Pulse distortion in linear slow light systems: theoretical limits and compensation strategies

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ABSTRACT

We study analytically pulse distortion in linear slow light systems, and provide some useful limits on these devices. Additionally, we also show that the contributions of phase and amplitude broadening can be de-coupled and quantified. It is observed that phase broadening is generally smaller than amplitude broadening in conventional slow light media (lorentzian gain profile) except for very large fractional delays, where it becomes larger. Upon these expressions, we may envisage new strategies to minimize the distortion in the delaying of pulses, depending on the specific application and the required fractional delay. To overcome the residual distortion, we show that nonlinear systems can lead to a re-sharpening of the pulses and a re-generation of the filtered frequencies.

Keywords: slow light, linear systems, nonlinear regeneration.

1 INTRODUCTION

Slow light has important potential implications in modern technologies oriented towards high capacity networks such as all-optical signal processing, optical buffers, optical memories and quantum computing [1]. In recent years, a number of slow light systems have been experimentally realized based on different physical phenomena in optical media, such as electromagnetically-induced transparency, coherent population oscillations, optical parametric amplification and stimulated scattering processes [2]. All these slow light schemes share a common feature making the essence of the slow and fast light generation in optical media: the presence of one or multiple strong spectral resonances to obtain a highly dispersive material. Unfortunately, the sharp change in refractive index is also accompanied with an inevitable dispersion (in amplitude and phase) that mostly manifests as pulse broadening. As a result, the larger the delay in the slow light medium, the more broadening in the output pulse. In order to overcome this delay-distortion trade-off, several linear slow light schemes have been theoretically and experimentally investigated [2-13]. In particular, tailoring the shape of the spectral resonance to optimize the dispersive properties of the material could partially reduce the induced distortion while keeping the fractional delay. However, this approach could not fully eliminate the induced distortion, thus the maximum achievable pulse delay remains limited to a few pulse durations.

This paper provides an analytical study of pulse broadening in a linear slow light medium. The main result of the paper is a procedure that allows quantifying amplitude and phase distortions accumulated in the slow light medium. The paper is structured as follows: in section 2, we review a general representation of linear slow light media using Fourier analysis and Kramers-Kronig relations, and we apply this definition to various systems of interest; in section 3 we show how to quantify the pulse broadening, decoupling the contributions of amplitude and phase distortion, and showing that amplitude distortion is generally much larger than phase distortion; in section 4 we show that the broadening induced by the linear slow light medium on the signal can be completely compensated (without cancelling the delaying effect) by a nonlinear medium inserted after the slow-light medium; finally, we will derive some conclusions of our study, and we provide some perspectives for future developments.

Advances in Slow and Fast Light IV, edited by Selim M. Shahriar, Philip R. Hemmer,
Proc. of SPIE Vol. 7949, 79491B · © 2011 SPIE · CCC code: 0277-786X/11/$18
doi: 10.1117/12.880796

2 REPRESENTATION OF A LINEAR SLOW LIGHT MEDIUM

Mathematically, a linear system can be completely characterized by its impulse response, i.e. by the output of the system when it is fed with a Dirac delta at the input. The response of the medium to an arbitrary input can then be calculated as the convolution of the input waveform with the impulse response. In the Fourier domain, the transform of the impulse response is given by the transfer function $H(\omega)$, and the output of the system ($\tilde{A}(\omega)$) for an arbitrary input ($S_{in}(\omega)$) can be obtained as $\tilde{A}(\omega)=H(\omega)\cdot S_{in}(\omega)$. In particular, optical systems are described by a transfer function of the type $H(\omega)=\exp(g(\omega)+i\Phi(\omega))$, where g and Φ are related by the well-known Kramers-Kronig (KK) relations. KK relations basically impose causality in the response of the system, i.e. for a delta impulse at the input at $t=0$, the response at the output remains nonzero only for $t\geq 0$.

