Fluid Avalanches in the Laboratory

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Swiss Group of Rheology
Annual Meeting

October 24, 2008 - ETHZ

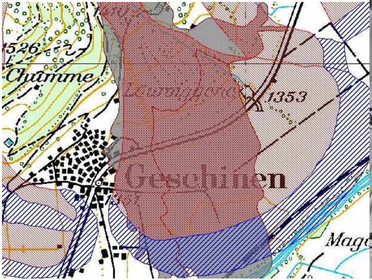
Contents

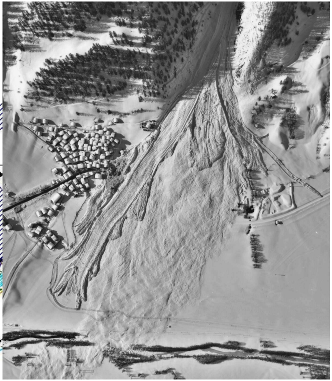
Societal motivation : property damage

Airolo (TI, 11 Feb. 1951) ; Montroc (Chamonix, France, 9 Feb. 1999).

Avalanches cause substantial property damage on average every 10 years.

Hazard mapping





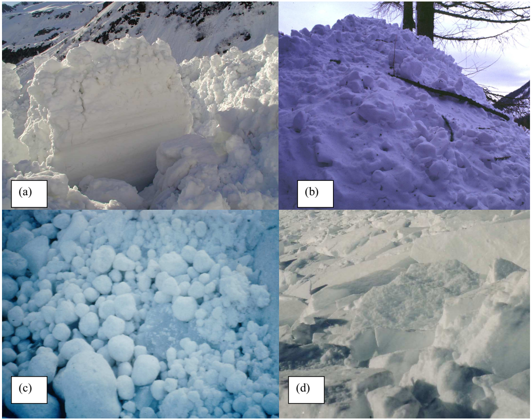
Geschinen (VS) : catastrophic avalanche of 23 Feb. 1999 and avalanche maps (zoning)

A wide range of volumes

Natural flows exhibit a wide range of flow and material features.
Avalanche volume : from a few cubic meters to 10^6 m^3 (or more)

Rheology

Snow : wide range of physical characteristics and rheological properties



Scientific objectives

Much of our work is concerned with a better understanding of gravity-driven, time-dependent flows over irregular topographies and involving complex fluids. We work on the laboratory scale by carrying out experiments with different materials, which are assumed to account for some essential features of natural materials, while being simpler to characterize and handle.
Our goals : with our experiments on model fluids, we would like to address and answer the following point

- Can we derive a compact set of governing equations that describe the behavior of an avalanching mass of material down a slope ?
- For complex flow geometries, is flow dynamics controlled by rheological properties or flow self-organization (levee, front, etc.) ? Do other processes (mass balance, segregation, boundary conditions) play an essential part ?
- Is there any link between the physical/rheological properties for a bulk material at rest (i.e., quasi-static to low-deformation domain) and those exhibited by the same material in a flow ?

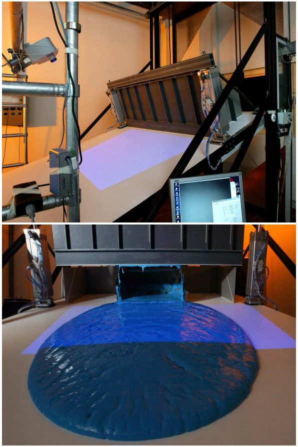
Laboratory versus field investigation

Our philosophy is well summarized by Iverson

“The traditional view in geosciences is that the best test of a model is provided by data collected in the field, where processes operate at full complexity, unfettered by artificial constraints. (...) If geomorphology is to make similarly rapid advances, a new paradigm may be required : mechanistic models of geomorphic processes should be tested principally with data collected during controlled, manipulative experiments, not with field data collected under uncontrolled conditions.”

Iverson, R.M., How should mathematical models of geomorphic processus to be judged ?, in *Prediction in Geomorphology*, edited by P.R. Wilcock, and R.M. Iverson, pp. 83–94, American Geophysical Union, Washington, D.C., 2003.

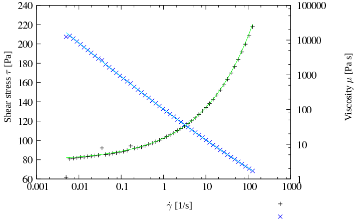
Avalanches in the laboratory : the dam-break problem



Experiments : Carbopol (polymeric gel) colored in blue.

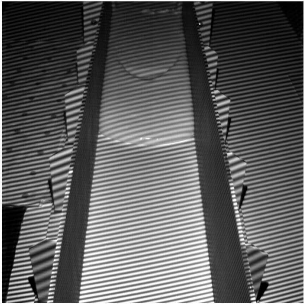
Small-scale experiments : balance between pressure gradient, inertia, and viscous dissipation

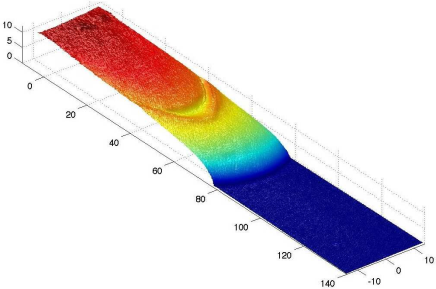
- rheological behavior : imposed (and controlled rheometrically).



- initial and boundary conditions : known and controlled.

