On a Markov Modulated Chain Exhibiting Self-Similarities Over Finite Timescale

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Abstract

Recent papers have pointed out that data traffic exhibits self-similarity, but self-similarity is observed only on a finite timescale. In order to account for that, we introduce the concept of pseudo long-range dependences. In this paper, we describe a Modulated Markov process producing self-similarity on a finite timescale; the process is quite easy to manipulate and depends only on three parameters (two real numbers and one integer). An advantage of using it is that it is possible to re-use the well-known analytical queuing theory techniques developed in the past in order to evaluate network performance. A quantitative method based on the decomposability *theory of Courtois is given to evaluate the domain of validity where the process exhibits pseudo long-range dependences. The validation on a queuing problem is also discussed. Finally, we analyze the inputs of a statistical multiplexer in the context of a project called Scalability Enhancements for Connection-Oriented Networks (SCONE).*

1 Introduction

The aim of this study is to generate a source model reflecting some properties of a real source and analyze its impact on a multiplexer. Here, we want to model data networks. Recent work [1, 2] has pointed out that such a traffic exhibits self-similarities on a finite timescale. In this introduction, we will try to understand intuitively from where this specific behavior comes, because modeling is based on understanding what is essential [3]. If we consider the measured Local Area Network (LAN) traffic as a source, we must take into account that many people are working on the network, having their own schedules. Human behavior has a big influence on the network utilization but the inverse is true too: the patience of the human being is not unlimited. The influence of the protocol used should be taken into consideration. Here we consider only data traffic in a first step: voice, sound and video traffic is ignored in our model. In the following, we consider the measurements performed on the Bellcore network (the files are available via anonymous ftp from `flash.bellcore.com`, directory *lan/pub*). The measured Bellcore traffic is 99.5% Internet Protocol (IP). Data traffic consists of a large variety of service types (file-transfers, workstations communications, terminal communications, ...). IP can support different types of protocols. If we consider TCP (Transmission Control Protocol)/IP protocol [4], it has a very specific behavior. Manthorpe [5] studied the influence of the transport layer on traffic modeling. This protocol uses a sliding window to ensure the efficiency of the transmission. The flow sent on the network depends on the window size but also on the network load and size. Maybe the most complex aspect in TCP is embedded in the way it handles time-out and retransmission. Every time it sends a segment, TCP starts a timer and waits for an acknowledgment. If the timer expires before data in the segment has been acknowledged, TCP assumes that the segment was lost or corrupted and retransmits it. TCP is intended for use in an internet environment. In such an environment, a segment traveling between a pair of machines may traverse a single, low-delay network or it may wind across multiple intermediate networks through multiple gateways. Thus it is impossible to know a priori how quickly acknowledgments will return to the source. Furthermore, the delay at each gateway depends on the traffic. TCP monitors the performance of each connection and deduces values for time-outs and uses an adaptive retransmission algorithm which allows it to adapt itself if the performance of the connection changes. Furthermore, TCP reacts to congestion. When congestion occurs, TCP normally uses two techniques: slow start and multiplicative decrease of the window size. Ethernet, when it has something to transmit, waits until the channel is free. Furthermore, it is able to detect collisions on the network. In other words, if two workstations sense the channel to be idle and begin to transmit simultaneously they will both detect the collision almost immediately. The two stations should abruptly stop transmitting as soon as a collision is detected. Therefore, our model for Ethernet will consist of alternating contention and transmission periods.

To resume, we roughly assume that LAN traffic measured on Ethernet can be examined at three major levels of behavior corresponding to a certain resolution of time:

- The connection level describes the human behavior. The connection duration is determined by the file sending time and the file length. The duration between calls on an Ethernet network is typically in the time range of 10 ... 1000 sec.

- The TCP/IP level describes the transport level. As we have seen before, the traffic sent on the network depends of an uncontrollable number of parameters but the major influences on it is the network behavior. The time to transmit a TCP/IP packet and receive the corresponding acknowledgment typically varies from 0.001 to 10 sec.

- The Ethernet network level where the sent traffic depends essentially on the local traffic flowing on the network. The time between sending and not sending a frame is typically in the range 1 ... 50 msec.

At the human level, we consider two operational modes: sending or not sending a file. The change between the two modes depends uniquely on the human behavior and how he/she reacts when a congestion occurs in the network. At the TCP/IP level, the protocol is principally controlled by the network behavior. As seen before, the analysis of the jumps between the two states is difficult because of the dependences between the protocol and the network. At the lowest level, Ethernet waits for the channel before transmitting data. A Markov model has been built on these considerations [6, 7]. It has been found that such a model exhibits self-similarities on a finite timescale.

The aim of this paper is to discuss some properties of the Markovian model we propose here and to consider the statistical multiplexing of these sources. The paper is organized as follows. Section 2 introduces the concept of pseudo long-range dependences and gives some useful definitions for the rest of the paper. The Markov model (exhibiting self-similarities on a finite timescale) we propose is introduced in Section 3. The validation on a queuing problem with such a source is also discussed. The inputs of a statistical multiplexer are analyzed in Section 4 (in the context of SCONE).

