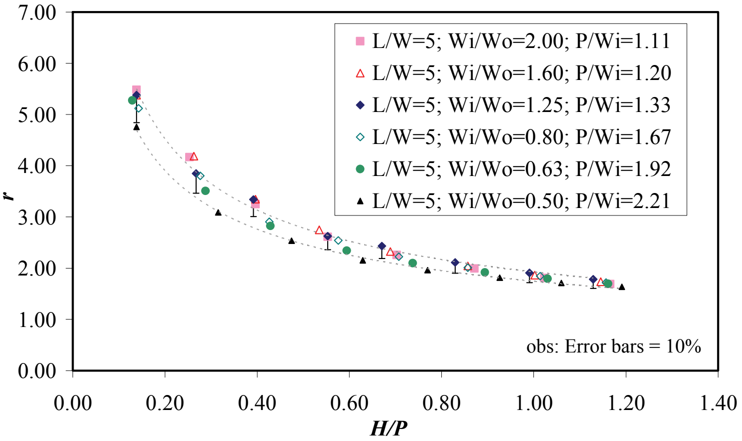


Figure 5. Discharge enhancement ratio *r* as function of *H/P* for different values of W_i/W_o.

4.4 *Influence of L/W*

As expected, the most important parameter influencing the efficiency of a PKW is the developed length ratio L/W. Experimental values let appear differences of about 50% between tests with L/W= 7 and L/W= 3 for low ratios H/P. However, with the increase of H/P, the differences tend to decrease (Fig. 6).

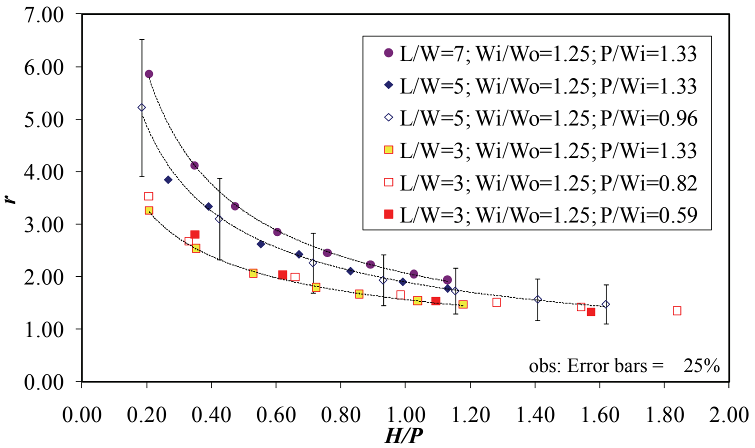


Figure 6. Discharge enhancement ratio *r* as function of *H/P* for different values of *L/W*.

5 HYDRAULIC DESIGN

In order to provide a mathematical formulation for the hydraulic design of a PKW, a non-linear global stepwise regression approach was applied to fit the most influent parameters. The Evolutionary Polynomial Regression (EPR) toolbox (Laucelli et al, 2009) was applied to all the measurements (*r versus H/P*) performed on the models with ratio P_d/P=2.3 (P_d=0.50 m).

5.1 *Universal formula*

The finally adopted mathematical form, for the preliminary design of a PKW consists of an exponential function, where the main geometrical mentioned parameters are considered.

$$r = e\left(\begin{array}{l} -0.25945\left(\frac{P}{Wi}\right)^{1.4}\left(\frac{H}{P}\right)^{0.15} + 1.0056\left(\frac{L}{W_i}\right)^{0.1}\left(\frac{P}{W_i}\right)^{0.5}\left(\frac{H}{P}\right)^{0.7} \\ +0.067404\left(\frac{L}{W_i}\right)^{0.3}\left(\frac{P}{W_i}\right)^{0.1}\left(\frac{W_i}{W_o}\right)^{0.25}\left(\frac{H}{P}\right)^{0.2} + 13.9156\left(\frac{L}{W}\right)^{0.35}\left(\frac{H}{P}\right)^{0.15} \\ -14.0239\left(\frac{L}{W}\right)^{0.35}\left(\frac{H}{P}\right)^{0.2} + 0.094 \end{array}\right) - 1 \tag{4}$$

