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Geometrical Description and Structural Analysis of a Modular Timber Structure

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ABSTRACT: The ambitious goal of the ongoing research at IBOIS, the Laboratory of timber constructions at the Ecole Polytechnique Fédérale de Lausanne (EPFL) is to develop a next generation of timber constructions made out of innovative timber-derived products, through applying textile principles on the building scale. The presented structure is a modular composition of timber folded panels, notably demonstrates an example of applying the geometric techniques used to produce modular patterns and lattices to timber construction context. Effectively, it is shown that complex space structures can be designed using simple connection technology between elements. Moreover, by taking advantage of advanced CAM process, complex planar timber elements are cut in large scale and assembled with high precision as for the prototype of the structure presented in this paper. Here, the basic module is consisted of two mutually supporting timber folded panels which are slipped in, consecutively, along their cuts, to build up an arch. The folding concept corresponds to a new family of reciprocal frame structures, where instead of traditional linear elements, mutually supported planar members are employed. The inter-module connection's stability is provided by contact boundary condition over the slide joints. The fundamental mechanical properties of the structure are examined using Finite Element Method and considering the non-linear contact boundary condition. The static behavior is studied under the self-weight load case as well as the modal dynamic response. According to analysis results, and by aid of a CAD parametric model, structural and geometrical alternatives are proposed to improve the structural performance. A prototype based on this geometric principal has been fabricated and assembled to explore feasibility of the concept in the building scale.

Key Words: Timber space structure; planar reciprocal frame structure; structural system improvement; Finite Elements analysis; parametric design

1. INTRODUCTION

1.1. IBOIS, the re-interpretation of timber construction

In recent years, the necessity of using renewable and sustainable resources in the building sector has become obvious, and interest in timber as a building material has revived [1–3]. Novel timber-derived products, such as massif block panels, have emerged and the use of such products is spreading [4, 5].

Since timber can be viewed as a fiber-derived product, it follows that the analogy between micro scale fiber structures and timber-derived wooden structures can be explored at micro and macro-scale. The key to this approach is the underlying notion that timber's fibrous nature, historically perceived as a liability for a construction material, is in fact a precious feature that should be exploited to increase both the material's functional and aesthetic value. In fact, this

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inherent flexibility of timber-derived panels permits it to be folded into robust, lightweight structures that use material very sparingly. This materiality is explored within a current research direction at IBOIS. The idea is to inspire from techniques of the textile industry and to re-interpret the basic units of a fabric as nodes and knots using thin flexible timber elements to propose entangled (woven) forms as timber space structures. An innovative "timber knot" as well as a generalization of the concept into a "timber fabric" is presented in [6]. The structure presented in this article comes from a different inspiration of textile techniques, tightly related with the geometric techniques used to describe patterns and tailings.

1.2. Geometry of space structures: a survey on "form-element" realationship

During the two past decades, space structures with multiple forms and various materialities have been designed, where the free-form practice has been the favorite stream. In context of timber construction, "local" inter-element relationships and the link between the "local" condition and the "global" forms are of grand importance. That's why the origin of the "form" for space structures is questioned and surveyed in the following lines, looking forward to understand the important link between the global geometry and its local consequences on the discretized elements.

Effectively, four main directions dealing with the geometry of the space structures can be recognized as follows.

- **Space structures issue of a From-finding practice:** Two main structural families of this category are tensile membranes and vaulted masonry construction. Multiple techniques have been introduced in the literature and practiced in order to determine a tensile or compressive geometry: physical models of Otto for the tensile roofs, computer-based methods as Force-density method and Dynamic relaxation for tensile roofs [7–9] and Thrust network analysis for masonry vaults as in [10, 8]. The term, "form-finding" is also used to recall a form-generation process used to find the relaxed form of grid-shells under a certain boundary condition as in [11]. Nevertheless, what is common between these various techniques is the fundamental property that the final form is the direct result of the equilibrium and is influenced intensively by the materiality and the boundary condition applied.
- **Free-form design:** Here the goal is not to give reader a review of the free form design practice but rather to reveal an important "local" consequence of such design on structural elements. According to free-form approach, the design surface comes as the input and then by means of geometric-mathematical models, it's approximated by a typologically uniform (or at least modular) planar mesh. Although single or double curved panels are also allowed, but planar elements constitute the major part of the approximated mesh. The mesh generation is clearly constrained by tolerances of the approximation problem from the reference surface, as well as the economic-industrial external constraints [12–14]. The supporting structure follows closely this approximated mesh and thus its spatial form is indeed less influenced from materiality the connection technology. In other words, "local" conditions are determined by the "global" form.
- **Topology optimization:** The space structures belonging to this family have their form generated by resolution of a topology or shape optimization problems for which a review can be found in [15].

  These methods combined with evolutionary algorithms, often result in free-form spatial objects, which although they might have been generated based on a structural criteria, fall into the second category, where they are geometrically approximated in order to be built.
- **Reciprocal frame structures:** This family of modular space structures is constituted of interlaced linear elements, where the final form is a result of a basic module as well as a connection technology. The concept exists since centuries, for which a review can be found in [16] and a technical decomposition of the structure is proposed in [17].

  The interest of such structural system lays in the fact that using identical linear beams and simple connection techniques, it's possible to reproduce rather complex three-dimensional space structures.