2 Pseudo Long-Range Dependences

Long-range dependent processes have been at the center of a debate quite recently. The aim of this Section is to define a new class of models, the *pseudo long-range dependent* models which are based on finite Markov chains (it is necessary to consider infinite Markov chains to obtain truly long-range dependent processes). Let us give some definitions, useful for following this text. We consider the process of cell arrivals on a slotted link; call X_t the random variable representing the number of cells during the t^{th} time slot, namely during time interval $[t-1, t)$. Let $X = (X_t : t = 0, 1, 2, 3, ...)$ be a covariance stationary stochastic process with an autocovariance function $Cov\{X_t, X_{t+\tau}\}$ and $X^{(m)} = (X_k^m : k = 0, 1, 2, 3, ...)$ a new averaged series over non-overlapping blocks of size m (m is a time interval in our case) with an autocovariance function $Cov\{X_t^{(m)}, X_{t+\tau}^{(m)}\}$. $X_k^{(m)} = \frac{1}{m}\sum_{t=km-m+1}^{km} X_t$ and let N_m be the number of arrivals during m timeslots. Mathematically [8], the differences between the short-range dependent processes and the long-range dependent processes are reported in table 1. There is another category: the processes of long-range dependences of index β, but they don't have a degenerate correlational structure as $m \to \infty$. All stationary autoregressive-moving average (ARMA) processes of finite order, all finite Markov chains fall into the first category. In the second category, we have the fractional Brownian motion [9], ARIMA processes [10], chaotic maps [11] which have long-range dependences. However if we look at this definition, we see that a process having "long term dependences", but over a limited timescale is considered as a short term dependent

	short-range	*long-range*
$\sum_{\tau=0}^{\infty} Cov\{X_t, X_{t+\tau}\}$	convergent	divergent
spectrum at 0	finite	infinite
$Var\{X^{(m)}\}$ is for large m asymptotically of the form ($0 < \beta < 1$)	$1/m$	$m^{-\beta}$

Table 1: Differences between the short-range dependent processes and the long-range dependent processes

process. We see for example that Ethernet measurements have long term dependences, at least over 4 or 5 orders of magnitude. In other words, if we represent the number of Ethernet packets arriving in a time-interval of 1 s, then the statistics of the number of packets looks the same for 10 s, 100 s, 1000 s, 10000 s. Researchers at Bellcore have observed a stabilization of the index of dispersion [12] indicating a lack of self-similarity. According to the definition, a short term dependences process would be sufficient to model LAN traffic. The difference with other processes (Poisson, ON-OFF, ...) is striking and they should be categorized differently. Therefore, we propose to name them: *pseudo long-range dependences processes.* A pseudo long-range dependent process is able to model as well as an (exactly) long-range dependent aggregated traffic over several timescales. In fact, this definition of long term dependences processes comes historically from the importance of self-similar processes which are able to give an elegant explanation to an empirical law (*Hurst effect*). Because of the difficulty of tracting fractal-based models, Anderson [13] and Robert [6, 7] proposed LAN traffic with Markov chains having pseudo long-range dependent modeling. In the next Section, we will give the characteristics of the used Markov model.

3 Markov model

Here, we investigate the use of a simple, discrete time Markov modulated model for representing self-similar data traffic. X_t is assumed to be 0 or 1 during the t^{th} time slot, namely during time interval $[t-1, t)$. The Markov modulated chain is assumed to be stationary and homogeneous. Let $Y_t = i$ be the modulator's state i, $i \in 1, 2, 3, ..., n$ at time t. The arrivals of cells are modulated by a n-state discrete time Markov chain with transition probabilities $a_{ij}(t_1, t_2) = Pr[Y_{t_2} = j | Y_{t_1} = i], a_{ij}(t, t+1) = a_{ij}$. Let ϕ_{ij} denote the probability of having j cells given that the modulator's state is i which we assume independent of t. More specifically $\phi_{ij} = Pr[X = j | Y = i]$. The Markov modulated chain state probabilities are noted as $\pi_i(t) = Pr\{Y_t = i\}$, i is referred to as the modulator's state and t as the time. All the moments are equal and no longer time dependent.

$$E\{X_t^k\} = E\{X\} = \vec{\pi}\mathbf{\Lambda}\vec{e},\ \forall k = 1, 2, \cdots,\ \forall t = 0, 1, 2, \cdots \quad (1)$$

with $\vec{\pi} = (\pi_1, \pi_2, ..., \pi_n)$ and $\vec{e}$ the unity vector. $\mathbf{\Lambda}$ is defined as

$$\mathbf{\Lambda} = diag(E[X|Y=1], E[X|Y=2], ..., E[X|Y=n]) \tag{2}$$

If N_m represents the number of arrivals in a window of m time intervals, the variance of N_m can be written

$$\begin{aligned} Var\{N_m\} = \quad & mE\{X\} - m^2(E\{X\})^2 + \\ & 2(\textstyle\sum_{i=1}^{m-1}(m-i)(\vec{\pi}\mathbf{\Lambda}\mathbf{A}^{i+1}\mathbf{\Lambda}\vec{e})) \end{aligned} \tag{3}$$

Now, we propose a family of models

$$\mathbf{A} = \begin{pmatrix} 1 - 1/a - ... - 1/a^{n-1} & 1/a & \cdots & 1/a^{n-1} \\ b/a & 1 - b/a & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ (b/a)^{n-1} & 0 & \cdots & 1 - (b/a)^{n-1} \end{pmatrix} \tag{4}$$

and $\phi_{11} = \phi_{20} = \phi_{30} = \phi_{40} = \cdots = 1$

$$\mathbf{\Lambda} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{5}$$

So, the Markov chain has only 3 parameters: a, b and the number of states in the Markov chain n. The studied Markov chain is finite, irreducible, with recurrent states. There is a unique stationary distribution and the solution to the system $\vec{\pi} = \vec{\pi}\mathbf{A}$ exists. The resolution of this system leads to $\pi_i = \pi_1(1/b)^{i-1}$ with $i = 2, \cdots, n$. On the other hand, $\sum_{i=1}^{n} \pi_i = 1$. Finally

