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Active tensegrity structures with sliding cables – static and dynamic behaviour

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Abstract — Tensegrities are spatial, reticulated and lightweight structures that are increasingly investigated as structural solutions for active and deployable structures. Tensegrity systems are composed only of axially loaded elements providing opportunities for actuation and deployment through changing element lengths. In cable-based actuation strategies, the deficiency of having to control too many cable elements can be overcome by connecting several cables. However, replacing sequences of cable elements by continuous cables sliding over joints significantly changes the mechanics of classical tensegrity structures. Furthermore, challenges emerge for structural analysis, control and actuation. In this paper a modified dynamic relaxation (mDR) algorithm is employed to investigate actuated tensegrity structures with continuous cables. The dynamic behavior of such structures is also investigated.
Keywords — Tensegrity structures, sliding cables, active and deployable structures, dynamic relaxation

1 Introduction

Tensegrities are spatial, reticulated and lightweight structures that are composed of struts and cables. Stability is provided by a self-stress state established in tensioned and compressed elements. The tensegrity concept has applications in fields such as sculpture, architecture, aerospace engineering, civil engineering, marine engineering and biology [1]. Tensegrity structures have a high strength-to-mass ratio leading to strong but lightweight structural systems [2, 3]. Furthermore, tensegrities are flexible and easily controllable with low energy requirements [4]. Therefore, tensegrity structures are particularly attractive for active and deployable structures.

As a special type of prestressed pin-jointed framework, tensegrity structures are composed of axially loaded elements providing opportunities for actuation and deployment through changes element lengths. Length changes can be made to struts and/or cables through various actuation strategies. Strut-based actuation, employing telescopic members, has already been used in active tensegrity control applications. Fest et al. [5] experimentally explored shape control of a five-module large-scale active tensegrity structure. The actuation strategy was based on controlling the self-stress state of the structure through small movements of ten telescopic struts. This actuation was also used for self-diagnosis, self-repair and vibration control [6, 7]. Tibert and Pellegrino [8] numerically and experimentally investigated use of telescopic struts for the deployment of tensegrity reflectors. Generally, strut-based actuation becomes difficult to implement under conditions where internal forces are substantial, and required changes in shape are large. Furthermore, when strut-actuation is used for deployment, the structure may have no stiffness until it is fully deployed.

Cable-based actuation has been investigated in many research projects involving active and deployable structures. Bouderbala and Motro [9] studied folding of expanded octahedron assemblies and showed that cable-mode folding was less complex than strut-mode, although the latter produced a more compact package. Djouadi et al. [10] developed a cable-control strategy for vibration damping of a tensegrity structure. Sultan and Skelton [11] proposed a tendon-control deployment strategy for tensegrity structures. Actuation is conducted in such way that the structure goes throughout successive equilibrium configurations. Smaili and Motro [12] investigated folding of tensegrity systems by activating finite mechanisms. A cable-control strategy was applied to a double-layer tensegrity grid. The proposed strategy was extended to the folding of curved tensegrity grids [13].

Most research studies of deployment of tensegrity structures showed that cable-actuation strategy directs tensegrity structures to maintain stiffness as they move from one equilibrium position to another. There are, however, a few disadvantages with this approach. Tibert and Pellegrino [14] argued that controlling cables is complicated, because of all the additional mechanical devices that are necessary. The deficiency of having to control too many cable elements can be overcome by connecting several cables together and using only one motor to control them [11]. This suggests that groups of individual active cable elements could be combined into continuous active cables. A single continuous cable can slide over multiple nodes through frictionless pulleys. This strategy has the advantage that fewer actuators are necessary for control. However, using continuous cables significantly changes the mechanics of classical tensegrity structures. Specifically the number of self-stress states may decrease and the number of internal mechanisms may increase eventually resulting in unstable configurations [15]. This leads to significant challenges for structural analysis, control and actuation.