  In such family of structures, the global form is not an approximation of a design reference surface, but the result of the basic element's topology and the connection technology applied. Unlike the free-form design, the "global" form is determined by the "local" conditions.

In fact, in context of timber engineering it's recognized that the complexity of the connection technology, becomes an important feature in order to distinct different structural propositions both from financial and structural aspects. Usually, free-form timber frames end up with heavy and complex steel nodes, designed to resist external loads. In such context it follows that the connection technology is preferred to be simple, whereas, for timber elements, complex geometries can be realized by means of advanced CAM facilities. The analogy is thus interesting to be investigated with the reciprocal frames, where complex forms come from a basic repeated module and simple connection technology.

The core idea which follows in coming lines is an investigation on a family of space structures, where the focus, rather than on an irregular surface approximation, lays on materiality and related connection technology. According to this approach, the final form is a result of the geometry of connected members as well as the employed connection technology.

The modular structure proposed in this paper comes from such exploration on form-element relationship. It is not only geometrically treated, but also from mechanical point of view it is demonstrated that its structural behavior is understood. Moreover a prototype is realized to complete the investigation.

2. PRESENTATION OF STRUCTURE

2.1. The folding concept

The folding concept presented in this paper has been initially examined, during an architectural workshop, “The Atelier Weinand” at IBOIS-EPFL, turning around the discrete architectural geometry. A V-form basic module is fabricated through connecting two timber panels and is then spatially multiplied using consecutive spatial rotations and translations to form an arch (fig 1).

The structure can be decomposed into four principal typologies (fig 2) each consisted of two mirrored timber panels, joined together as a V-form module, through the bisector plan, by means of two hard wood dowels (fig 3). They are placed in the middle as shear keys and two oval head screws inserted nearer to the borders in order to avoid relative translation and rotation. Fasteners are inserted in direction of the normal to the bisector plan. These modules are then slipped in consecutively along their U shape cuts, to form an arch. The inter-panel stability is provided by roto-rigidity of slide connection and axial contact of reciprocal panels.

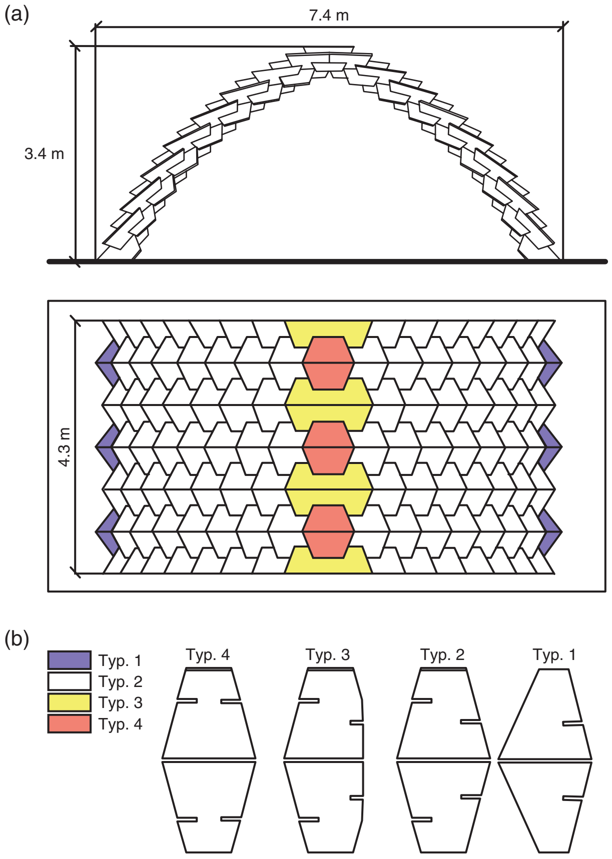


Figure 2. (a) Front and top view of the prototype (b) Principal typologies of timber panels (unfolded top view).

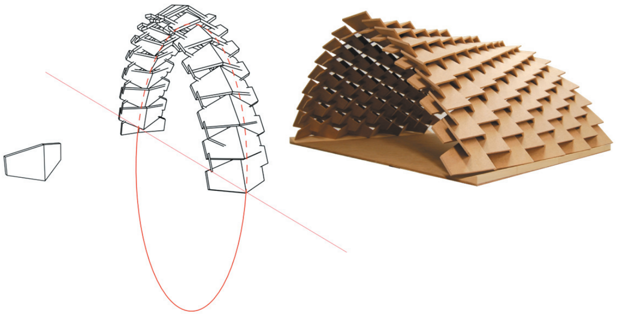


Figure 1. Folding concept for the modular structure designed in the Atelier Weinand workshop © Bastien Thorel.

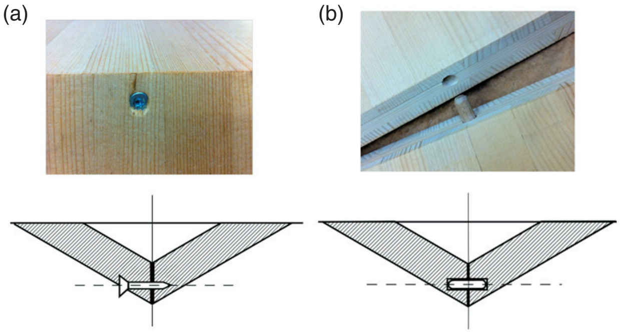


Figure 3. Mirrored panels connection fasteners: (a) Oval head screws (b) wooden dowels inserted in prefabricated holes.