$$\pi_1 = \frac{1 - 1/b}{1 - (1/b)^n} \tag{6}$$

and $E[X] = \pi_1$ with the proposed Markov chain because of the particular form of $\mathbf{\Lambda}$. The moments are all equal $E[X] = E[X^2] = E[X^3] = \cdots$ and the variance is equal to $E[X^2] - E^2[X]$. Figure 1 shows us the evolution of $E[X]$ as a function of b for different values of n. Notice that for $n = 1000$, the $E[X]$ values are extremely low for $b < 1$. When $n \to \infty$, $b \geq 1$ $\forall E[X]$. On the other hand, with a small number of states, b can have values between 0 and 1 for "reasonable" $E[X]$. Now, there is a problem because $\pi_1 = E[X]$ is not determined for $b = 1$. In applying l'Hospital rule, we find that for $n \geq 2$, $\pi_1 = E[X] = 1/n$ when $b = 1$. The interarrival distribution T_A is the following, for $k = 1$

$$prob(T_A = 1|Y_t = 1) = 1 - \sum_{i=1}^{n-1}(1/a)^i$$

and for $k \geq 2$

$$prob(T_A = k|Y_t = 1) = \sum_{i=1}^{n-1}(b/a^2)^i(1 - (b/a)^i)^{k-2}$$

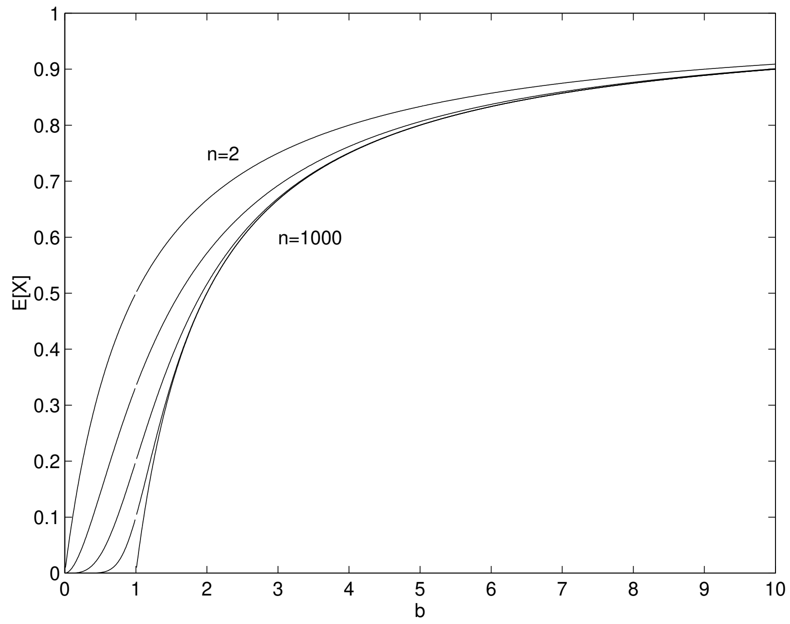

Figure 1: $E[X]$ as a function of b when $n = 2/3/5/10/1000$

this distribution is a sum of geometric distributions and has a large variability. The interarrival expected value is [14]

$$E[T_A|Y_t = 1] = \frac{1-(1/b)^n}{1-1/b} \tag{7}$$

Notice that if $n \to \infty$, $b > 1$. The second moment of T_A is given by

$$E[T_A^2|Y_t = 1] = 1 + \frac{1-(1/b)^n}{1-1/b} + 2\frac{1-(a/b^2)^n}{1-a/b^2} \tag{8}$$

If $b > 1$ and $a/b^2 < 1$

$$E[T_A^2|Y_t = 1] = 1 + \frac{1}{1-1/b} + 2\frac{1}{1-a/b^2} \tag{9}$$

for $n \to \infty$ but if $b < 1$ or $a/b^2 > 1$, $E[T_A^2|Y_t = 1] \to \infty$ when $n \to \infty$. The burst length distribution T_R is geometric.

3.1 Domain where the process exhibits self-similarities

To build our model, we have considered a theory which was developed approximately 20 years ago by Courtois, the theory of *decomposability*. Courtois's analysis [15] is based on the important observation that large computing systems can usefully be regarded as nearly completely decomposable systems. Systems are arranged in a hierarchy of components and subcomponents with strong interactions within components at the same level and lower interactions with other components. Near decomposability has been observed in other domains than computing: in economics, in biology, genetics, social sciences. The pioneers in this domain are Simon and Ando who studied several study-cases in economics and in physics [16, 17, 18]. What they stated is that aggregation of variables in a nearly decomposable system must separate the analysis of the short term and long term dynamics. They proved two major theorems. The first says that a nearly-decomposable system can be analyzed by a completely decomposable system if the intergroup dependences are sufficiently weak compared to intragroup ones. The second theorem says that even in the long term, the results obtained in the short term will remain approximately valid in the long term, as far as the relative behavior of the variables of the same group is concerned. In our study, the problem is inverse: we postulate the LAN traffic is composed of different timescales. The Markov chain we propose to analyze here is in fact decomposable at several levels. In a first step, the development is done for only one level of decomposability. The presented development deviates from those of Simon, Ando and Courtois.

The Markov chain to be studied is characterized by its transition matrix $\mathbf{A}$ and its state probabilities $\vec{\pi}$ ($\vec{\pi}_{t+1} = \vec{\pi}_t\mathbf{A}$), $\mathbf{A}$ is nearly completely decomposable. Let $\mathbf{A}^*$ be completely decomposable, then $\mathbf{A}^*$ is composed of squared submatrices placed on the diagonal:

$$\mathbf{A}^* = \begin{pmatrix} \mathbf{A}_1^* & \cdots & 0 & 0 \\ 0 & \mathbf{A}_2^* & \cdots & 0 \\ 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \mathbf{A}_M^* \end{pmatrix} \tag{10}$$

