Optimal Polarization Synthesis of Arbitrary Arrays subject to Power Pattern Constraints

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Abstract — **The optimal polarization synthesis of arbitrary arrays with focused power pattern is addressed. The proposed approach determines the weightings of the array elements to achieve a pattern whose power is subject to arbitrary upper bounds, and whose polarization is, at the same time, optimized over an angular region. The search for the solution is formulated as a convex optimization problem that can be efficiently solved using freely available software. Examples of the synthesis of linear and circular polarizations with planar arrays are presented to validate the proposed method and illustrate its potentialities.**

1 INTRODUCTION

Using polarized waveforms has several benefits such as, for instance, the increasing of the capacity of communication systems [1] and the improvement of the performance of active sensing systems [2]. However, though the synthesis of antenna arrays has received much attention over the years [3, 4], the way to generate polarized waveforms with an array is still an open and interesting topic. The joint synthesis of the waveform polarization and the spatial power pattern via convex optimization has been addressed only very recently in [5].
A method to synthesize the spatial power pattern and the polarization of arrays is presented. Arbitrary arrays, i.e. arrays of any geometry composed of elements that can have arbitrary and differing radiation patterns, can be handled and any state of polarization can be synthesized.

2 PROBLEM FORMULATION AND RESOLUTION

Let us consider an antenna array composed of N given elements placed at arbitrary but known locations. The problem is described for a one-dimensional pattern synthesis. This synthesis is performed over the polar angle θ in a fixed azimuthal plane $\varphi = \varphi_0$ (see Fig. 1), that is omitted in the notations. The extension to a two-dimensional (2-D) pattern synthesis, i.e. a synthesis over both angular directions θ and φ, is straightforward and examples of 2-D pattern synthesis are shown in Section 3.

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The electric field $\boldsymbol{E}$ radiated by the array is:

$$\boldsymbol{E}(\theta) = \begin{bmatrix} E_\theta(\theta) \\ E_\varphi(\theta) \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_\theta(\theta)^H \boldsymbol{w} \\ \boldsymbol{a}_\varphi(\theta)^H \boldsymbol{w} \end{bmatrix}, \qquad (1)$$

where $\boldsymbol{a}_\theta$ and $\boldsymbol{a}_\varphi$ are the antenna array responses according to θ and φ respectively. The weighting vector $\boldsymbol{w}$ is composed of N complex scalars w_i that control the vectorial field $\mathbf{E}(\theta)$ radiated by the array. These weightings w_i are the unknowns to determine.

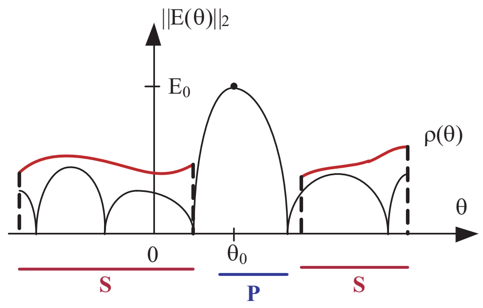

Figure 1: Schematic view of the pattern synthesis problem in a plane $\varphi = \varphi_0$. The magnitude of the electric field must reach E_0 at the direction θ_0 and remain below an arbitrary upper bound $\rho(\theta)$ over the angular region S while the polarization is optimized over P.

The array synthesis problem amounts to find the weighting vector $\boldsymbol{w}$ in (1) to achieve a pattern that has both spatial power and polarization requirements. More precisely, the spatial power is taken care of by constraints whereas the polarization is the optimization goal. As represented in Fig. 1, the goal is to synthesize a pattern having:
• a main beam in the direction θ_0 with sidelobes below a given upper bound $\rho(\theta)$ over an angular region S and
• a wave polarization, characterized by (γ_0, δ_0), optimized over an angular region.