2.2. Geometric and parametric decomposition of global form

A trapezoidal plate is introduced to be the generator member of the form, noted as β_0 and referred to as the "Base panel". From mathematical point of view, this single panel is transformed by means of two classes of operators in order to give shape to the global form. The first operations are the set of Boolean operators, representing the machining, connecting and assembling process. Whereas the second type of operators, referred to as Geometrical operators, introduce rigid body movements and consist of congruent maps, employed to place the object in the space. Among Boolean operations, union, intersection and remove are used. While among Geometrical operators, we may enumerate rotation around a space vector, reflection against a plan, and inversion against a spatial point or translation in direction of a space vector as examples of simple isomorphisms.

The parametric geometric transformation permitting to determine the slide cuts between two consecutive modules is illustrated in fig 4. The

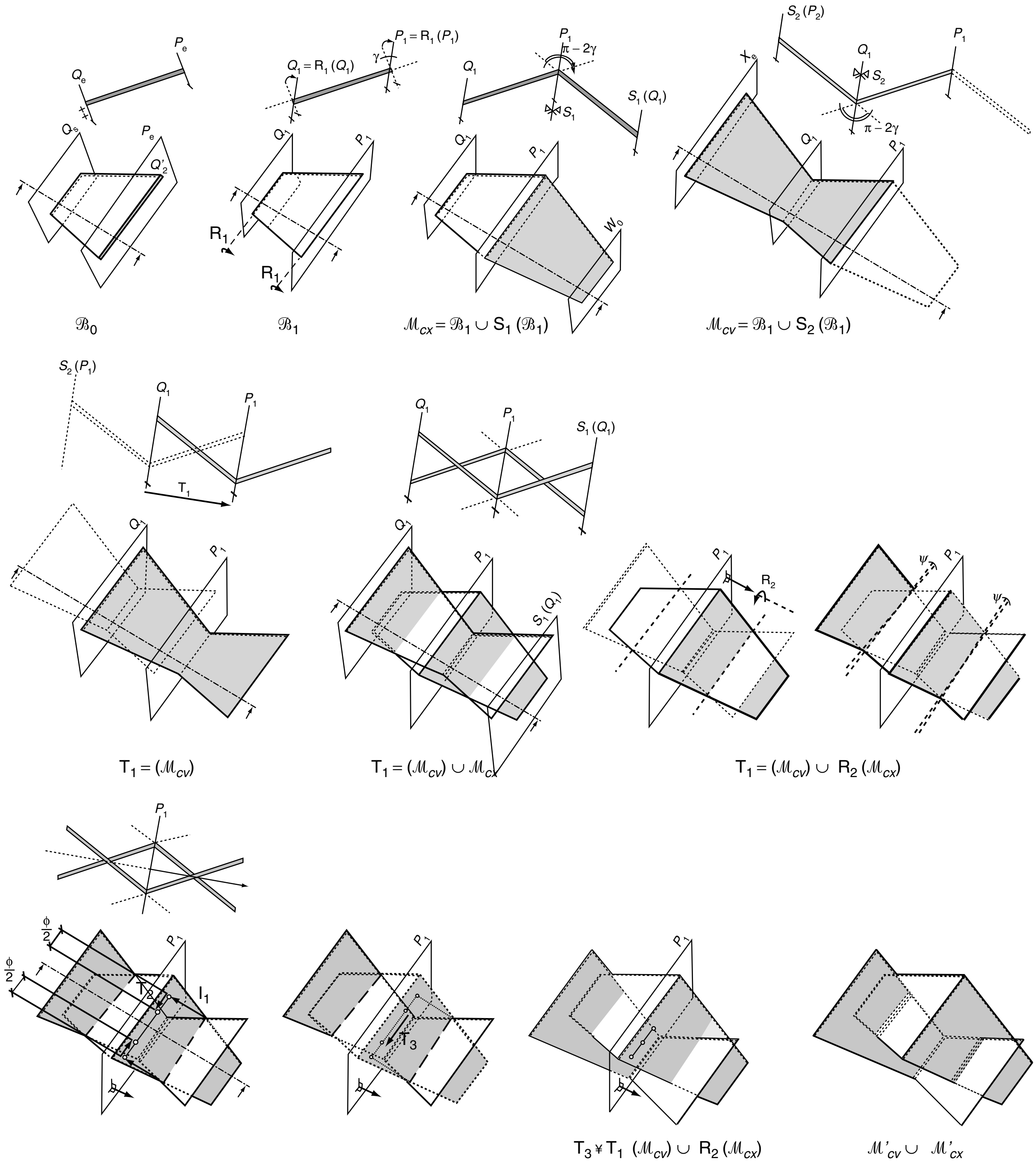


Figure 4. Transformation of basic trapezoidal panel (β_0) into two *V*-form modules ($M'_{cv} \cup M'_{cx}$).

manuscript letters (β, M, etc) stand for geometric objects and bold capital letters as (**S**, **R**, **T**, etc) symbolize isomorphisms, respectively the symmetry, rotation and translations. Boolean operations are represented by their mathematical symbols of union, etc. By repeating the same geometric transformation, a double cut module is obtained shown in fig 5, called the "Base module", which is the base brick of the modular structure.