The remaining elements are equal to zero. $\mathbf{A}_{IJ}^*$ is a sub-matrix of $\mathbf{A}$ at the intersection of the I^{th} set of rows and the J^{th} set of columns and $a_{i_Ij_J}$ the element at the intersection of the i^{th} row and the j^{th} column of $\mathbf{A}_{IJ}$. To $\mathbf{A}^*$ is associated a new $\vec{\pi}^*$: $\vec{\pi}^*_{t+1} = \vec{\pi}^*_t\mathbf{A}^*$. For simplification $\mathbf{A}_{II} = \mathbf{A}_i, i = 1, \cdots, M$ and is a square matrix $n(i) * n(i)$ with $\sum_{i=1}^{M} n(i) = n$. Each submatrix of $\mathbf{A}_i^*$ has its own set of eigenvalues $\lambda^*(i_I)$. For convenience, we suppose they are ordered: $\lambda^*(1_I) = 1 > \lambda^*(2_I) \geq ... \geq \lambda^*(n(I)_I), I = 1, ..., M$. $\lambda^*(1_I) = 1$ because the matrices are stochastic [19]. With the matrix $\mathbf{A}$, the situation is different because only one eigenvalue ($\lambda(1_1)$) is equal to 1. We suppose the eigenvalues are ordered as well. Suppose now $\mathbf{A}$ is diagonalizable, so

$$\mathbf{A} = \mathbf{P}^{-1}\mathbf{D}\mathbf{P} \tag{11}$$

$\mathbf{P}$ is the passage matrix and $\mathbf{D}$ can be written

$$\mathbf{D} = \sum_i \lambda_i \mathbf{P}_i = \sum_{i=1}^{n(I)} \sum_{I=1}^{M} \lambda(i_I)\mathbf{P}_I(i) \tag{12}$$

with $\mathbf{P}_l$ = projector (i.e. $p_{ij} = 0\ \forall i, j \neq l$, $p_{ll} = 1$). So

$$\begin{aligned} \mathbf{A} = \quad & \mathbf{P}^{-1}\mathbf{P}_1(1)\mathbf{P} + \textstyle\sum_{I=2}^{M} \lambda(1_I)\mathbf{P}^{-1}\mathbf{P}_I(1)\mathbf{P} + \\ & \textstyle\sum_{I=1}^{M} \sum_{i=2}^{n(I)} \lambda(i_I)\mathbf{P}^{-1}\mathbf{P}_I(i)\mathbf{P} \end{aligned} \tag{13}$$

$\lambda(i_I)\mathbf{P}^{-1}\mathbf{P}_I(i)\mathbf{P}$ can be replaced by $\mathbf{Z}(i_I)(z_{kl}(i_I)$ are the elements of $Z(i_I))$. The properties of $\mathbf{Z}(i_I)$ are given in [15].

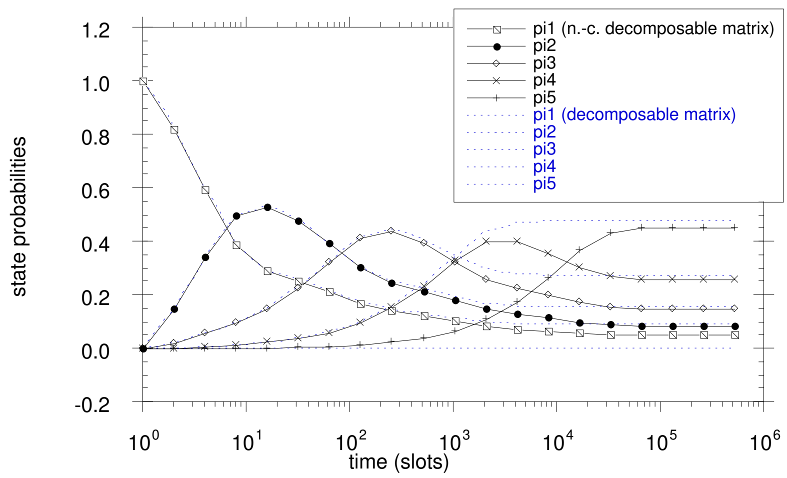

Figure 2: Behaviors comparison of $\vec{\pi}_t$ and $\vec{\pi}^*_t$, $a = 6.7$, $b = 0.576$, $n = 5$

Similarly for $\mathbf{A}^*$, we have

$$\begin{aligned}\mathbf{A}^* = \quad & \mathbf{P}^{-1}\mathbf{P}_1(1)\mathbf{P} + \textstyle\sum_{I=2}^{M} \lambda^*(1_I)\mathbf{P}^{-1}\mathbf{P}_I(1)\mathbf{P} + \\ & \textstyle\sum_{I=1}^{M}\sum_{i=2}^{n(I)} \lambda^*(i_I)\mathbf{P}^{-1}\mathbf{P}_I(i)\mathbf{P}\end{aligned} \tag{14}$$

Here we will give the first theorem of Simon and Ando [16] without demonstration:

Theorem 1 *For an arbitrary positive number ς, there exists a number ϵ_ς such that for $\epsilon \le \epsilon_\varsigma$,*