In an ideal scenario, slow light systems should exhibit a flat amplitude response and a linear phase response with large tunable slope, hence $g=g_0$ and $\Phi=\Phi_1\omega$. While this response would be desirable, it also turns out to be impossible except using passive (non-tuneable) delaying elements with long propagation lengths. Tunable delaying elements may introduce significant delays at the expense of working close to or within a resonance. This means that the system operates only in a narrow frequency band (i.e. the system is bandwidth-limited) and that distortion in the signal is always present and should be bounded depending on the actual application requirements. This also implies that broadening is inevitable in linear slow light systems: any system that introduces a low-pass filtering of the signal (a reduction in the signal root-mean-square spectral width), always causes pulse broadening due to the uncertainty principle [14].

Linear slow light systems are tailored to have a symmetric spectral response around the central frequency of the pulse. In a resonance, the center is the region that exhibits larger delay with less distortion. Therefore, g exhibits even symmetry and consequently Φ exhibits odd symmetry. In terms of a series expansion, g and Φ can be then written as:

$$g = g_0 + g_2\omega^2 + g_4\omega^4 + \ldots \tag{1}$$

$$\phi = \phi_1\omega + \phi_3\omega^3 + \phi_5\omega^5 + \ldots \tag{2}$$

Where, for simplicity, the resonance is supposed to be centered at $\omega=0$. In terms of a physical interpretation of the coefficients of the series expansion, g_0 can be interpreted as the amplitude gain/loss of the system in the center of the resonance and Φ_1 is the delay associated to the slow light effect. All the higher-order terms in the expansion are undesired (although mostly inevitable) and bring distortion in the pulse: the higher-order terms in g (g_2, g_4, etc) introduce *amplitude* distortion and the higher-order terms in Φ (Φ_3, Φ_5, etc) introduce *phase* distortion.

With regards to these expressions it is important to have in mind two important concepts:

- Since g and Φ represent purely real and imaginary components, their distortion effect can never mutually cancel (they add distortion "orthogonally"). In other words, phase and amplitude distortion cannot mutually compensate. In a more detailed manner, we will see later on that the pulse broadening in the slow light medium can be written as $B^2_{signal}=B^2_{amplitude}+B^2_{phase}$.
- Since Φ shows only odd terms in its expansion, its broadening effect cannot be compensated with conventional second-order chromatic dispersion (which introduces even phase distortion).

We can now evaluate these expansions for several interesting cases. First, we will compute this expansion for the case of the Lorentzian gain profile, since this is the conventional spectral response in slow light media. Then we will compute two other cases: the case of "zero" amplitude distortion (flat amplitude response within a certain frequency band) and the case of "zero" phase distortion (linear frequency response within a certain frequency band). These two extreme cases are helpful and provide a good physical insight on the main sources of broadening in slow light systems.

Starting with the lorentzian profile, we can write g and Φ as:

$$g = g_0 \frac{1}{1+\left(\dfrac{\omega}{\Delta/2}\right)^2} = g_0\left[1-\left(\frac{\omega}{\Delta/2}\right)^2+\left(\frac{\omega}{\Delta/2}\right)^4+\ldots\right] \tag{3}$$

$$\phi = g_0 \frac{\frac{\omega}{\Delta/2}}{1+\left(\frac{\omega}{\Delta/2}\right)^2} = g_0\left[\left(\frac{\omega}{\Delta/2}\right)-\left(\frac{\omega}{\Delta/2}\right)^3+\left(\frac{\omega}{\Delta/2}\right)^5 \ldots\right] \tag{4}$$

Where Δ is the full-width at half maximum of the Lorentzian. The delay associated to the slow light process is proportional to the logarithmic gain divided by the bandwidth of the interaction, as expected. Additionally, we can see that both amplitude and phase distortions are present, and become larger for larger gain and smaller bandwidth.