Measurement system : 3D surface reconstruction

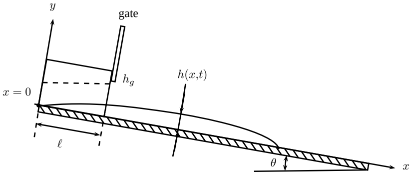




Steve Cochard's thesis, Sébastien Wiederseiner, Martin Rentschler, & Nicolas Andreini (EPFL/ENAC/LHE).

Governing equations : a shallow world

Most models used for computing the behavior of an avalanching mass are based on the shallow-flow approximation : $\epsilon = H/L \ll 1$.

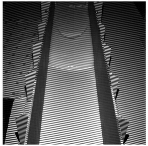


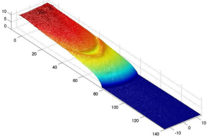
There are two approaches

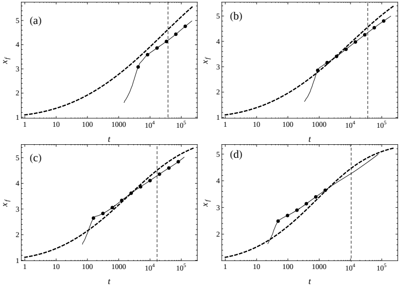
- Flow-depth averaged equations : historical approach used by Saint-Venant (floods), Savage & Hutter (granular flows), Iverson & Denlinger, Mangeney & Bouchut and many others...
- Lubrication approximation : pioneering work conducted by Reynolds and subsequent authors (boundary layer theory), renewed interest with the work done by Mei & Liu, Huppert, Balmforth & Craster.

Application to viscoplastic avalanches

The 3D surface measurement technique can be applied to viscoplastic materials.







Variation in the front position with time for $\theta = 24^\circ$. Experiments done with Carbopol at various concentrations.

Dashed curves : theoretical prediction given by a zero-order nonlinear convection equation (modeling the behavior of an avalanching mass of Herschel-Bulkley fluid).

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Geophysical flows

Complex fluids

- Particles
  - Material
  - Shape
  - Size distribution
  - Roughness
- Interstitial fluids
  - Viscosity

$\Rightarrow$

How do we measure the rheological properties ?

- Yield stress
- Shear-thinning, Shear-thickening
- Thixotropy, rheopexy

Geophysical flows

Complex fluids

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How do we measure the rheological properties ?

- Yield stress
- Shear-thinning, Shear-thickening
- Thixotropy, rheopexy

Consequences for the rheologist

1. **Wide gap** (because of the size distribution)

2. $\begin{pmatrix} T \\ \omega \end{pmatrix} \dashrightarrow \begin{pmatrix} \tau \\ \dot\gamma \end{pmatrix}$

T : Total Torque
ω : Angular velocity
τ : shear stress
$\dot\gamma$: shear rate

$\Rightarrow$

Solve the Couette inverse problem

$$\tau(r) = \frac{T}{2\pi r^2 h}$$

$$\omega = \int_{R_{in}}^{R_{out}} \frac{\dot\gamma(r)}{r} dr$$

r : Radius
h : Height of fluid
$R_{in/out}$: Radius of the inner/outer cylinder

Associated Couette inverse problem

Solving methods :

- Infinite series approach
  $\dot{\gamma}(\tau) = \frac{\omega}{\ln s}\left[1 + \ln s \frac{d \ln \omega}{d \ln \tau} + \frac{(\ln s)^2}{3\omega}\frac{d^2\omega}{d(\ln \tau)^2} + \ldots\right]$
- Least square approach $\quad \min ||\omega - K\dot{\gamma}||$
- Projection approach $\quad < K\dot{\gamma}, u_i > = < \omega, u_i >$
- Adjoint operator approach $\quad \dot{\gamma} = \sum_{i \in J} < K\dot{\gamma}, u_i > \Psi_i$

$$K^{\star} u_i = \Psi_i$$

Associated Couette inverse problem

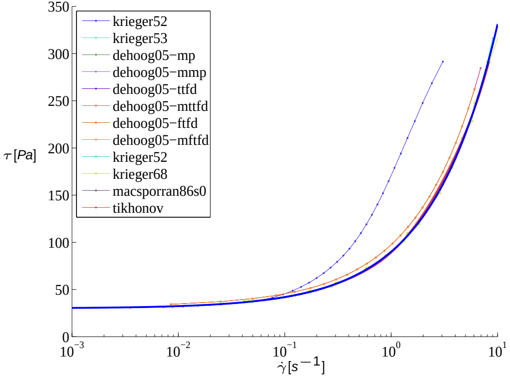
Solving methods :

- Mooney (1931)
- Krieger & Maron (1952)
- Krieger & Elrod (1953)
- Krieger (1968)
- Yang & Krieger (1978)
- Mac Sporran (1986)(1989)
- Nguyen (1992)
- Yeow (2000)
- Ancey (2005)
- De Hoog & Anderssen (2005)(2006)