$$\max_{k,l} |z_{kl}(i_I) - z^*_{kl}(i_I)| < \varsigma \tag{15}$$

with $2 \le i \le n(I), 1 \le I \le M, 1 \le k, l \le n$

Let us now focus our attention on the implication of this theorem. The discussion is intuitive but very important in this context. The time behavior of π_t and the comparison with the time behavior of π^*_t is central to this concept. Due to the eigenvalues' ordinance, the first terms of (14) will not imply a big variation in the short term ($t < T_1$), because the $\lambda(1_I)$ $I = 1, ..., M$ are close to unity. Thus, for $t < T_1$, the predominantly varying term of $\mathbf{A}$ is the last one, so π_t and π^*_t evolve similarly. For $T_1 < t < T_2$, the time behaviors of π_t and π^*_t are defined by the last terms of $\mathbf{A}$ and $\mathbf{A}^*$ respectively, a similar equilibrium is being reached within each subsystem of $\mathbf{A}$ and $\mathbf{A}^*$. For $T_2 < t < T_3$, the most significantly varying term of $\mathbf{A}$ is the second one. For $t > T_3$, the first term of $\mathbf{A}$ dominates all the others. A global equilibrium is attained in the whole system. The whole nearly completely decomposable system moves towards equilibrium, but the short-term equilibrium relative system moves towards equilibrium; the short-term equilibrium relative values of the variables within each subsystem are approximately maintained. This dynamic behavior of a nearly decomposable matrix may be dissociated into four distinct periods that Simon and Ando [16] call, respectively, 1) short-term dynamics, $t < T_1$ 2) short-term equilibrium, $T_1 < t < T_2$ 3) long-term dynamics, $T_2 < t < T_3$ and 4) long-term equilibrium, $t > T_3$. Here, we want to analyze matrices nearly completely decomposable with the form described in Section 3. $\mathbf{A}^*$ is built based on the Courtois's theory, with $b/a < 1$.

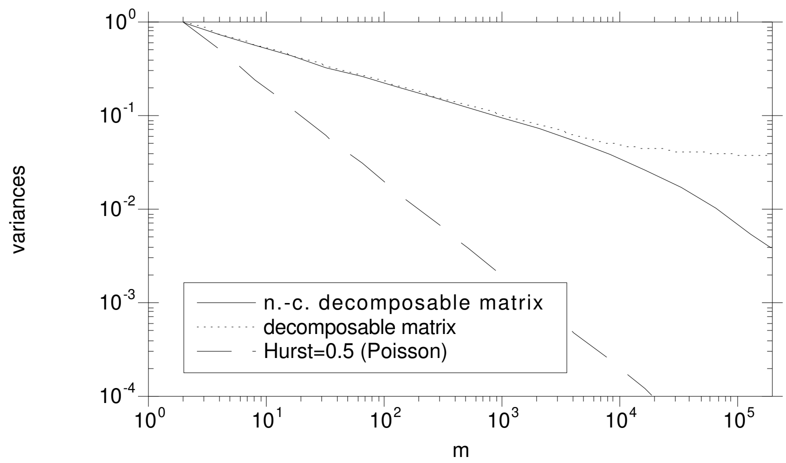

Figure 3: Behavior of a nearly completely decomposable matrix and a decomposable matrix versus window of size m, $a = 6.7$, $b = 0.576$, $n = 5$

$$\mathbf{A}^* = \begin{pmatrix} 1 - 1/a - ... - 1/a^{n-2} & 1/a & \cdots & 0 \\ b/a & 1 - b/a & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{16}$$

Here, we have only two submatrices placed on the diagonal, $\mathbf{A}_1^*$ and $\mathbf{A}_2^*$. $\mathbf{A}_1^*$ is a square matrix $(n-1) * (n-1)$ and $\mathbf{A}_2^*$ a square matrix $1 * 1$. In this case, $n(1) = n - 1$ and $n(2) = 1$, $n(1) + n(2) = n$, $M = 2$. Note that $b < a$ and that $\mathbf{A}^*$ is a non-ergodic matrix. The time behavior of π_t and π_t^* are given in Figure 2 and the variance-time plot of these two Markov chains is given in Figure 3. By definition [20], a process having only one Hurst parameter H to describe it is called exactly second-order self-similar. The process X_t and the averaged processes $X_t^{(m)}$ have identical correlational structures. With the Markov chains we will analyze, we are not exactly in this case because all finite Markov chains have a limit. Therefore, we propose to use the name of "local" Hurst H_l parameter instead of Hurst parameter for our Markov chains. The Markov chains we examine have pseudo long-range dependences. The self-similarity tests performed on these chains (by the variances method and the visual test) and the fitting problem can be found in [21].

3.2 Validation on a queuing problem

Here, we consider an ATM link buffer; the process of cell arrivals on a slotted link. The service time is equal to one slot and can only start at the beginning of a slot. No priority level is considered here and the service strategy of the queue is First In First Out (FIFO). The buffer has a capacity of c cells. If cells arrive when the buffer is full, they will be discarded. We assume here that the output link is slower than the input one. The arrival of cells is modulated by the n-state Markov chain described previously. The time unit is one slot, therefore, at the

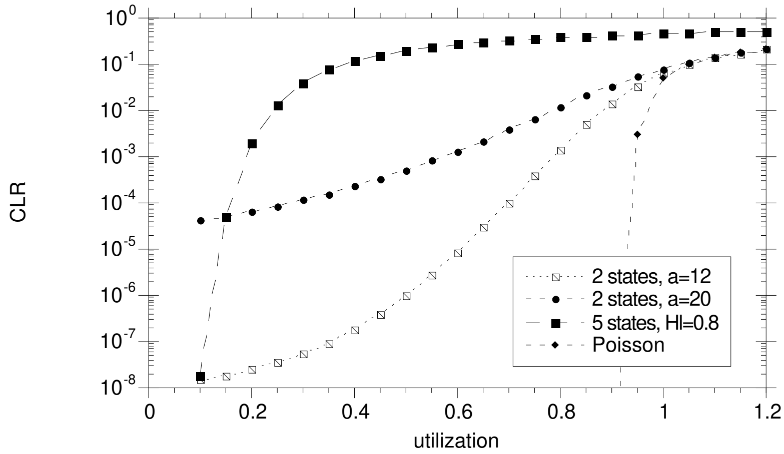

Figure 4: CLR comparison of two different Markov chains

input of the queue we have to define the *minislot* concept: the time spent between the begin of the arrival of the first minislot and the begin of the arrival of the K^{th} one is equal to one time unit. The rate ratio between the input and the output link is equal to K. In the next paragraph, we describe how to obtain the distribution of the random variable $X(K)_t$ as a function of K.