Finite-element formulations for sliding cable-elements have been developed for modeling of suspension systems [16-19] and fabric structures [20]. Chen et al [21] presented a formulation of multi-node sliding cable-element for the analysis of Suspen-Dome structures. Genovese [22] investigated form-finding and analysis of tensegrity structures with sliding cables. The complete formulation of such systems was provided by Moored and Bart Smith [23]. Moored and Bart-Smith [23] formulated the potential energy, equilibrium equations and stiffness matrix for tensegrity structures with continuous (clustered) cables. The equilibrium equations of a tensegrity structure are non linear. Analysis can thus be carried out in an iterative manner through use of the transient stiffness method. Matrix methods generally require iterative assembling and inversion of large stiffness matrices. As a vector-based method, the dynamic relaxation method (DR) does not require such complexity. This method introduced by Otter [24] and Day [25] in the mid-1960s is particularly attractive for modeling nonlinear structural behaviour. DR is an explicit iterative method for the static solution of structural-mechanics problems [26]. When the DR method is used, the static problem is transformed into a pseudo-dynamic one by introducing fictitious inertia and damping terms in the equation of motion. DR traces the motion of each node of the structure until it comes to rest in static equilibrium due to artificial damping. One of the advantages of this method is that the formulation of the structure stiffness matrix is not needed and hence the method is particularly suitable for problems with material and geometrical nonlinearities. DR has been used by many researchers to solve a wide variety of engineering problems. Furthermore, Barnes [27] showed that DR is particularly efficient for form finding and analysis of tension structures. Hundreds, perhaps thousands of structures such as cable-stayed bridges and large tent structures have been designed and then analyzed using DR. For a tensegrity structure with continuous cables, the uncoupled nature of the DR process makes it particularly straightforward to implement [28].

In this paper, a modified dynamic relaxation (mDR) algorithm applicable to the analysis of tensegrity structures with continuous cables is proposed. The algorithm is validated by simulating load response and actuation of an active tensegrity beam. Results are compared with those obtained employing a stiffness-based algorithm to show effectiveness of the proposed methodology. The effects of introducing continuous cables sliding over frictionless pulleys into tensegrity structures are also investigated.

2 Modified dynamic relaxation method

The DR method follows from augmenting static equilibrium equations (Eq. (1)) by including inertial and damping terms (Eq. (2)):

$$\mathbf{F}_{int}(\mathbf{u}) = \mathbf{F}_{ext} \tag{1}$$

$$\{\mathbf{M}\,\dot{\mathbf{v}} + \mathbf{C}\,\mathbf{v}\} + \mathbf{F}_{int}(\mathbf{u}) = \mathbf{F}_{ext} \tag{2}$$

In Eq. (1) and (2), u and v are the vectors of nodal displacements and velocities, **M** and **C** are the mass and damping matrix, $\mathbf{F}_{\mathrm{int}}$ is the vector of internal forces and $\mathbf{F}_{\mathrm{ext}}$ is the vector of external forces. Introducing the residual force vector **R** as the difference between external and internal forces at any time *t*, Eq. (2) becomes

$$\mathbf{M}\,\dot{\mathbf{v}}^t + \mathbf{C}\,\mathbf{v}^t = \mathbf{R}^t \tag{3}$$

In DR, Eq. (3) is iteratively resolved. Due to damping, nodal velocities and accelerations decay to zero as the solution is approached. The transient response is attenuated leaving the steady state solution for the applied load. The static equilibrium is thus attained and the out-of-balance or residual forces come to zero.

To obtain the DR basic equations, the following central difference approximations are used for temporal derivatives:

$$\mathbf{v}^t = \frac{\mathbf{v}^{t+\Delta t/2} + \mathbf{v}^{t-\Delta t/2}}{2}, \quad \dot{\mathbf{v}}^t = \frac{\mathbf{v}^{t+\Delta t/2} - \mathbf{v}^{t-\Delta t/2}}{\Delta t} \tag{4}$$

Using these approximations, Eq. (3) can be re-arranged to give the recurrence equations for nodal velocities where subscript *i,x* refers to the i^{th} node and direction *x* (respectively, for directions *y* and *z*):

$$v_{i,x}^{t+\Delta t/2} = v_{i,x}^{t-\Delta t/2}\left(\frac{M_{i,x}/\Delta t - C_{i,x}/2}{M_{i,x}/\Delta t + C_{i,x}/2}\right) + R_{i,x}^{t}\left(\frac{1}{M_{i,x}/\Delta t + C_{i,x}/2}\right) \tag{5}$$

The velocities are then used to predict displacements at time ($t + \Delta t$):

$$u_{i,x}^{t+\Delta t} = u_{i,x}^{t} + \Delta t \,.\, v_{i,x}^{t+\Delta t/2} \tag{6}$$

The iterative process of DR method consists of a repetitive use of Eq. (5) and (6). The process continues until the residual forces are close to zero. The values of masses **M** and damping **C** have to be chosen to ensure that the recurrence scheme converge to the static equilibrium [26]. Generally, a diagonal mass matrix is used along with a mass proportional damping matrix. This strategy involves the determination of a critical viscous damping coefficient. An alternative damping approach is the use of kinetic damping. This approach is adopted in this study [29].