The application of Equation 4 to the entire data set shows that the accuracy of the calculated r-values ranges between ±5% with respect to the measured ones (Fig. 7). The coefficient of determination *CoD* is equal to 99.56%. In the *CoD* equation, expressed in Figure 7 (left), *N* is the number of data; *avg*(y_{exp}) is the average value of observations; $\hat{y}$ is the value predicted by the mathematical equation and y_{exp} is the corresponding observation.

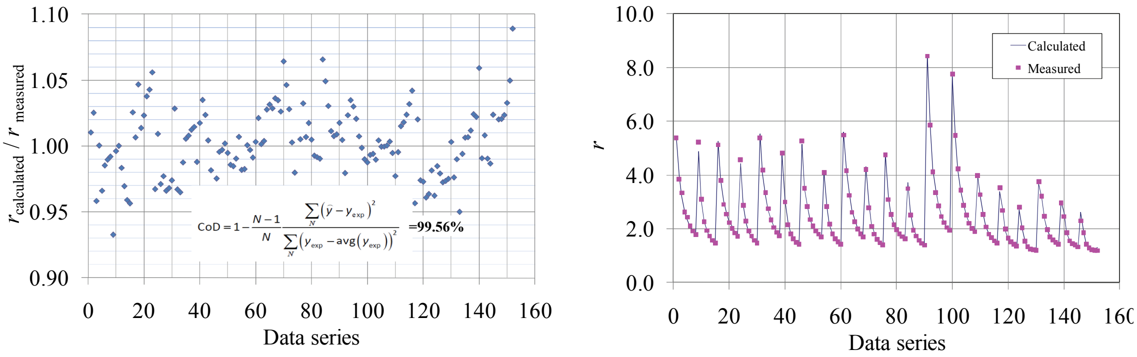


Figure 7. Ratio between the measured and calculated *r*.

5.2 *Application domain*

It is important to notice that the proposed empirical equation is based on a limited number of configurations and parameters. Therefore, its application is rigorously valid inside the corresponding application domain. Figure 8 illustrates the curves W_i/W_o versus *L/W* (left) and P/W_i versus *L/W* (right) where the coefficient of determination (*CoD*) shown in Figure 7 is valid. Moreover, it is applicable for $0.2<H/P<1.2$ and for $P_d/P\geq 2.3$ (no significant influence of the dam height).

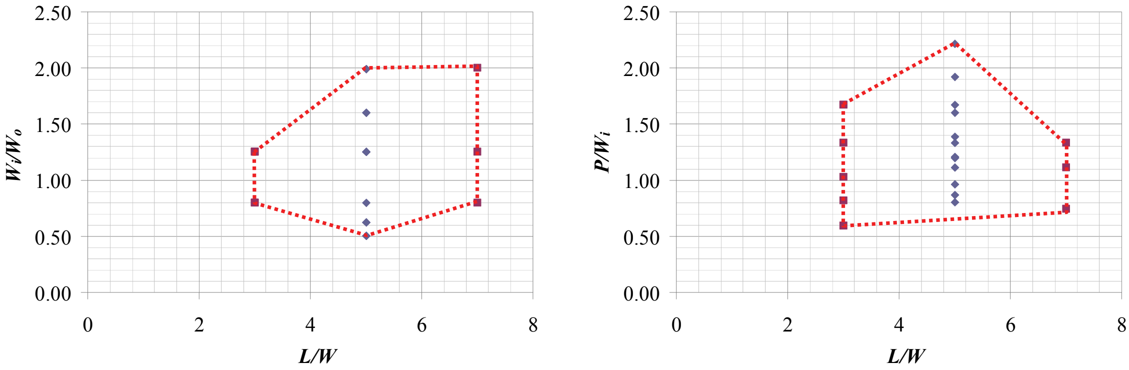


Figure 8. Application domain of the universal formula.