Here, we want to calculate the distribution of the random variable $X(K)_t$ as a function of K. This distribution can be calculated by a recurrence formula. Let K be the number of slots and k the number of "1"s during l minislots and let i be the modulator's state. Then we observe the instants $K, K+1, K+2, \cdots$. Let $D_k^l(i)$ be the distribution of cell arrivals between two instants (K and $2 \times K$ for example) when the modulator is in state i at the beginning of the slot (K here). The recurrence formula is given by

$$\begin{aligned} D_k^{l+1}(i) = \quad & Pr[X_t = 1|Y_t = i] \textstyle\sum_{j=1}^{n} a_{ij} D_{k-1}^{l}(j) + \\ & (1 - Pr[X_t = 1|Y_t = i]) \textstyle\sum_{j=1}^{n} a_{ij} D_k^l(j) \end{aligned} \tag{17}$$

The initial conditions are given by

$$D_k^1(i) = \begin{cases} k = 0 & 1 - Pr[X_t = 1|Y_t = i] \\ k = 1 & Pr[X_t = 1|Y_t = i] \\ k \geq 1 & 0 \end{cases} \tag{18}$$

The transition matrix, when we observe the instants $K, 2 \times K, 3 \times K, ...$ is given by $\mathbf{A}^K$. The transition matrix of the queue $\mathbf{Q}$ is *upper-block Hessenberg*. The loss probability can be calculated by the MBH algorithm [22]. Figure 4 shows us the evolution of the Cell Loss Ratio (CLR) as a function of the queue's utilization ($K.E\{X\}$) for different Markov chains. Here, $E\{X\} = 0.05$ and the queue's length is 500. The 5-state chain has a local Hurst parameter $H_l = 0.8$ ($a = 6.7$, $b = 0.5764$). With the 2-state Markov chain, we have tried to find a common point with the 5-state Markov chain. If the utilization = 0.1, CLR values of the two sources are equal but their

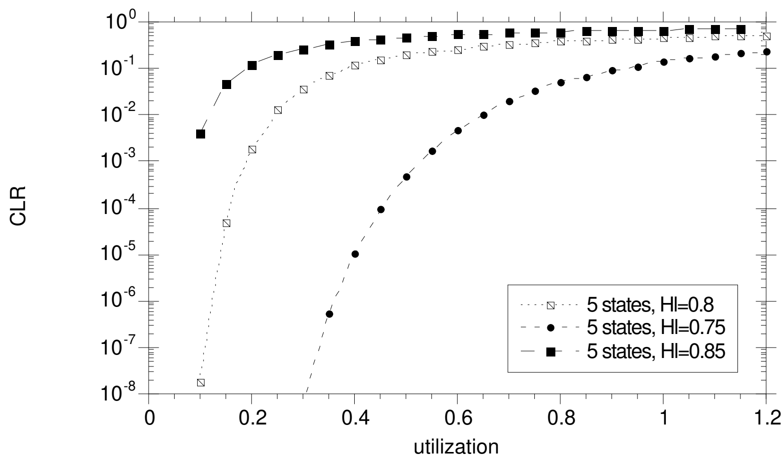

Figure 5: Variation of the local Hurst parameter for a 5-state Markov chain, $E\{X\} = 0.05$

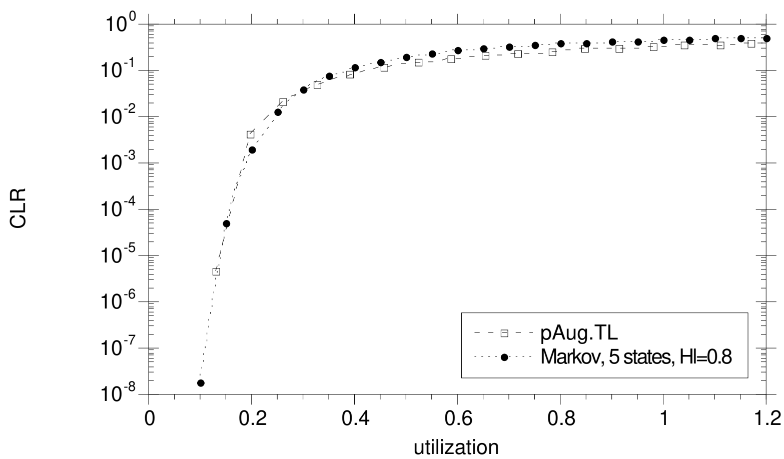

Figure 6: Comparison of the queuing behavior with a 5-state Markov chain and Bellcore measurements

evolution is completely different with higher utilization. Increasing the a parameter of the 2-state Markov chain is not a good solution because the evolution remains the same. We observe only a shift of the curve. On the same Figure, the evolution of a Poissonian source is given. The local Hurst parameter has a big influence on the queuing behavior. Here, we have considered a 5-state Markov chain in varying the local Hurst parameter H_l but we must not forget that the domain of validity where the process exhibits self-similarities increases with H_l. Figure 5 shows us the influence of the local Hurst parameter on the queue's behavior. Now, it is quite important to know how a queue reacts to measured traffic in order to know if the sources we have considered are realistic or not. To verify that, we have taken a Bellcore file (`pAug.TL`), discretized it and put the resulting traffic in a queue having a constant service time. The file contains 1 million packets (the length of one packet varies from 2 to 29 cells). Figure 6 shows us the comparison. The two curves are close together, which indicates a good behavior of the proposed source versus utilization.