Base modules are then slipped in, one on top of the other, to build up a half-arch, which is trimmed by a vertical plan placed at the mid line of the span and is mirrored against the same plan to produce a complete arch. Next step will be to trim the arch using a horizontal plan, and to place it on level (See Fig 6a.). This arc is then translated in direction of vectors **w** and 2**w** to get the modular structure of the prototype (See Fig 6b.). Naturally, one can continue to expand the structure, by multiplying n**w** consecutive transversal translations.

From mathematical point of view, it's easy to show that each Base module can be mapped on the other slipped Base modules of the arch by means of an isomorphism. This transformation, similar to the logic followed in Fig 4, is a product of spatial translation and rotations, and thus an orthogonal map.

As a result of the implementation of the geometric transformation described above in a CAD environment, a parametric model of the modular structure is obtained where the geometry is controlled by means of a set of meaningful scalars. This parametric description of the structure, apart from the benefits from the design point of view, as shown in §4.2 may lead to alternative structural systems which show better performances in terms of dynamic behavior.

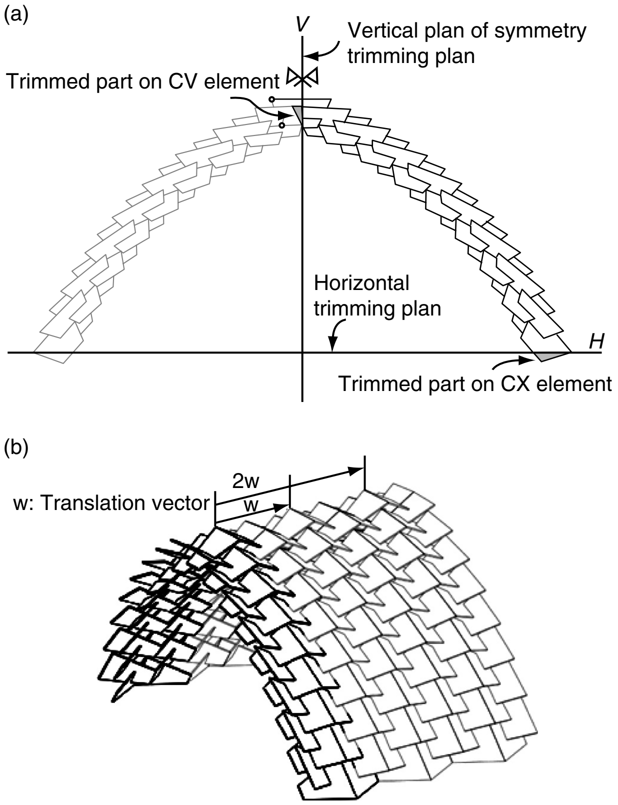


Figure 6. (a) Trimming of half arc by horizontal and vertical plans and mirroring (b) Translating arc to build up the geometry of the real scaled prototype.

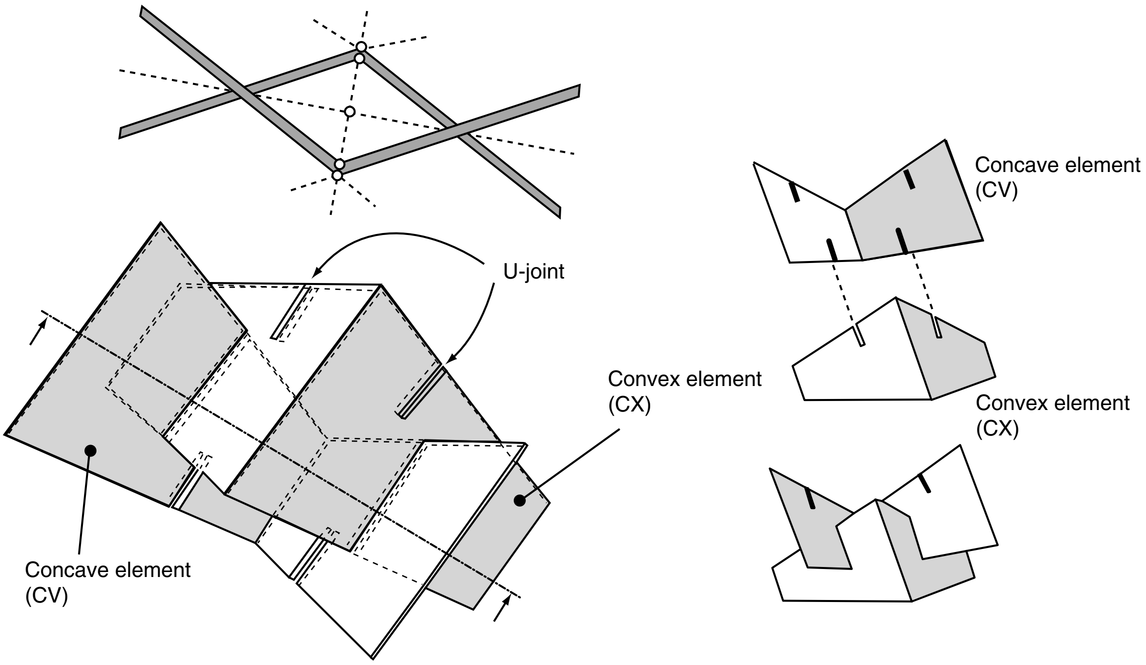


Figure 5. Base module.