The modification introduced to DR in order to adapt it to the analysis of tensegrity structures with continuous cables is concerned with the calculation of the residual forces. With the assumption that all sliding cables run over frictionless pulleys, each continuous cable in the structure can be assumed as a string of cable sub-elements all carrying the same tensile force. At each time step *t*, Eq. (6) is used to determine current node coordinates of the structure. The new member-length vector is thus easily determined. For continuous members, current element lengths can be calculated using Eq. (7).

$$\bar{\mathbf{l}} = \mathbf{S}\,\mathbf{l} \tag{7}$$

where **S** is a transformation matrix. Moored and Bart-Smith [23] called this transformation matrix the *clustering matrix* and showed that this matrix can be defined as follow:

$$S_{ij} = \begin{cases} 1, & if\ sub-element\ e_j\ is\ part\ of \\ & continuous\ element\ \overline{e}_i \\ 0, & otherwise \end{cases} \tag{8}$$

Subsequently, current internal forces in the m[th]-member of the tensegrity may be determined as follows:

$$\overline{t}_m^t = \frac{\overline{E}_m \overline{A}_m}{\overline{l}_{0,m}} \left(\overline{l}_m^t - \overline{l}_{0,m} \right) + \overline{t}_m^0 \tag{9}$$

where $\overline{l}_{0,m}$ and $\overline{l}_m^{t+\Delta t}$ are rest and current length of continuous member *m*. $\overline{E}_m$, $\overline{A}_m$ and $\overline{t}_m^0$ are Young modulus, cross-section area and initial prestress of continuous member *m*. Once the vector of internal forces in the tensegrity elements ($\overline{\mathbf{t}}$) is determined, the vector of internal forces in the tensegrity sub-members ($\mathbf{t}$) can be computed employing Eq. (10).

$$\mathbf{t} = \mathbf{S}^{\mathbf{T}} \overline{\mathbf{t}} \tag{10}$$

The residual forces can thus be calculated. For any node *i*, the residual force in x-direction $\mathbf{R}_{i,x}$ is calculated as the sum of the external force $\boldsymbol{f}_{ext,i,x}$ and the x-component of the resultant force induced by the contributions of the N members meeting at node *i* (Eq. (11)). Similar equations may be written for the *y* and *z* coordinate directions.

$$R_{i,x}^t = f_{ext,i,x} + \sum_{m=1}^{N} \frac{t_m^t}{l_m^t} \left(x_{j,m}^t - x_{i,m}^t \right) \tag{11}$$

A stiffness based mass matrix and a kinetic damping are adopted in this work [29].The iterative process continues until static equilibrium is attained when the norm of the vector of residual forces goes below a fixed precision value.

3 Case study: a tensegrity beam

The performance of the proposed modified DR algorithm is demonstrated using the topology of a tensegrity structure studied by Moored and Bart-Smith [23]. The structure is a tensegrity beam composed of an assembly of three prismatic modules commonly known as quadruplex modules. A quadruplex unit comprises four struts held together in space by 12 cables. When only rigid-body movements are blocked, this tensegrity unit has a unique state of self-stress and three infinitesimal mechanisms. The quadruplex unit can be reinforced by adding four reinforcing cables which remove mechanism modes from the structure.

Three reinforced modules are connected together with no bar-to-bar connections, forming a class I tensegrity structure. A perspective view of the tensegrity beam is given in Fig. 1 where grayed lines denote bars and thin lines denote cables. Three of the end nodes of the structure (nodes 1, 3 and 6) are pinned forming a cantilever beam. Ten cable elements of the top surface and ten cable elements of the bottom surface of the structure are grouped into four continuous cables. In Fig. 2, sliding cables are shown in dashed lines in a top view of the tensegrity beam. Each sliding cable is attached to two end nodes and runs frictionnlessly through four intermediate nodes. Details of the four continuous cables are given in Table 1.