Associated Couette inverse problem

Example : an artificial Herschel-Bulkley fluid $\tau = \tau_y + K\dot{\gamma}^n$

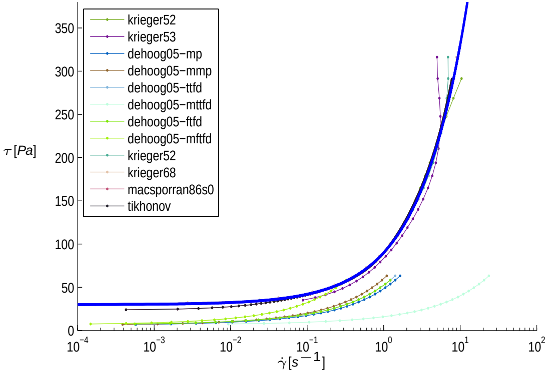
$$s = \frac{R_{in}}{R_{out}} = 0.9$$



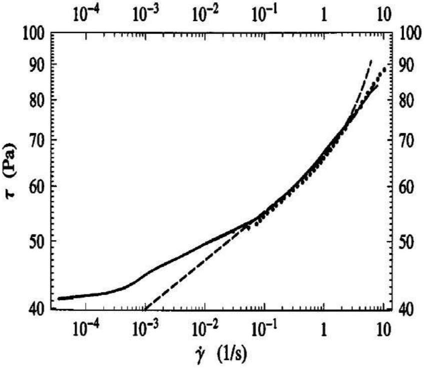
Associated Couette inverse problem

The same fluid with a wide-gap geometry

$$s = \frac{R_{in}}{R_{out}} = 0.2$$

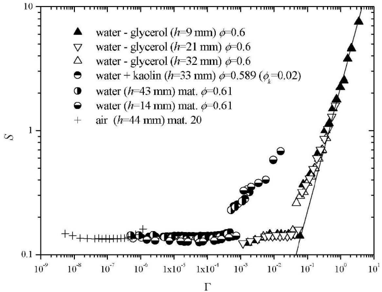


Example : a polymeric gel



Ancey, *J.Rheology* **49** (2005) 441-460

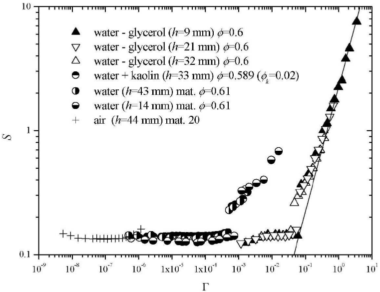
Example : a particle suspensions



Ancey, *J. Rheol.* **45** (2001)1421-1439

S : adimensionalized shear stress
Γ : adimensionalized angular velocity

Example : a particle suspensions



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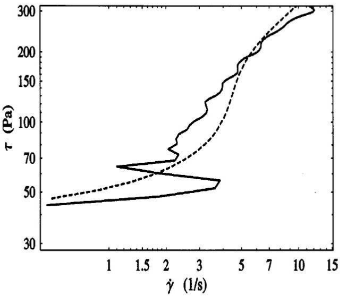


Ancey, *J.Rheology* **49** (2005) 441-460

S : adimensionalized shear stress
Γ : adimensionalized angular velocity

Rheology and rheophysics

- Shear localization ?
- Particle segregation ?
- Particle migration ?
- Ordering ?
- Particle roughness ?
- Particle Shape ?
- Slipping ?

Do we measure material's physical properties...

... or disturbing effects ?

Classical and optical rheometry

Continuum mechanics approach

$\Downarrow$

Classical rheometry

$\Downarrow$

T and ω

$\Downarrow$

Solve the Couette inverse problem

$\Downarrow$

τ and $\dot{\gamma}$

Rheophysical approach

$\Downarrow$

Clear suspensions

$\Downarrow$

Particle motion (FPIV / FPTV)

$\Downarrow$

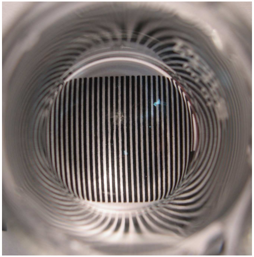
Differentiate the velocity profile

$\Downarrow$

τ and $\dot{\gamma}$

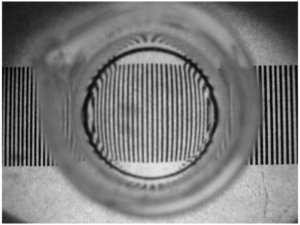
Where do the properties come from ?

Studied flows



Optical methods

$\Longrightarrow$



concentrated particle suspensions (25mm thickness)

The simplest complex fluid

- Iso-index $\Rightarrow$ Clear suspension
- Iso-density $\Rightarrow$ No gravity effects
- Molecular tagging of the particles
  $\Rightarrow$ the laser excites fluorescence

Particles

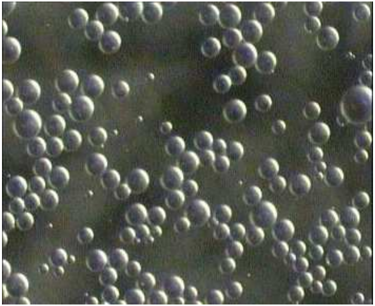
- Shape : spherical
- Granulometry

Fluid

- Three fluids mixture
- Newtonian
- Viscosity : variable

Non colloidal and highly concentrated particle suspensions

- Spherical PMMA particles with a diameter of 50 to 350 μm
- Mixture of three newtonian fluids (Lyon & Leal 1997)



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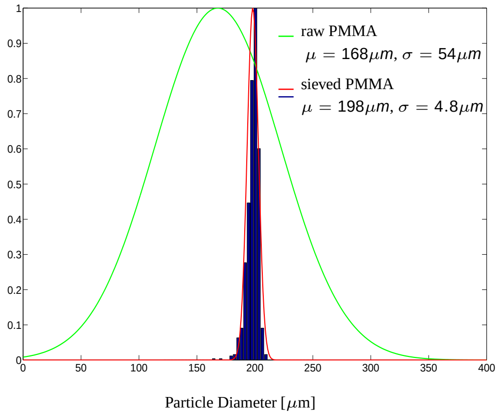
Wet sieving