Now we can set to calculate the adequate profile for minimizing either amplitude or phase distortion. It is clear that we cannot minimize both at the same time, since the two quantities are related by KK relations. Once we impose one, the other has to be We can set the amplitude distortion to zero or close to zero by utilizing a flat-top spectral profile, where g_2, g_4, etc become zero. In this case $g=g_0\cdot\mathrm{rect}(\omega/\Delta)$ (see figure 1). The phase response of this system can be found via the KK relations:

$$\phi = \frac{g_0}{\pi}\left[\ln\left(1+\frac{\omega}{\Delta/2}\right)-\ln\left(1-\frac{\omega}{\Delta/2}\right)\right] = \frac{2g_0}{\pi}\left[\left(\frac{\omega}{\Delta/2}\right)-\frac{1}{3}\left(\frac{\omega}{\Delta/2}\right)^3+\frac{1}{5}\left(\frac{\omega}{\Delta/2}\right)^5 \ldots\right] \tag{5}$$

It is interesting to compare this case with the standard lorentzian profile. We can see that, for equal delays in both cases, the leading term in the phase distortion of the flat-top spectral profile is smaller by a factor of 3. This means that in practical situations the lorentzian profile is certainly not a good choice if one seeks to minimize distortion. The only advantage of the lorentzian profile over the flat-top spectral profile is that slightly smaller amounts of gain are required to achieve the same amount of delay.

Lastly, we analyze the case of an ideal (purely linear) phase response where the phase distortion becomes zero or close to zero. The ideal phase response is described by $\Phi=g_0\cdot(2\omega/\Delta)\cdot\mathrm{rect}(\omega/\Delta)$ (see figure 1). The corresponding gain profile can be found again reverting to KK relations:

$$g = \frac{g_0}{\pi}\left[2\ln 2-\left(1+\frac{\omega}{\Delta/2}\right)\ln\left(1+\frac{\omega}{\Delta/2}\right)-\left(1-\frac{\omega}{\Delta/2}\right)\ln\left(1-\frac{\omega}{\Delta/2}\right)\right] = \frac{g_0}{\pi}\left[2\ln 2-\left(\frac{\omega}{\Delta/2}\right)^2-\frac{1}{6}\left(\frac{\omega}{\Delta/2}\right)^4-\ldots\right] \tag{6}$$

It is interesting to observe that a positive delay in the signal leads to a convex response in the gain. In other words, a positive delay leads to a low-pass contribution in the gain response. Incidentally we can also verify that, of the three profiles analyzed here, this profile is the one that shows the largest delaying efficiency (delay per gain in the center frequency), however this comparison is rather biased since the bandwidth of this configuration is slightly smaller than in the previous two cases.

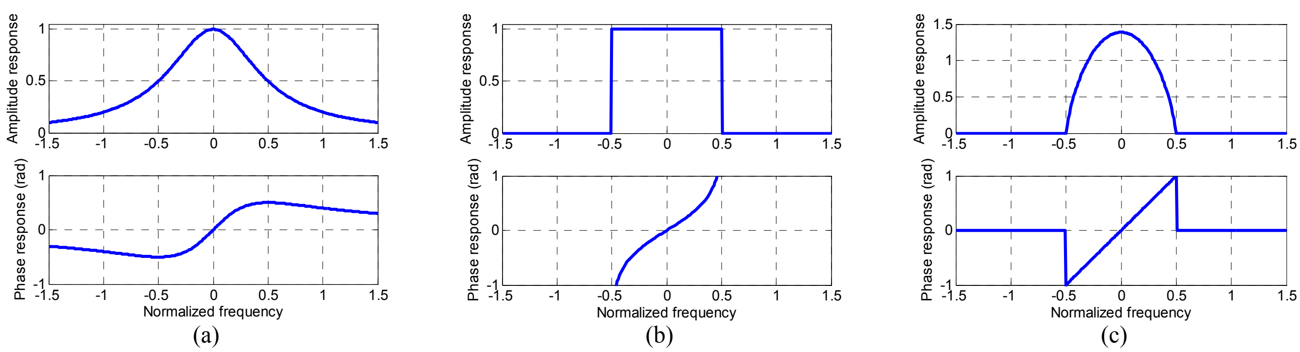


Fig. 1: amplitude and phase response of the three systems considered: (a) conventional lorentzian profile; (b) flat-top profile and (c) linear phase response