5.3 *Generation of efficiency curves*

Figure 9 illustrates a series of efficiency curves (*r versus H/P*) generated with Equation 4. The curves represent PKWs with developed length ratios *L/W* of 3, 4, 5, 6 and 7 and constant values of P/W_i=1.33, W_i/W_o=1.25 and P_d/P=2.3. The interpretation of Figure 9 shows that there is not only one optimal solution of a PKW, from the hydraulic point of view. PKWs with different developed length ratios *L/W* can evacuate the same discharge for different values of *H/P*.

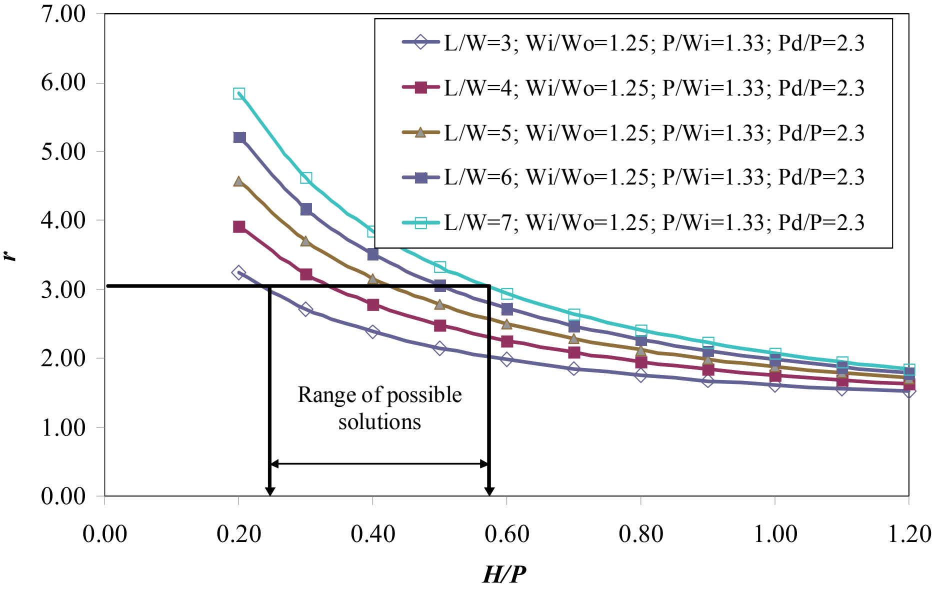


Figure 9. Generated efficiency curves for PKWs with different developed length ratios *L/W* and P/W_i=1.33, W_i/W_o=1.25 and P_d/P=2.3 and example of solutions for a discharge enhancement ratio *r*=3.

6 CONCLUSIONS

In the present paper, a parametric study on the main geometrical parameters of a PKW was carried out by means of experimental investigations. The main conclusions to be highlighted are:

- The height of the dam on which the PKW is installed can have a significant influence on the PKW discharge capacity. Results revealed that for a PKW with L/W=5, values of P_d/P near to zero can reduce the efficiency of the PKW in about 15%.
- The vertical-to-horizontal dimensions ratio (P/W_i) does not influence considerably the efficiency of the PKW for a same value of H/P.
- PKW with ratios $W_i/W_o>1$ are more efficient. However, no remarkable differences could be pointed out between values of 1.25, 1.60 and 2.0.
- The developed length ratio L/W is the most influent parameter on PKW capacity.

From the available experimental dataset, an empirical equation was established for the calculation of the discharge enhancement ratio. The equation consists of an exponential function that is rigorously valid inside an application domain. Its usefulness was demonstrated by the generation of several PKW efficiency curves with different values of *L/W*. The interpretation of these curves demonstrates that no single optimal PKW solution exists from the hydraulic point of view. The ideal solution can only be selected when considering local and economical constraints related to excavation, materials, construction techniques and particularly to the downstream energy dissipation system.

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