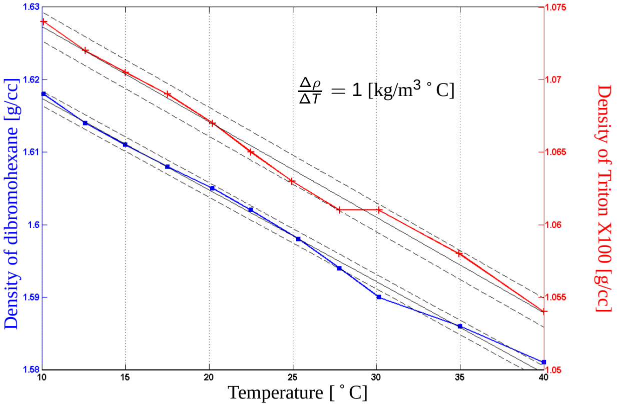
raw PMMA
$\mu = 168\mu m, \sigma = 54\mu m$
sieved PMMA
$\mu = 198\mu m, \sigma = 4.8\mu m$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
50
100
150
200
250
300
350
400
Particle Diameter [μm]

Temperature and wavelength effects

- Temperature effects
- Wavelength effects

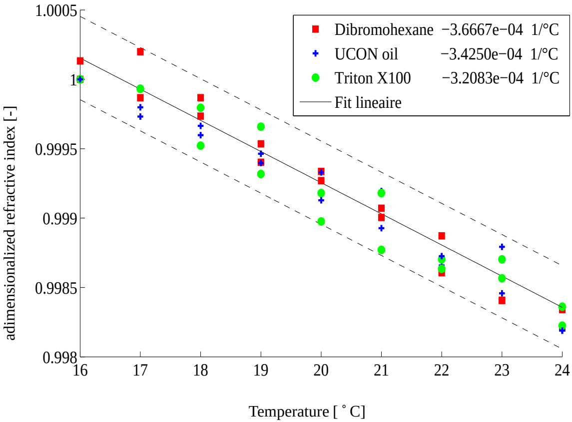
Temperature effects

- Density
- Refractive index

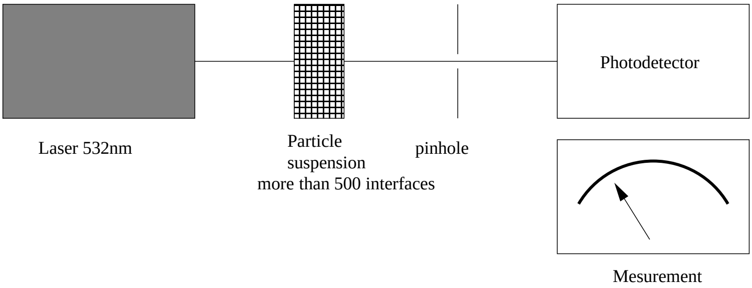


Temperature effects

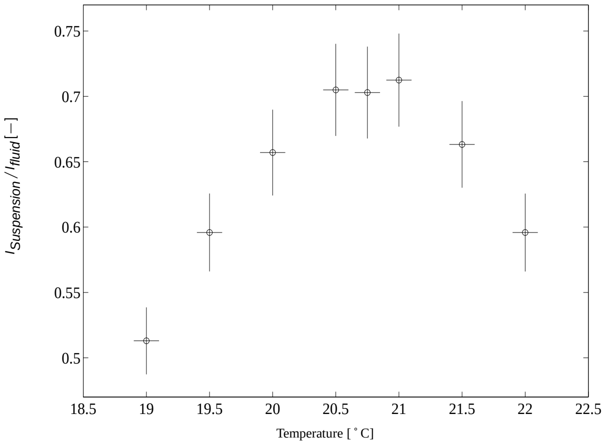
- Density
- Refractive index



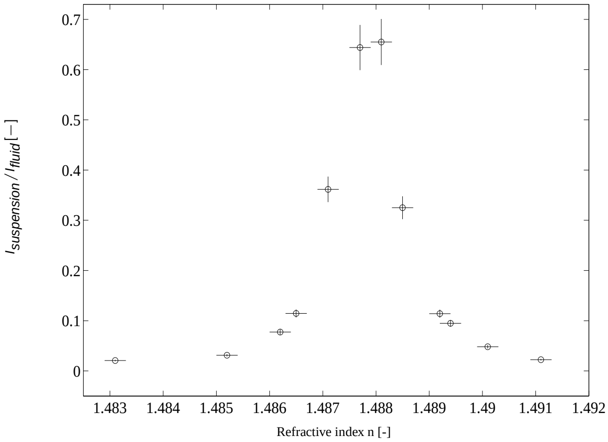
Temperature effects on light transmission



Temperature effects on light transmission



Effects of mismatch in the Refractive index on transmission



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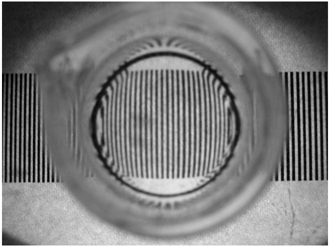
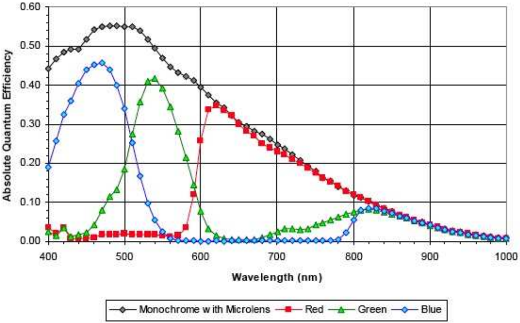
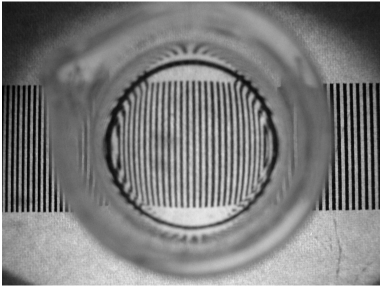
Wavelength effects