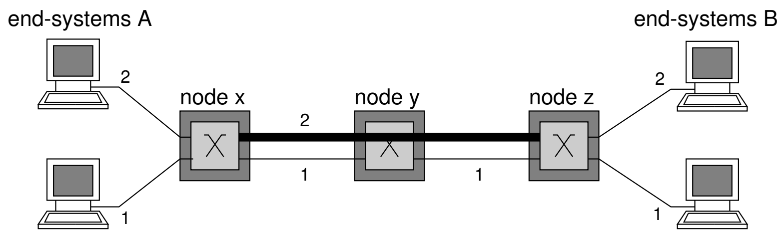

Figure 7: Concept of SCONE

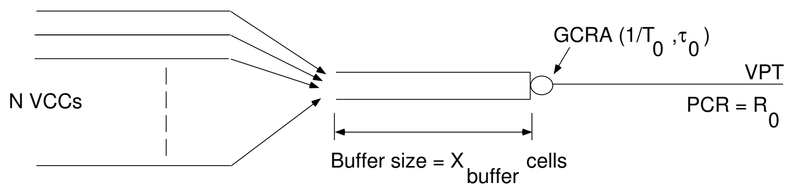

Figure 8: Multiplexeur

4 SCONE

In this Section, we are interested on statistical multiplexing it is possible to obtain with a superposition of Markovian sources having pseudo long-range dependences. The aim is to know if it is possible or not to earn statistical multiplexing in spite of the different behavior the sources could have [23]. This subject is discussed [24, 25] because one has to know how ATM has to be displayed. The context in which the multiplexing problem is studied is SCONE. SCONE proposes a new architecture for the ATM networks. The aim of it is to reduce the computing costs per connection or flow in multimedia networks. This cost is associated with the processing time and the storage of informations. The proposed solution with SCONE is to bundle connections in the same transit network, in other words, in using dynamic connections (virtual paths) between ATM access nodes. Characteristics of SCONE about the basic idea are reported in [26]. In [27], a proposition concerning the signaling is presented. It concerns ATM networks which have to support simultaneously the setup of a *Virtual Channel Connection* (VCC) and a *Virtual Path* (VP). In [28], the traffic control in SCONE is presented. In SCONE, the concept of *Virtual Path Trunk* (VPT) is introduced. A VPT is a path connection established by the network in order to reduce the awareness of the connection at the transit nodes. VCCs are established on VPTs. VCC connections are *Variable Bit Rate* (VBR). VPT connections are *Constant Bit Rate* (CBR), *Available Bit Rate* (ABR) or *Unspecified Bit Rate* (UBR). We will only consider the CBR and VBR classes. CBR is a particular case of VBR where the connection parameter is simply given by the *Peak Cell Rate* (PCR). If VPT is CBR, it is not possible to have statistical multiplexing between VPTs. SCONE gives the possibility of carrying non-CBR traffic. Here, we will study the VCC connections with self-similar traffic on a finite timescale. Figure 8 shows us a description of the situation. N identical connections, VBR, having the same connection attributes enter in the multiplexer made up with a buffer and a server. The time is discrete; its basic unit is the ATM cell. The buffer length is X_{buffer}. The VPT connection is VBR and the server serves cells conforming to GCRA($1/T_0$, τ_0) and GCRA(R_0, CDV_0).

4.1 Multiplexer input

In this Subsection, we want to model the *Generic Cell Rate Algorithm* (GCRA). At the multiplexer input, N VCC connections must be conform to the GCRA parameters. For a given set of parameters, we have to adjust the GCRA parameters in order to have a cell loss ratio less than a given value. To begin, let us recall the GCRA role. ATM Forum [29] and ITU [30] have specified a mechanism for the *User Network Interface* (UNI) which controls the traffic flowing through a VCC connection. This mechanism is the GCRA: it defines a relationship between the PCR and the *Cell Delay Variation* (CDV) and a relation between the *Sustained Cell Rate* (SCR) and the *Burst Tolerance* (BT). GCRA depends on two parameters, its increment (T) and its limit (τ). This algorithm defines a *Theoretical Arrival Time* (TAT) which is the "nominal" arrival time of the cell [31]. If the arrival time is not less than $TAT - \tau$, then the cell is conformed and the algorithm updates the TAT value to $max(t, TAT) + \tau$, otherwise, the cell is not conform. The equations of the GCRA counter are the same as the unfinished work in a G/D/1/c queue. The behavior of the GCRA is the same as a queue concerning the rejection of cells if we assume the same maximum rate at the input and at the output of the GCRA. However, one big difference between the two mechanisms is that the GCRA does not modify the traffic shape but the queue does. Here we examine only the rejection or marking of cells. The occupancy of the queue is equal to (unfinished work/D) with D being the service time. Cells are eliminated if the queue is full, if (unfinished work/D) exceeds the queue length. With the GCRA, the cells are marked if the counter goes beyond a certain limit. The equations of the GCRA counter and the equations of the queue's unfinished work are the same. The GCRA limit=(queue's length.D) for sufficiently long queues (3.3% error for a queue having 30 positions).

ATM-Forum permits CBR or VBR traffic transmission on a VCC connection. For the CBR connections, the user doesn't declare a SCR but only a PCR. In other words, the cells must be conform to GCRA($1/T$, CDV) with small CDV. For the VBR traffic, the source has to declare $PCR_{VCC} = R$, $SCR_{VCC} = 1/T$, $BT = \tau$ and CDV. In other words, arriving cells have to be conform to two consecutive GCRAs, GCRA(R, CDV) and GCRA($1/T, \tau$). The advantage of using VBR connections is that it is possible to send back-to-back cells on the connection during a time interval depending on the BT value. One should know that the mechanism is more complex. We have seen that the GCRA could be replaced by a queue. In this Subsection, we want to examine the loss probability in the GCRA as a function of given parameters with a given source (5-state Markov chain with different local Hurst parameters). Figures 9, 10 and 11 show us the value of τ and SCR we have to declare for loss probabilities from 10^{-6} to 10^{-10}. Figure 12 shows us the influence of the CLR on τ.