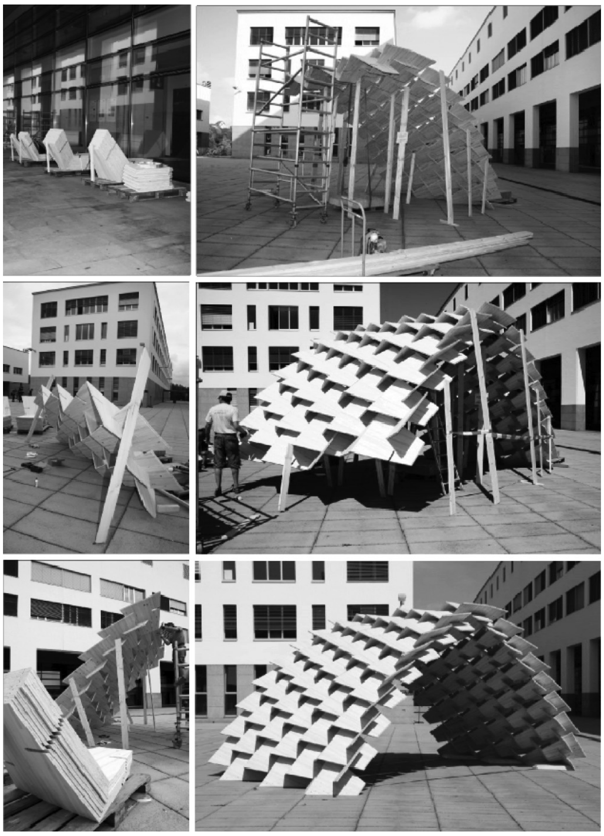


Figure 7. Prototype realization at EPFL © Alain Herzog.

2.3. Prototype realization

A prototype of this structure has been realized at EPFL in order to test the structural feasibility of the concept as well as to investigate the architectural ambiance. A selection of photos illustrating the project is shown in fig 7. The project included development of relative NC codes for 5-axis machining, as well as designing constructive details and fabrication of the structure for exposition. All V-form folded modules have been manufactured from 21 mm thick three-layer cross-laminated panels and cut by means of CNC machines at EPFL.

3. STRUCTURAL ANALYSIS

Here the objective is to understand the structural system considering the geometric non-linearity by the aid of appropriate numerical models and to propose eventual improvements. The diagnostics about the structural system is consisted of the linear static analysis under the self-weight and the modal dynamic analysis, in order to examine the rigidity. Based on these observations, improvements are proposed in §4.

3.1. Local boundary condition analysis of the Base module connections

In each Base module two types of connection are distinguished: TYPE I of connections refer to the slide connection between *CX* and *CV*, a geometrical lock between panels. Instead the connection between mirrored panels in *CX* or *CV* module along their joint is noted as TYPE II. In fact this type is a more mechanical connection and depending on which type of fastener used, the modeling considerations will be different (See Fig 9).

In order to describe the TYPE I of connections, for each slide, a local axis of coordinates is introduced: (See Fig 8) let 1 be the direction along the slide, 3 be the normal direction to *CX* panel and 2 defined by means of right hand rule. Therefore, supposing contact boundary condition, the degrees of freedom, 1 in compression, 2, 3, 11, 22, 33 are constrained according to the geometric configuration of panels which blocks these relative movement and rotations. The only unconstrained degree of freedom is 1 in tension. This means that panels can be slipped out.

For TYPE II of connections, let 1' be the direction along the connection between mirrored panels, 3' the vertical direction and 2' be defined by right hand rule. (See Fig 8) In such a configuration, direction 2' would measure the relative move away of mirrored panels. The constraint scheme in TYPE II depends closely to construction detail. In a preliminary modeling we suppose that panels are tied together and thus degrees of freedom 1', 2', 3', 1'1', 2'2', 3'3' are constrained.

3.2. Modeling hypothesis and local boundary condition analysis

According to the adapted modeling approach, the thickness of panels enters in the reality of modeling, permitting to model the slide connection as it is defined from the geometric configuration. For each slide connection, five normal contacts are defined between two pair of surfaces as master and slave