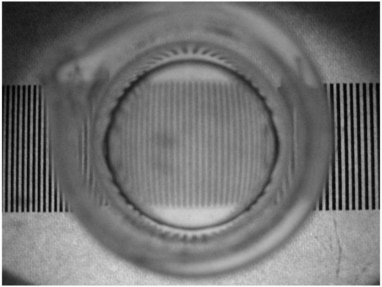


Wavelength effects

RGB picture with a color CCD camera :

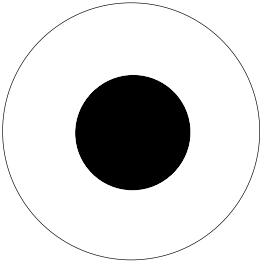


Blue component

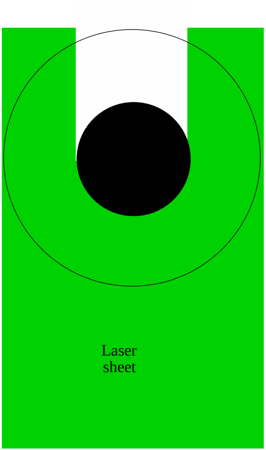


Red component

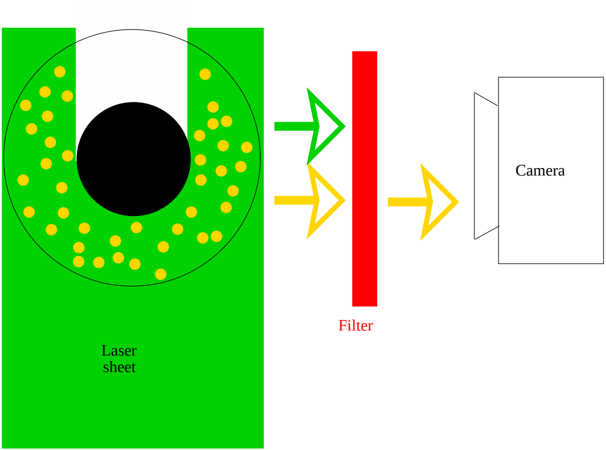
Measurement methods



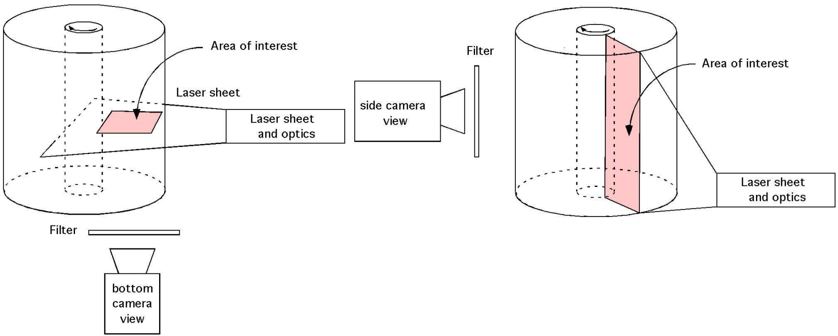
Measurement methods



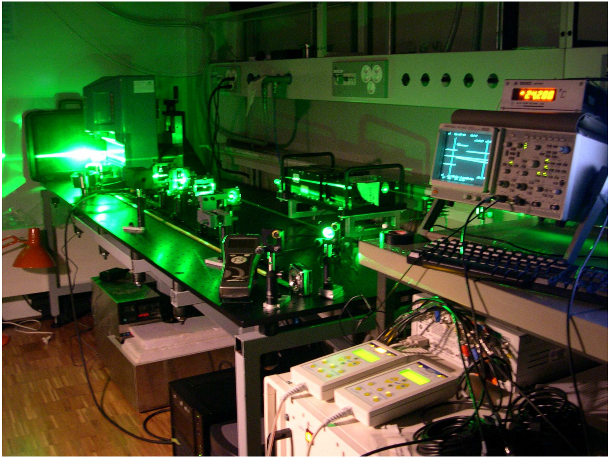
Measurement methods



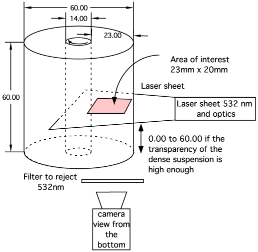
Measurement methods



The setup

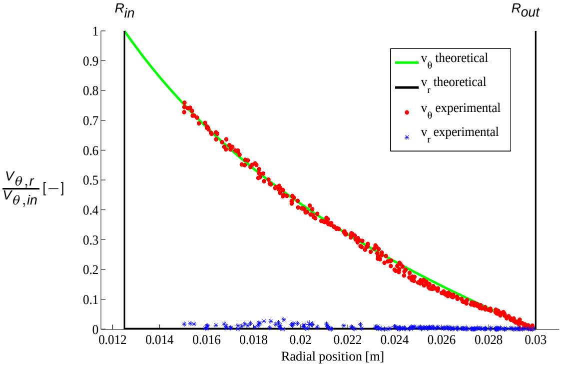


FPIV Images

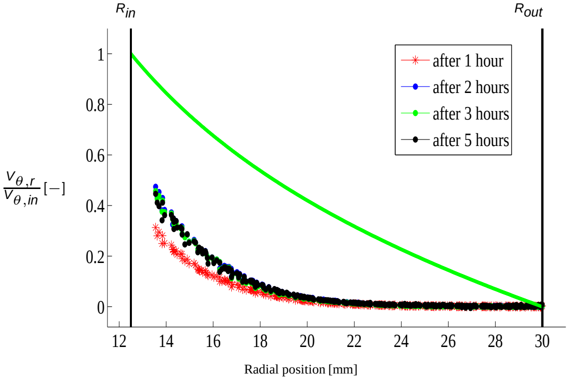

60.00
14.00
23.00
60.00
Area of interest
23mm x 20mm
Laser sheet
Laser sheet 532 nm
and optics
0.00 to 60.00 if the
transparency of the
dense suspension is
high enough
Filter to reject
532nm
camera
view from
the
bottom