The magnitude of the vectorial electric field is equal to: $\|\boldsymbol{E}(\theta)\|_2 = \left[|E_\theta(\theta)|^2 + |E_\varphi(\theta)|^2\right]^{1/2}$.
Let us now focus on the polarization constraint. As suggested in [6], a given polarization (γ_0, δ_0) can be

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synthesized over a range of directions ($\theta \in P$) by imposing:

$$\left|\frac{E_\varphi(\theta)}{E_\theta(\theta)} - \gamma_0 e^{j\delta_0}\right| \leq \epsilon, \; \forall \; \theta \in P \qquad (2)$$

where ϵ allows to tune the degree of accuracy with which the polarization is achieved. Setting $(\gamma_0, \delta_0) = (1, 90 \text{ deg})$ allows the synthesis of a circular polarization, whereas a linear polarization according to θ is synthesized if γ_0=0.

After some manipulations detailed in [7], the synthesis problem described in Fig. 1 can be written as:

$$\min_{\boldsymbol{w}} \;\; \epsilon \qquad (3)$$

$$\text{s.t.} \begin{cases} \left|E_\varphi(\theta) - \gamma_0 e^{j\delta_0} E_\theta(\theta)\right| \leq \epsilon |E_\theta^-(\theta)|, \; \forall \; \theta \in P \\ \left[|E_\theta(\theta)|^2 + |E_\varphi(\theta)|^2\right]^{1/2} \leq \rho(\theta), \quad \forall \; \theta \in S \\ \Re\{E_\theta(\theta_0)\} = E_0/[1+\gamma_0{}^2]^{1/2} \end{cases}$$

The optimization problem (3) is convex and it can be transformed into a Second Order Cone Program (SOCP) [8]. There are many free software that solve efficiently SOCP, without any specific tuning, in roughly the same time as a linear programming problem of equivalent size. To solve the SOCP, the optimization toolbox SeDuMi [9] is used.

3 NUMERICAL RESULTS

Numerical examples are presented to validate and illustrate the potentialities of the proposed synthesis approach.

3.1 Validation: Sequentially Rotated Array

An array composed of 16 infinitesimal dipoles, represented in Fig. 2(a), is considered. This array is synthesized to radiate a pattern having 2-D spatial constraints namely:

• a main beam in the broadside direction $\theta_0 = 0$ deg, i.e. $(u_0, v_0) = (0, 0)$ since $u = \sin\theta\cos\varphi$ and $v = \sin\theta\sin\varphi$, with sidelobe levels $\rho(\theta, \varphi)$ below -12 dB for $\theta \geq 25$ deg, i.e. outside the white circle in Fig. 2(b)

• a circular polarization optimized over the angular range $P = \theta \leq$25 deg.

The synthesized far field pattern and Axial Ratio (AR), obtained solving (3), are plotted in Fig. 2(b,c). The power pattern (Fig. 2(b)) complies to the requirements and the AR plot (Fig. 2(c)) clearly shows that a circular polarization is synthesized over the region P. The optimized weightings are given in Table 1.

	P = 0 deg		P = [-25 deg,25 deg]	
feed	$\|w\|$	$\angle w$ [deg]	$\|w\|$	$\angle w$ [deg]
1	0.42	168	0.59	-173
2	0.65	-85	0.95	-90
3	1.00	0	1.00	-4
4	0.65	94	0.85	90
5	0.66	-176	0.84	-180
6	0.41	-103	0.59	-83
7	0.65	4	0.96	0
8	1.00	90	1.00	86
9	1.00	180	1.00	176
10	0.65	-86	0.85	-90
11	0.42	-12	0.59	7
12	0.65	94	0.95	90
13	0.65	-176	0.96	180
14	1.00	-90	1.00	-94
15	0.66	4	0.84	0
16	0.41	77	0.59	97

Table 1: Optimal weightings of the sequentially rotated array synthesis.