3 PULSE BROADENING IN LINEAR SLOW LIGHT MEDIA

Minimizing pulse distortion in slow light media has always been of interest for the scientific community working in the field. The most successful approach to minimize distortion in a systematic way was developed by Pant et al. [6]. The optimal profiles were found by minimizing the pulse-spectrum-weighted deviation of the physically achievable slow-light system transfer function from the ideal system transfer function (constant gain and linear phase), over the pulse bandwidth. This approach is very powerful (in the sense that the optimal performance is found regardless of the specific distortion metric used for the pulse) but does not give a good physical insight on the sources of distortion and the strategy to minimize them.

In this paper we provide a method to compute the amplitude and phase broadening suffered by the pulse in the slow light medium. Our procedure allows to isolate (and quantify) the relative contributions of phase and amplitude broadening, and this allows to identify the good strategies to minimize the distortion. Our procedure starts by defining a shape-independent metric of pulse width and pulse broadening. The best metric that we find is the rms pulse width:

$$\sigma_t^2 = \frac{1}{E}\int_{-\infty}^{+\infty} t^2 |A(t)|^2 dt \tag{7}$$

where $A(t)$ is the temporal amplitude of the pulse at the output of the slow light system and

$$E = \int_{-\infty}^{+\infty} |A(t)|^2 dt \tag{8}$$

is the total energy inside the pulse. For simplicity, we have considered that the pulse is always centered at $t=0$. This simplifies the relations, and the results do not lose any generality. The rms pulse width is the definition of pulse width that is conventionally used to quantify pulse distortion fiber-optic communication systems. This definition of pulse width measures how much the pulse energy is temporally "spread", regardless of the shape.

We can connect these expressions of pulse broadening in the time domain to the frequency domain (where our slow light systems are better described) by using Parseval's theorem and the well-known properties of the Fourier transform:

$$\sigma_t^2 = \frac{1}{E_\omega}\int_{-\infty}^{+\infty} \left|\frac{d\tilde{A}}{d\omega}\right|^2 d\omega \tag{9}$$

Where $\tilde{A}(\omega)$ is the Fourier transform of $A(t)$. Let us now consider that the Fourier transform of the pulse envelope reads $\tilde{A}(\omega)=X(\omega)\cdot\exp[i\varphi(\omega)]$. In terms of the slow light system, can recall here that $\tilde{A}(\omega)=H(\omega)\cdot S_{in}(\omega)$, hence $X(\omega)$ accumulates all the amplitude distortion introduced by the slow light system and $\varphi(\omega)$ accumulates all the phase distortion. We can finally obtain an expression that relates the pulse width to the amplitude and phase response of the system [9,10]:

$$\sigma_t^2 = \frac{1}{E_\omega}\left[\int_{-\infty}^{+\infty}\left|\frac{dX}{d\omega}\right|^2 d\omega + \int_{-\infty}^{+\infty}|X(\omega)|^2\left|\frac{d\varphi}{d\omega}\right|^2 d\omega\right] = \sigma_X^2 + \sigma_\varphi^2 \tag{10}$$

where, for simplicity, it has been assumed that the mean group delay has been subtracted from the output pulse (as stated before, the pulse is centered in $t=0$). Equation (10) shows that there are two main contributions to the temporal broadening of the pulse: one is due to the variations in the spectral amplitude of the signal (σ_X), which can be modified through the slow light system by the spectral filtering effect, the so-called amplitude broadening. The other (σ_φ) is due to the variations of the spectral phase in the pulse, and can thus be related to the phase distortion introduced by the medium. As it is visible, both contributions are positive (they add "orthogonally"), and therefore, they cannot cancel each other. We can also see that the "amplitude" contribution to the pulse width is the only term that never vanishes unless the energy is zero or the pulse is spectrally flat (the pulse is a Dirac delta).