Validation measurements

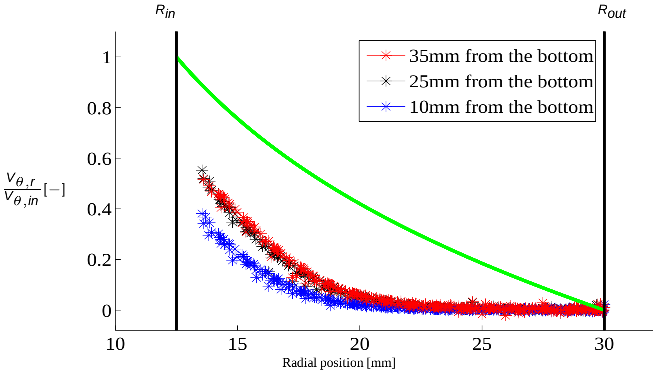
$$V_\theta(r) = \frac{A}{r} + Br \text{ with } A = \frac{R_{in}^2 R_{out}^2 \omega}{R_{out}^2 - R_{in}^2}, \quad B = \frac{R_{in}^2 \omega}{R_{in}^2 - R_{out}^2}$$



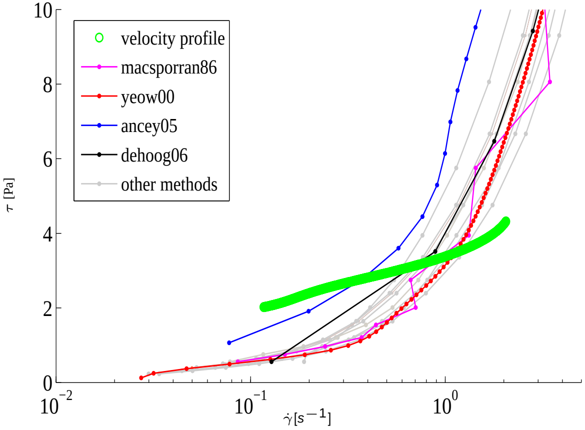
Time evolution of the suspension



Bottom end effects



Flow curve comparison



Future work

We want to use the same techniques to carry out experiments on the dam-break problem (sudden release of a finite volume of fluid down a plane) and measure the cross-stream velocity profile inside the bulk within the head.

Summary

- We investigate the rheological properties of concentrated particle suspensions.
- For that purpose, we have built a new facility to study the local (velocity and concentration profiles) and the bulk behavior of the particle suspension in a Couette rheometer
- The Couette inverse problem associated with a wide-gap concentric cylinder geometry has been analysed and different solving methods have been compared to the benchmark data obtained by differentiation of the velocity profile
- Using the same techniques, we are going to carry out dam-break experiments of clear and concentrated particle suspensions

- Iso-index $\Rightarrow$ transparency
- Iso-density $\Rightarrow$ No gravitation effects
- not toxic

Particles

- Sphericity
- Good optical properties
- Granulometry
- Fluorecent molecular tagging

Fluide

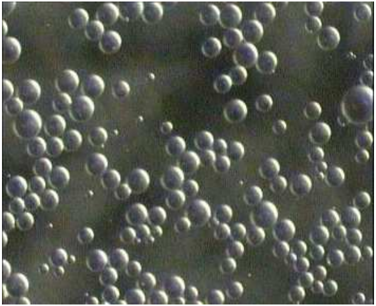
- No evaporation
- Wet the PMMA
- Should not disolve PMMA
- Low absorption
- No excitation
- Variable viscosity

Fluides

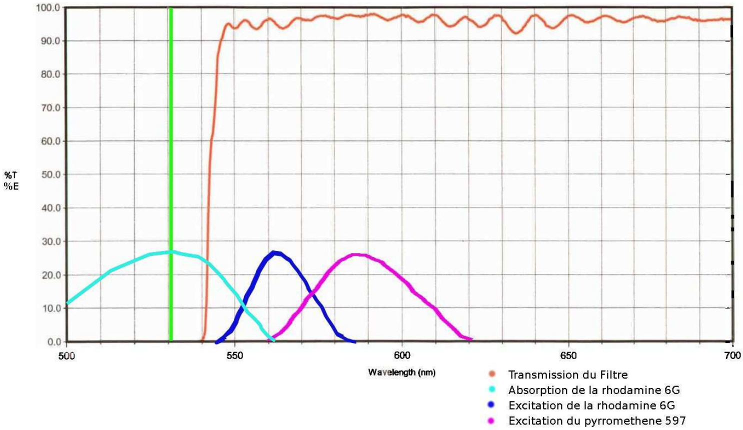
- **Lyon (1997)**
- Dibromohexane
- Triton X 100
- Huile UCON 75H

Transparent concentrated noncolloidal suspensions

- Spherical particles : 200 to 600 μm
- Iso-index and iso-density fluid mixture



Why Rhodamine 6G ?



How much rhodamine 6G ?