Referring to Fig. 2(a), the 16-element array can be seen as 4 sub-arrays composed of 4 sequentially rotated dipoles, namely (1,6,11,16), (2,7,12,13), (3,8,9,14) and (4,5,10,15). Within each sub-array, the optimized weightings have almost the same magnitude and a phase shift close to 90 deg, which is consistent with the concept of sequentially rotated arrays [10].

3.2 Planar Array for 2-D Pattern Synthesis

A planar array is used to synthesize a pattern having spatial power constraints and an optimized polarization. This array is composed of 3 × 3 dual-polarized patches having a fixed randomly chosen orientation, as represented in Fig. 3(a). The active element pattern method, detailed in [11], is applied to calculate the array pattern radiation in order to consider the mutual coupling effects.

3.2.1 Circular Polarization Synthesis

The constraints of the synthesis problem are the following:

• a main beam in the direction $(u_0, v_0) = (0.2, 0.3)$, i.e. (θ_0, φ_0)=(21 deg,56 deg), with an upper bound $\rho(\theta, \varphi)$ of -10 dB outside the circle in Fig. 3(b) and

• a circular polarization optimized inside the circle in Fig. 3(c).

The contour plots Fig. 3(b) show that the spatial

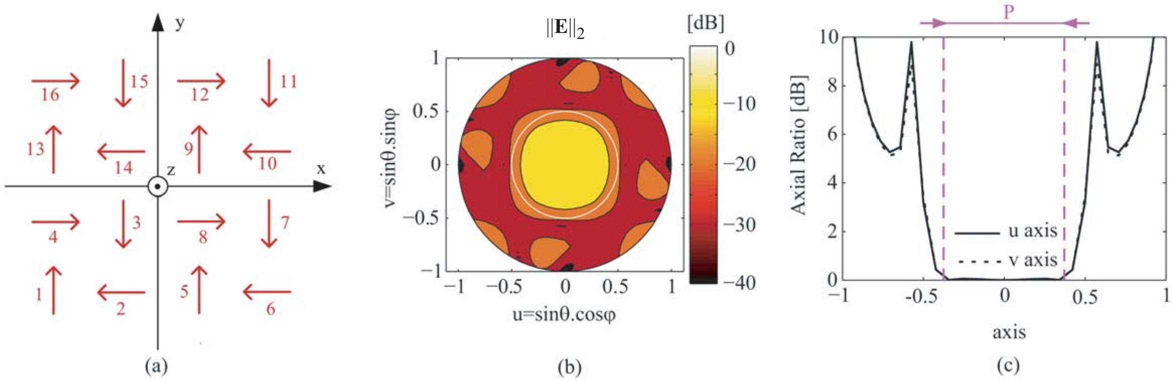

Figure 2: (a) Schematic view of the sequentially rotated array composed of 16 dipoles spaced by $\lambda/2$ with the results of the synthesis procedure (b) magnitude of the total electric field is lower than 12 dB outside the white circle. The AR, optimized over P is plotted on both axis $u = 0$ and $v = 0$, defined in (b).