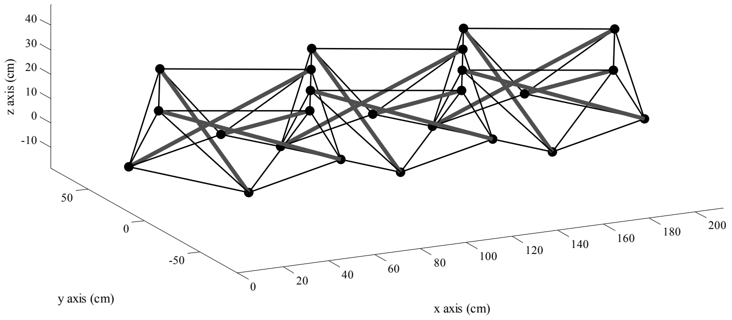

FIG. 1 – A perspective view of the tensegrity beam

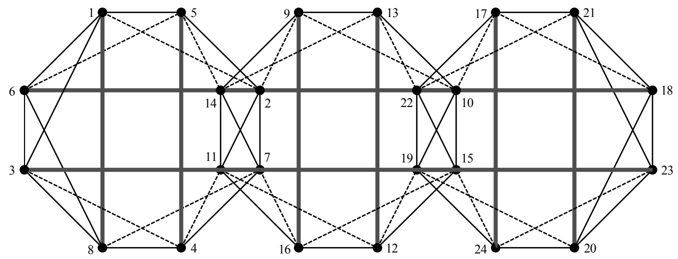

FIG. 2 – Continuous cables of the tensegrity beam

TAB. 1 – Details about continuous cables

Cable	Position	End nodes	Intermediate nodes
1	Top surface	6 and 21	5, 14, 13 and 22
2	Top surface	8 and 23	7, 16, 15 and 24
3	Bottom surface	1 and 18	2, 9, 10 and 17
4	Bottom surface	3 and 20	4, 11, 12 and 19

The tensegrity beam used in this study has a length of 212 cm, a width of 80 cm and a height of 30 cm. Struts are made of aluminum hollow tubes with a length of 85cm. Saddle, vertical and reinforcing cables have a length of 60, 48 and 40 cm, respectively. All cable members are made by stainless-steel. Detailed characteristics of used members are summarized in Table 2.

TAB. 2 –Material characteristics for the tensegrity beam

Member	Material	Cross-section area (cm^2)	Young modulus (kN/cm^2)	Specific weight (kN/cm^3)
Struts	Aluminum	2.55	7000	$2.7\ 10^{-5}$
Cables	Stainless-steel	0.5026	11500	$7.85\ 10^{-5}$

Actuated bending deformation of the tensegrity beam is first studied. Bending deformation can be obtained through antagonist actuation of the top and bottom sliding cables. Actuation is performed by changing the effective rest length of actuated cables. For example, a prescribed actuation stroke of 20% is defined as a change in the rest length of 20%. Prior to actuation, top continuous cables are contracted by 2% in order to introduce self-stress in the structure and counteract deflection induced by self-weight. The tensegrity beam is then actuated through modifying lengths of the four actuated cables. Top sliding cables are actuated with 10% contraction while the bottom sliding cables are expanded by 10%. Contraction and elongation of actuated cables is conducted progressively in steps of 1%. Note that actuation is deliberately performed through small and slow steps so that inertia effects can be neglected when the structure is in motion. The actuation response obtained by the proposed DR method and transient stiffness method based on Moored and Bart-Smith formulation [23] are compared in Fig. 3. Displacements at the top node 18 of the beam are displayed with respect to the actuation ratio in active elements (Fig. 3). It is observed that the result of the present study are in agreement with those obtained by Moored and Bart-Smith [23].

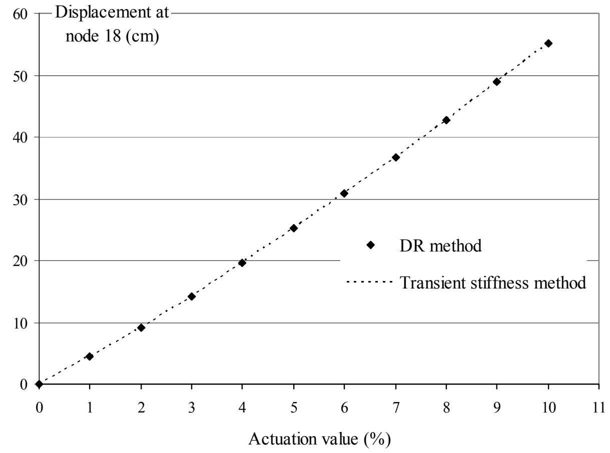

FIG. 3 – Actuation response of the tensegrity beam

When the top continuous cables have been actuated with 10% contraction while the bottom clusters have been expanded by 10%, this results in a 55cm tip deflection in the positive z-direction (Fig. 6).