High concentration

$\Downarrow$

More fluorescence

$\Downarrow$

COMPROMIS

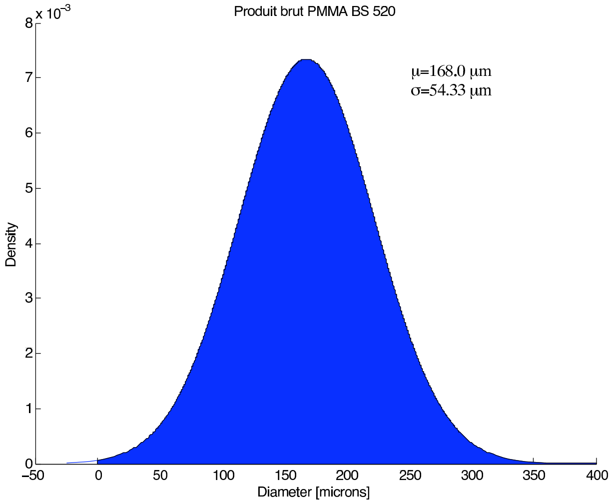
$\Uparrow$

Lower effect on the refractive index

$\Uparrow$

Low concentration

Produit brut



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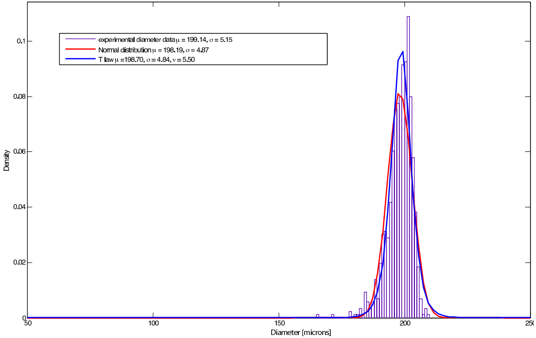
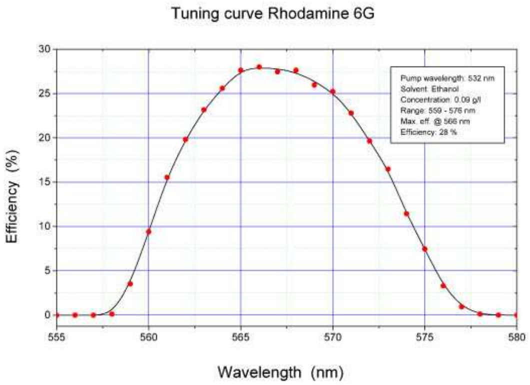
Produit tamisage par voie humide dans de l’ ?thanol



REPRODUCTIBILITE

Choix de la Rhodamine 6G

- Excellent efficacit ?
- suffisamment faible ”Stokes shift”



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Laboratoire Hydraulique Environnementale

Suspension properties

- **Iso-index $\Rightarrow$ transparency**
- **Iso-density $\Rightarrow$ No gravitation effects**
- **Non toxic**

Particules

- Sphericity
- Good optical properties
- Granulometry
- Fluorecent molecular tagging

Fluide

- No evaporation
- Wet the PMMA
- Should not disolve PMMA
- Low absorption
- No excitation
- Variable viscosity

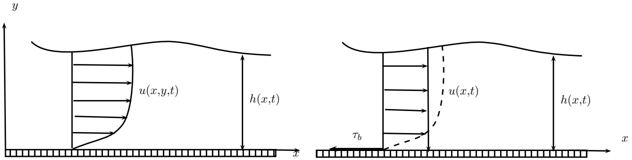
Shallow-flow equations

A versatile set of equations

$$\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} = E - D,$$

$$\frac{\partial h\bar{u}}{\partial t} + \beta\frac{\partial h\bar{u}^2}{\partial x} = \rho g h - kgh\frac{\partial h}{\partial x} - \frac{\tau_b}{\rho},$$

with β Boussinesq coefficient (usually set to unity), k a pressure coefficient, and τ_b the bottom shear stress, E and D entrainment and deposition rates.



Information is averaged when deriving the governing equations, which makes it difficult to properly define the coefficients that come up in the final equations.

Strength and weakness

The shallow-water equations offer a reasonably accurate physical framework for describing a host of natural phenomena. The governing equations are now well “tamed” by numerical methods. Numerical schemes for 1D and 2D models are reasonably fast and make it possible to simulate complex flows (e.g., tsunamis) on large scales.

However, when dealing with geophysical flows on steep slopes, we are faced with many issues :

- tracking the front position ;
- computing the internal dissipation and account for it through τ_p ;
- taking additional terms induced by irregular topography into account ;
- evaluating mass balance and its effect on the bulk dynamics ;
- estimating the change in the bulk composition (e.g., segregation) and local rheology.

Lubrication approximation

Starting with the Cauchy equations (mass and momentum balance equations), we scale the variables

$$\tilde{u} = u/U_*,\ \tilde{x} = x/L_*,\ \tilde{y} = y/(\epsilon L_*),\ \tilde{p} = p/P_*,\ \tilde{p} = p/P_*, \ldots$$

with $\epsilon = H_*/L_*$ and make a power ϵ-expansion of the scaled variables : $\tilde{u} = \tilde{u}_0 + \epsilon\tilde{u}_1 + \ldots$. Collecting together the terms associating the same power of ϵ, we end up with a hierarchy of equations. For instance, we have

$$\epsilon \mathrm{Re} \frac{\mathrm{d}u}{\mathrm{d}t} = 1 - \epsilon \cot\theta \frac{\partial p}{\partial x} + \epsilon^{n+1} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y}, \quad (1)$$

$$\epsilon^2 \mathrm{Re} \frac{\mathrm{d}v}{\mathrm{d}t} = -\cot\theta \left(1 + \frac{\partial p}{\partial y}\right) + \epsilon \frac{\partial \sigma_{xy}}{\partial x} + \epsilon^n \frac{\partial \sigma_{yy}}{\partial y}, \quad (2)$$