Now we can quantify the relative importance of each of these contributions in the broadening of a pulse travelling through a slow light medium. We consider then a Gaussian pulse at the input of the slow light system $s_{in}(t)=exp(-2t^2/\sigma_{in}^2)$ (defined in amplitude). By evaluating the previous expressions in the case of a lorentzian gain profile, we can compute the amplitude, phase and total broadening accumulated in a slow light medium for different widths of this pulse in relation to the bandwidth of the slow light device. The results are plotted in Fig. 2 for $g_0=5$ (21.7 dB gain in the signal). As it could be expected, the broadening overshoots rapidly for pulses below the bandwidth of the slow light device. Generally the amplitude contribution is larger except when $\sigma_{in}/(2\pi/\Delta)$ becomes <1.1 (in these conditions, this is equivalent to having a fractional delay >1.8). It is interesting to see that the phase contribution to broadening becomes more important for shorter pulses (or equivalently larger fractional delays). The growth rate of the phase broadening is larger in this case, so this situation becomes more severe when attempting larger fractional delays. This means that the linear phase case may be of technological interest in the limit of very large fractional delays, although this assertion should be investigated in more detail.

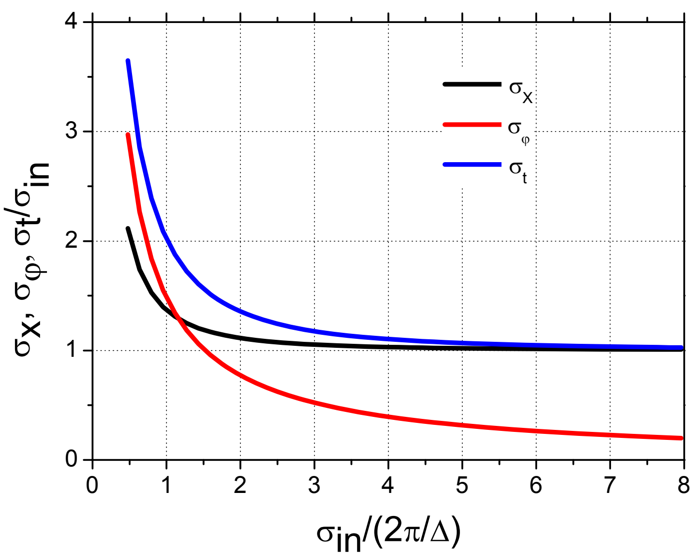


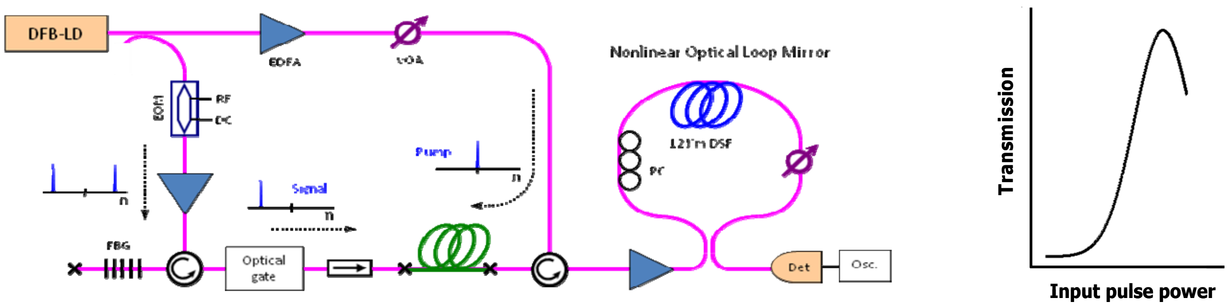
Fig. 2: amplitude (σ_X) and phase (σ_φ) broadenings accumulated in a Gaussian pulse as a function of the input pulse width σ_{in} (see text for the definition) normalized by the inverse bandwidth of the slow light system $2\pi/\Delta$.