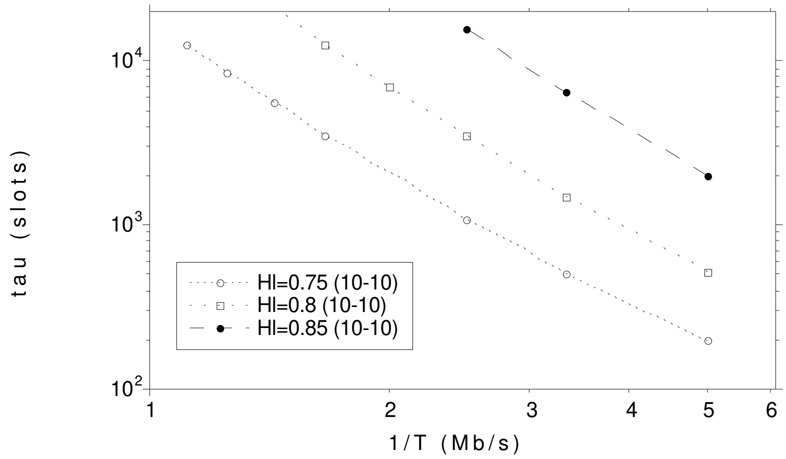

Figure 9: τ as a function of $1/T = SCR_{VCC}$ for different values of the local Hurst parameter, CLR= 10^{-10}

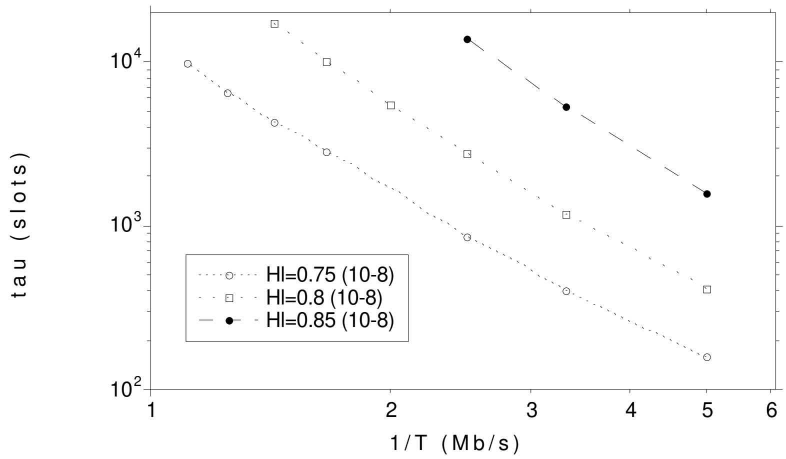

Figure 10: τ as a function of $1/T = SCR_{VCC}$ for different values of the local Hurst parameter, CLR= 10^{-8}

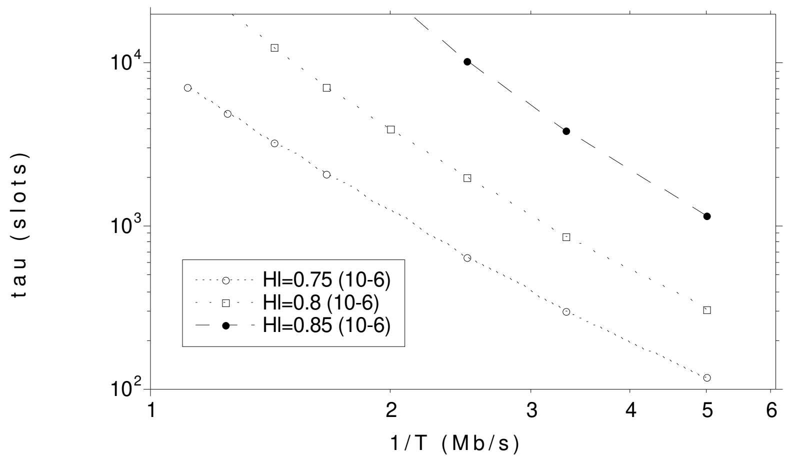

Figure 11: τ as a function of $1/T = SCR_{VCC}$ for different values of the local Hurst parameter, CLR= 10^{-6}

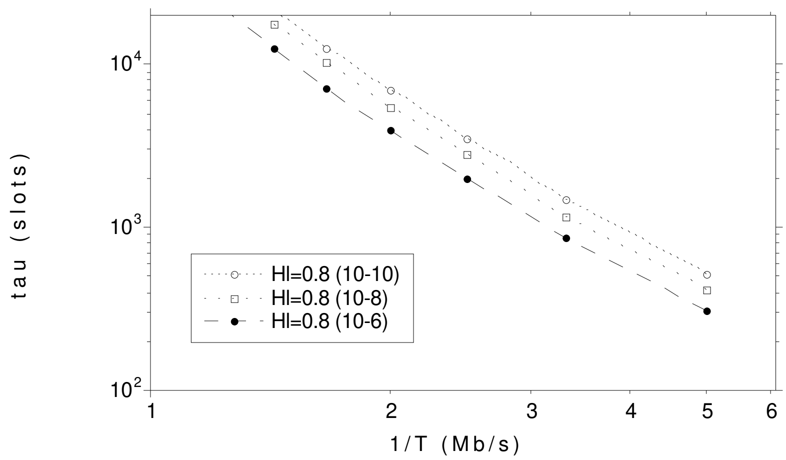

Figure 12: BT as a function of $1/T = SCR_{VCC}$ for different values of CLR, CLR $= 10^{-6}$ to 10^{-10}

5 Conclusions

In this paper, we have described a Markov chain producing self-similarity on a finite timescale which is quite easy to manipulate and depends only on three parameters. The validation on a queuing problem is also discussed. We think the theory of decomposability provides a fundamental explanation of the observed self-similarity on data networks. Furthermore, to consider only one relation between correlations over timescales seems to be unrealistic. It is surely necessary to take into account "local" self-similarities. With this kind of sources, we analyzed the inputs of a multiplexer in the context of SCONE.

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