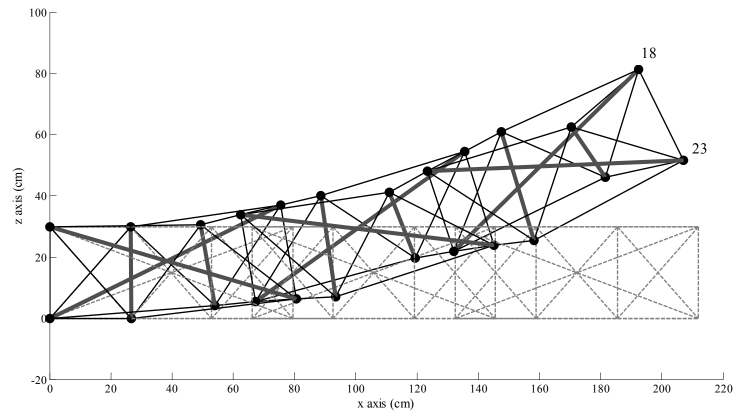

FIG. 4 – Deformed shape of the tensegrity beam due to 10% actuation of active cables

Load-response of the tensegrity beam is also investigated employing both DR and transient stiffness methods. The tensegrity beam is subjected to a vertical load applied at nodes 18 and 23. The load-displacement curves obtained by the proposed DR method and transient stiffness method based on Moored and Bart-Smith formulation [23] are compared in Fig. 5. It can be seen that the results predicted by the DR method are identical to those generated by the transient stiffness method.

Load-displacement curves of the tensegrity beam obtained employing modified and unmodified dynamic relaxation are compared in Fig.6. This comparison reveals that predicted displacement are under estimated if the effect of sliding cables is not considered in the analysis. Results indicate that the tensegrity beam with sliding cables is about 55% more flexible than an equivalent configuration having discontinuous cables. An eigenfrequency analysis showed also that including continuous cables induced a decrease in the tensegrity natural frequencies approaching 50% (Table 3). Furthermore, the nonlinear behaviour induced by large displacements is more accentuated in tensegrity structures with continuous cables.

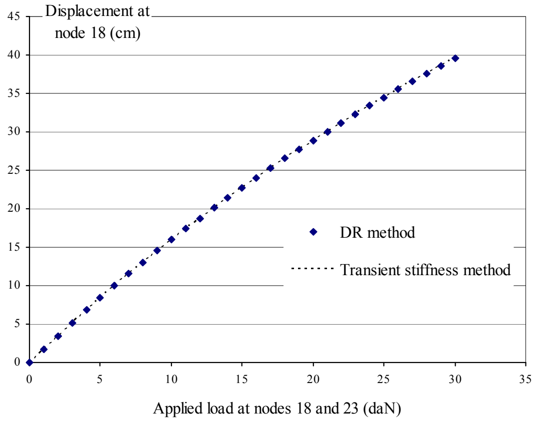

FIG. 5 – Load-displacement curves of the tensegrity beam

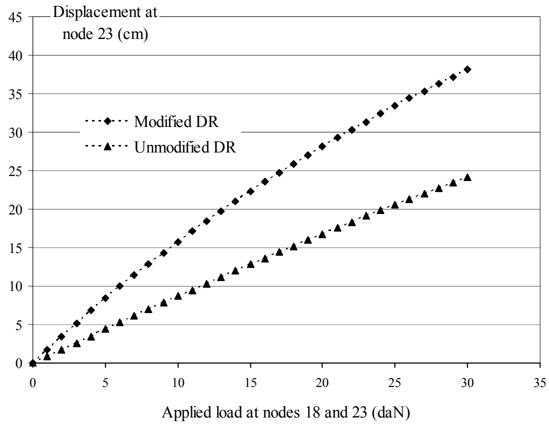

FIG. 6 – Load-displacement curves obtained employing modified and unmodified DR

TAB. 3 – Natural frequencies of the tensegrity beam (Hz)

Vibration mode	Without sliding cables	With sliding cables
1	21.6	12.4
2	30.3	17.1
3	42.0	20.9

4 Conclusion

Dynamic relaxation is an attractive static analysis method for tensile and tensegrity structures. The classic DR method is extended here to accommodate tensegrity structures with continuous cables. In cable-based actuation of tensegrity structures, the deficiency of having to control too many cable elements can be overcome by connecting several cables. Continuous cables are thus used instead of discontinuous members. Sliding cables are assumed to run without friction through structural nodes. The concept of cable grouping is a scalable solution that can be employed for active structures that incorporate many actuated elements in order to reduce the number of actuators needed for active control and deployment. However, grouping cables significantly changes the mechanics of classical tensegrity structures and this leads to new challenges for structural analysis, control and actuation. This study shows that the uncoupled nature of the DR process makes it particularly attractive to apply to structures with sliding cable elements.

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