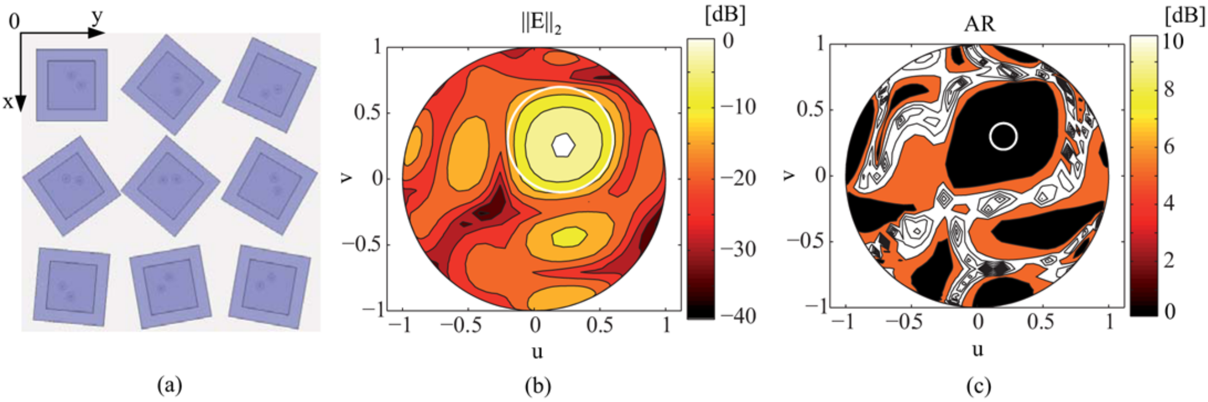

Figure 3: (a) Top view of the planar array composed of 3×3 dual polarized patches with random orientation. (b) Contour plot of the synthesized total electric field. The sidelobe levels are below -10 dB outside the white circle. (c) Contour plot of the axial ratio. The circular polarization has been optimized inside the white circle, where AR is lower than 0.2 dB.

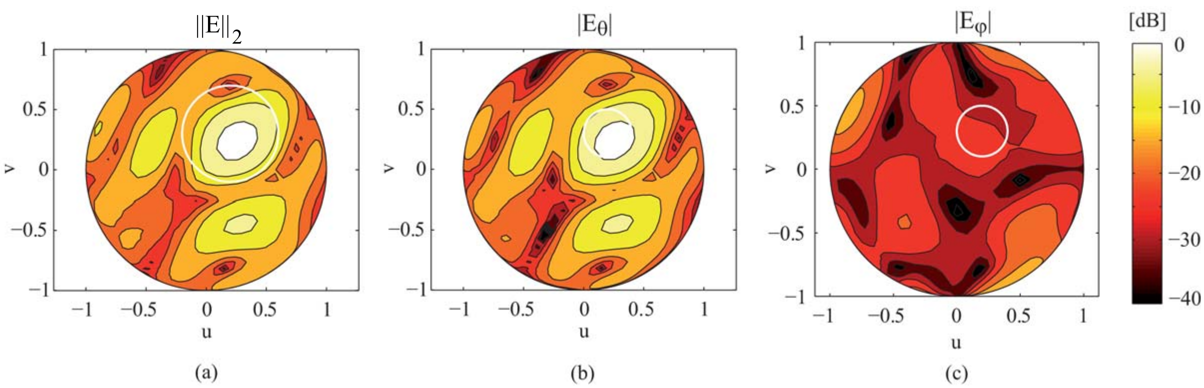

Figure 4: Far field patterns corresponding to the linear polarization synthesis with the planar array. (a) The magnitude of the total electric field is lower than -10 dB outside the white circle. A linear polarization according to θ is optimized inside the white circle plotted in (b) and (c).

power constraint is respected. The AR, represented in Fig. 3(c), is lower than 0.2 dB inside the circle where the polarization has been optimized.
The computation time of this synthesis problem is less than 5 seconds on a standard laptop.

3.2.2 Linear Polarization Synthesis

For this synthesis problem, the spatial power constraints are unchanged (u_0, v_0)=(0.2,0.3) and $\rho(\theta,\varphi) \leq$-10 dB outside the circle represented in Fig. 4(a). A linear polarization according to θ is optimized, by solving (3) with $\gamma_0 = 0$, inside the circle plotted in Fig 4(b,c).
The contour plots of the synthesized patterns confirm that, in this region, the cross-polarization $|E_\varphi|$ is at most 19 dB lower than the co-polarization $|E_\theta|$.

4 CONCLUSION

A synthesis method to design arrays that radiate a pattern, having both spatial power constraints (upper bounded sidelobes) and an optimized specified polarization over a given angular region, has been proposed. Note that the desired polarization is optimized, not only in a single direction, but over an angular region, which is a frequently encountered requirement.
The proposed synthesis method can easily be modified to yield an optimization problem that minimizes the sidelobe levels, while guaranteeing a specified accuracy of the wave polarization over an angular range.

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