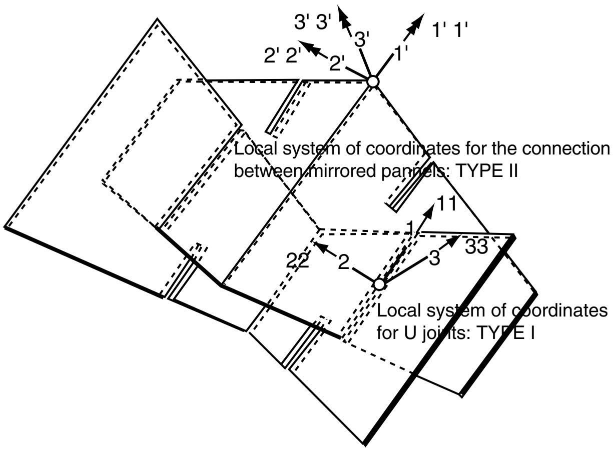
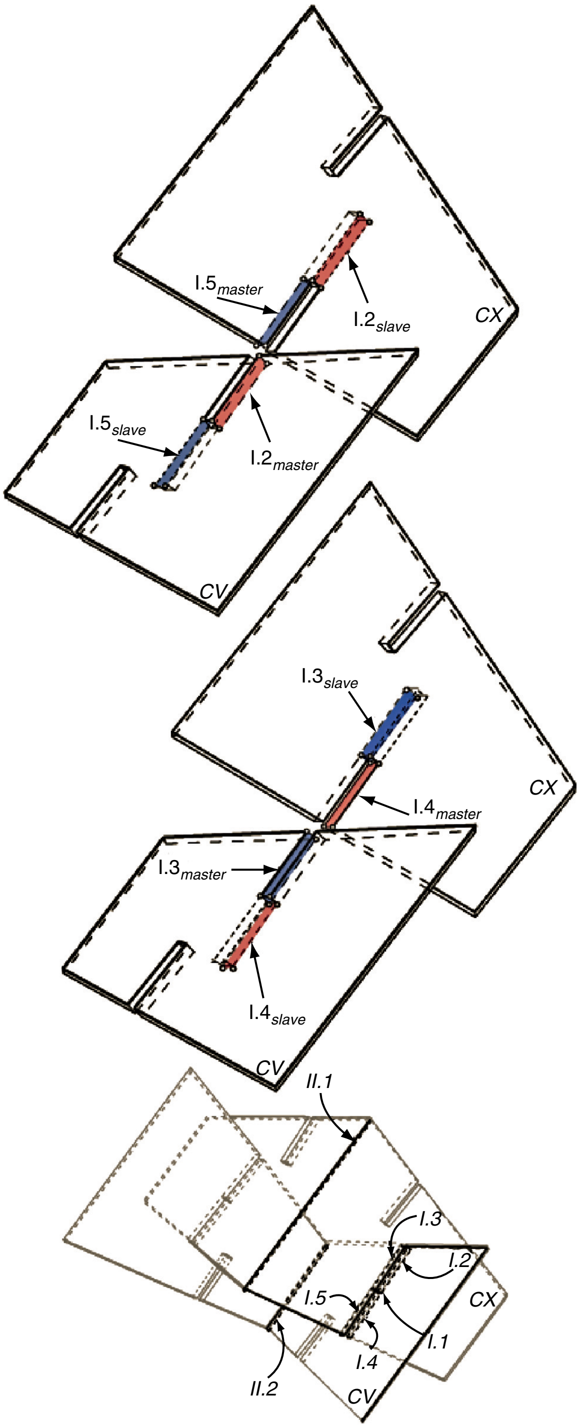


Figure 8. Local system of coordinates for connections between panels on a Base module.



TYPE I

I.1 Axial contact between coincident faces of U-joints
I.2 Contact between top face of CV-U-joint with upper face of CX
I.3 Contact between bottom face of CV-U-joint with lower face of CX
I.4 Contact between bottom face of CX-U-joint with lower face of CV
I.5 Contact between top face of CX-U-joint with upper face of CV

TYPE II

II.1 Joint along two mirrored panels of CX
II.2 Joint along two mirrored panels of CV

Figure 9. Base module connection typology.

surface. Contact property is considered to be frictionless. The contact boundary condition has to be satisfied along slide joints underlying finite deformations. A 10-node modified quadratic tetrahedron element is chosen to mesh the continuum model, referred to as C310DM ABAQUS® solid element.

The timber is considered to be an elastic homogeneous material throughout the whole thickness. The Young Modulus, $E = 8000\ Mpa$, Poisson's ratio, $\upsilon = 0.3$ and material density, $\rho = 500\ \text{kg/m}^3$ are concluded from documentation disposed by the industrial provider of cross-laminated panels [5].

3.3. Structural analysis of a single arch

Results for the global deformation field and the Von mises stress driven from a static non-linear analysis of a single arch under its self weight load case are shown in fig 10. It can be seen that the geometric configuration of the slided together modules leads to a concentrated distribution of stress at the location of slide cuts. This mainly happens because of bending behavior of the structure. Moreover, a modal dynamic analysis for the isolated modular arch has been realized to have a first estimation of structural rigidity in lateral and transversal loading conditions by comparing natural frequency values: the first global mode is a lateral one and it has a relatively small natural frequency of 0.59 Hz, comparing to practical guidelines, which advices a minimum natural frequency ranging between 1 to 4 Hz [18].

4. PROPOSITIONS FOR STRUCTURAL SYSTEM IMPROVEMENT

Based on our findings in §3, we proceed on this section with two goals: first, to obtain a more uniformed stress distribution in panels and to reduce the stress concentration in U-joints; Second, to increase structural rigidity, measured by means of natural frequency of the first global mode.

4.1. Toward a truss system

While having already a geometric superposition concept, one immediate remark would be to change the current beam-like system with a more truss-like one. Two main directions are tracked, as follows.