Lubrication approximation (continued)

To order ϵ^0, we have to solve

$$0 = 1 + \frac{\partial \sigma_{0,xy}}{\partial y}, \quad (3)$$

$$0 = -1 - \frac{\partial p_0}{\partial y}, \quad (4)$$

a much simpler set of equations than the full governing equations ! In the limit of $\mathrm{Re} \to 0$ and to order ϵ, we obtain

$$0 = -\cot\theta \frac{\partial p_0}{\partial x} + \frac{\partial \sigma_{1,xy}}{\partial y}, \quad (5)$$

$$0 = -\cot\theta \frac{\partial p_1}{\partial y} + \frac{\partial \sigma_{0,xy}}{\partial x}, \quad (6)$$

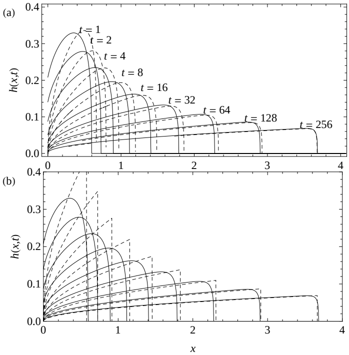
Application to Newtonian avalanches

To leading order, the governing equation for h writes

$$\frac{\partial h}{\partial t} + \underbrace{\frac{\partial h^3}{\partial x}}_{\text{convection}} = \underbrace{\frac{\partial}{\partial x}\left(h^3 \frac{\partial h}{\partial x}\right)}_{\text{diffusion}}. \tag{7}$$

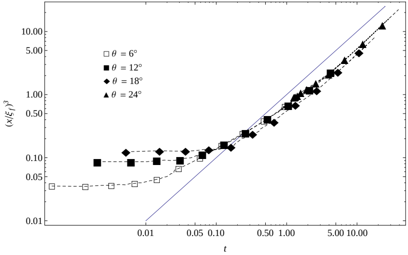
Analytical solutions can be worked out in terms of similarity solutions at late and early times : $h(x,t) = t^{-n}H(\xi,t)$ $\xi = x/t^n$, $n = 1/3$ (late time solution) or $n = 1/5$ (early time solution). Depending on the initial conditions, convergence towards the similarity solution can be slow.

Application to Newtonian avalanches (continued)



Flow-depth profiles provided by numerical solutions (solid line) of the nonlinear diffusion equation for $\theta = 6^\circ$ at dimensionless times $t = 1$, 2, 4, 8, 16, 32, 64, 128, and 256. In subplot (a), we plotted the analytical approximation obtained by composing the inner and outer similarity solutions (dashed line). In subplot (b), the analytical solution corresponding to pure convection is reported.

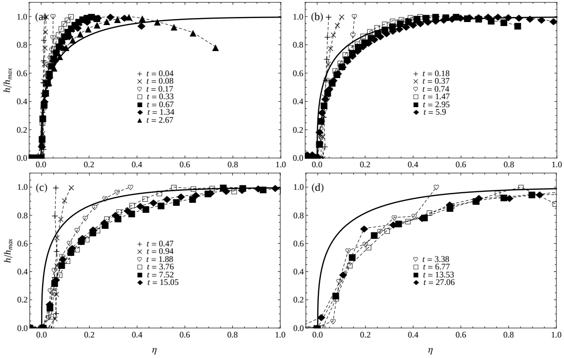
Application to Newtonian avalanches (continued)



Normalized front position $(x_f/\xi_f)^3$ as a function of time in a log-log representation : the experimental curves (dashed line marked with symbols) related to $\theta = 6^\circ$, 12°, 18°, and 24° slopes are indicated. The solid line represents the theoretical curve $(x/\xi_f)^3 = t$ corresponding to the outer similarity solution.

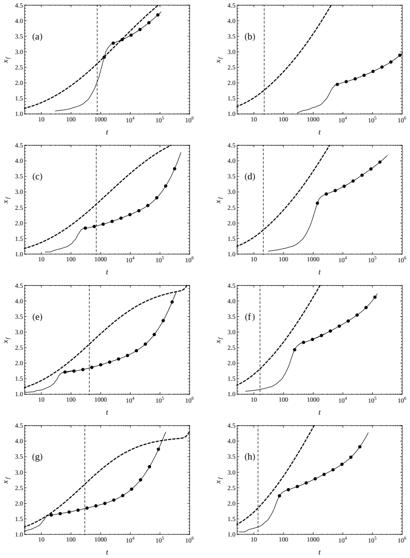
Fluid : glycerol $\mu \sim 345$ Pa.s (molten toffee)

Application to Newtonian avalanches (continued)



Flow-depth profiles $h(\eta, t)$ normalized by the maximum flow depth h_{max} for $\theta = 6^\circ$ (a), $\theta = 12^\circ$ (b), $\theta = 18^\circ$ (c), and $\theta = 24^\circ$ (d) at different dimensionless times. We also plotted the composite solutions (thick line).

Application to viscoplastic avalanches (continued)



Variation in the front position with time for $\theta = 12^\circ$. Experiments done with Carbopol at various concentrations. Dashed curves : theoretical prediction given by a zero-order nonlinear convection equation (modeling the behavior of an avalanching mass of Herschel-Bulkley fluid).

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