- **Addition of intermediate elements:** this could be realized by help of additional intermediate elements which are inserted properly at mid-plan of the arch to connect consecutive *CX-CX* and *CV-CV* modules between each other.
- **Opening the U-joint:** in the initial geometric configuration, each module's stability is provided by the locking effect of panels across the U-joints. In a truss system with additional

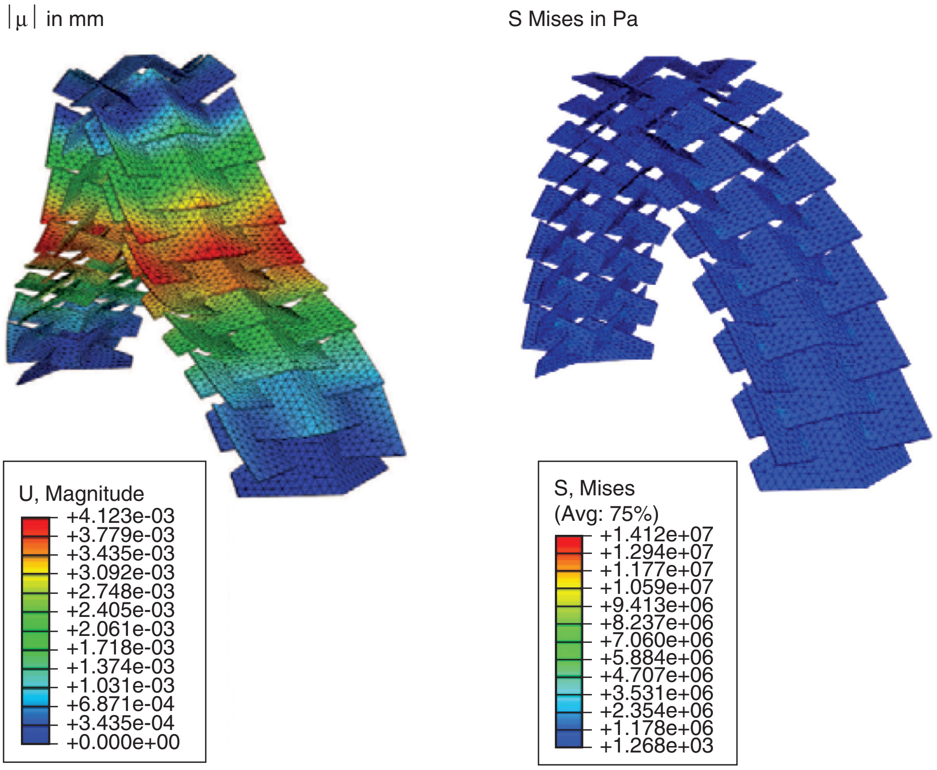


Figure 10. Global deformation field and von Mises stress in single isolated arc model..

intermediate panels, fixed between them and fixed to the slipped modules, it would be possible to open the angle of U-joints. This will help to reduce the concentrated stresses (fig 11a). Applying these two main modifications on the original structure, the maximum von Mises stress under the self weight load case is reduced from 14.1 Mpa for original configuration, to 1.66 Mpa, keeping the same order of magnitude of maximum total deformation. Furthermore, the stress has been led to intermediate elements rather than panels and consequently, the concentrated pattern of stress on panels has been resolved. The natural frequency for the first global mode of the structure is increased up to 0.98 Hz.

4.2. Increasing panel inter-locking effect

Let's consider a CX module, picked deliberately from the arch. The neighborhood of this module is as in (2) (fig 12a.). According to geometric principal exposed in fig 12, CX_1 is connected to CV_1 and CV_2 across two U-joints for each. The idea is to increase the length of these current U-joints to make CX_1 meet CV_{-1} (fig 12b). The whole two intersection cubes are then removed from CV_{-1} and it ends up with two extra U-joints on external part of the panels. Consequently, CV_1, connected currently with CX_0 and CX_1, in his place, intersects with CX_{-1}. Removing the two new intersection cubes from CV_{-1} provides two more extra U-joints connection, placed this time, in internal part of CV module.

If we resume, the general idea is to keep the cut-pattern for CX module unchanged, although for CV module there would be 4 more U-form cuts: two internal and two external (fig 12c).

To achieve this objective, the original geometrical concept has been implemented within a parametric computer-aided design interface. The important parameters determining the geometry of each typology of modules in the original design have been identified. Then, the geometrical configuration for the montage of the Base modules is set respectfully to the height and total span of the original structure.

$$CV_{-2} - CX_{-1} - CV_{-1} - CX_0 - CV_1 - CX_1 - CV_2 \qquad (2)$$

Increasing the slide length, while keeping constant total span and height of the structure, will increase the number of modules. The original design is consisted of 33 slided modules. In fact, by increasing length of joint by 103 mm to create two more U-joints between panels, 53 modules of nearly the same size is needed to respect the same height and span as the original

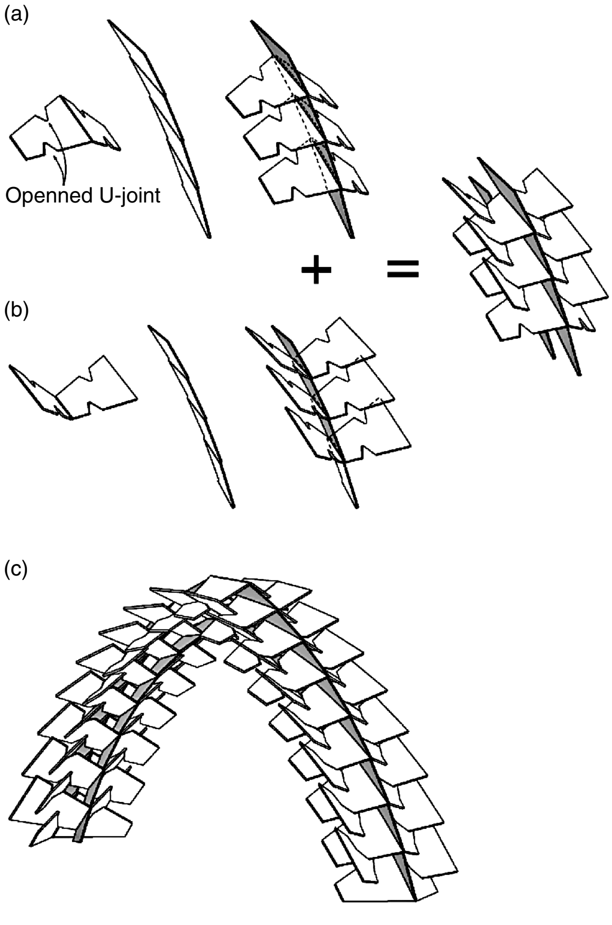
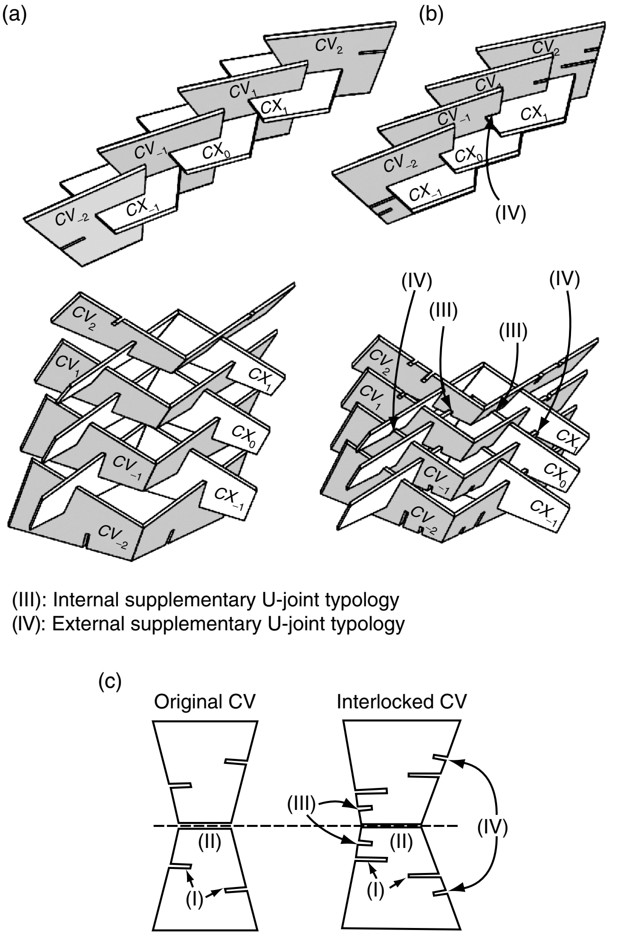


Figure 11. Toward a truss behaviour (a) CX module modification and intermediate panels (b) CV module modification and intermediate panels (c) Isolated arch reinforced with intermediate panels on top and bottom fibbers.



(III): Internal supplementary U-joint typology
(IV): External supplementary U-joint typology

Figure 12. Increasing panel interlocking effect: two geometric configurations (a) Original (b) Interlocked (c) Comparing geometric modification brought to CV module as well as its connection typology.

design. Therefore, it follows that the inter-locked version will be 53/33 ~ 1.7 times heavier than the original one. By multiplying number of slide joints and distributing their position across the all length of panel, we expect a more uniform load transfer between modules. Indeed, the results for von Mises stress, came from an elastic nonlinear analysis, confirms this idea. The maximum von Mises stress for self-weight load case, reduces to 1.15 Mpa for maximum total deformation of 1.3 mm, which is still acceptable. This is true even though the interlocked configuration is 1.7 times heavier than the original one. The main gain is on the structural rigidity, where the minimum natural frequency, calculated from a modal dynamic analysis is estimated to be 5.99 Hz. Using the values of natural frequency (f) and total mass (m) for the original configuration (marked with subscript 0 in (3)) and the improved inter-locked version (marked with subscript 2), one may compare the relative equivalent structural stiffness (k) between these configurations as represented in (3), concluding that the new slide locks make the original structure 165 times stiffer.

$$\frac{k_2}{k_0} = \frac{m_2}{m_0}\left(\frac{f_2}{f_0}\right)^2 \cong 165 \qquad (3)$$

5. CONCLUSION AND FURTHER WORK

The structure presented in this article shows an example for the design practice when the final form is driven by the connection technology, where complex modular global forms are proposed by means of mutually supported simple folded panels. The proposed slide connection scheme inspires a new family of reciprocal frames, where instead of linear members (beam or bar), planar (folded or elastically deformed) members are mutually supported. The connection between mutually supported members in this concept is integrated within the geometry of members, unlike the traditional reciprocal frame system where the connector members are employed.

As a further work, the same concept is employed where instead of folded panels curved thin deformed panels are used. The mutual supportiveness scheme of panels rests unchanged but the slide connection geometry needs to be determined based on a form-generation relaxation analysis. Moreover on such prototype, instead of single folded modules, continuous thin flat panels will be used where connections are milled and fabrication is realized by elastically deforming panels and sliding. As a consequence, transversal extension of the structure will be straightforward.

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