4 COMPENSATION OF PULSE BROADENING IN SLOW LIGHT SYSTEMS

To avoid the broadening effect inevitable in any linear slow light system, one can place a nonlinear regeneration element (for instance, a saturable absorber) at the output of the slow light system. The role of this nonlinear element can be interpreted in both time and frequency domains. A pulse passing through a saturable absorber should be sharpened, since the peak has a higher intensity than the wings and therefore experiences less distortion. Since this nonlinear element plays the role of pulse sharpening, the spectrum of the signal in the output of nonlinear element turns to be broader. As a result, the frequency components that have been filtered out by the slow light medium are regenerated, restoring the signal bandwidth. An experimental demonstration of this possibility has been carried out in our groups recently [15].

The experimental scheme is depicted in Fig. 3(a) and comprises two basic building blocks: an Brillouin slow light delay line and a nonlinear optical loop mirror (NOLM) that acts as a regeneration element [16]. Upon propagation through the Brillouin slow light line, the signal pulse essentially experiences distortion accompanied with signal delay. In the proposed configuration, the NOLM acts similarly to a fast saturable absorber. The transmission of the NOLM is larger for higher input powers and decays rapidly as the input power is reduced (see Fig. 3(b)). When a pulse enters into the compensation element, the peak of the pulse experiences a larger transmission coefficient than the wings, leading to a sharpening in its shape. By only using the input power to the NOLM as control variable, we can also accommodate the sharpening of the pulse in the NOLM, and potentially we can think on achieving a complete regeneration of the pulse. This way the system can give rise to a compensation of pulse broadening caused in the slow light line, so that one can achieve pulse delays with no broadening within a limited range of delay.

To produce time delays, the signal pulse was sent into the Brillouin delay line while the pump power was increased from 0 to 30 mW. Fig. 4(b) shows the normalized time waveforms of the signal pulses at the output of the Brillouin delay line. As in any typical Brillouin slow light system, it is clearly observed that the time delay increased as a function of the pump power. Also, the pulse exiting from the delay line was temporally both delayed and broadened with respect to the pump power. This way the largest pulse delay achieved was about 36 ns (corresponding to 1.3-bits delay) at the pump power of 30 mW and the delayed pulse was significantly broadened by factor 1.9. After passing through the delay line, the delayed pulse was amplified using another EDFA in order to saturate its power, and delivered into the nonlinear loop. In practice, the NOLM acts as a saturable absorber as previously described. Consequently, the shape of the broadened pulse was sharpened at the output, as shown in Fig. 4(a). Moreover, it must be pointed out that the unwanted background components imposed onto the pulse train (mainly amplified spontaneous emission from the EDFAs and the Brillouin amplifier) were cleaned up since they were rejected for transmission. It is clearly observed that the output pulse is compressed at the output of the loop, nevertheless fully preserving the time delays achieved in the delay line.



(a) (b)

Fig. 3: (a) experimental scheme used for the demonstration: as slow light medium, we use Brillouin gain in a 1-km-long standard single-mode fiber. The modulation technique was used to produce the pump and probe beams with the adequate frequency settings. A signal pulse train was generated downshifted in frequency with respect to the pump by the Brillouin shift. The signal pulses showed duration of 27 ns FWHM at a repetition rate of 200 kHz. EDFA; erbium doped fiber amplifier, EOM; electro-optic modulator, FBG; fiber Bragg grating, VOA; variable optical attenuator, DSF; dispersion shifted fiber, PC; polarization controller and α; an attenuation factor. (b) Transmission in the NOLM as a function of the input peak power, showing a saturable absorber-type of response for low input powers (which is the regime used in this case).

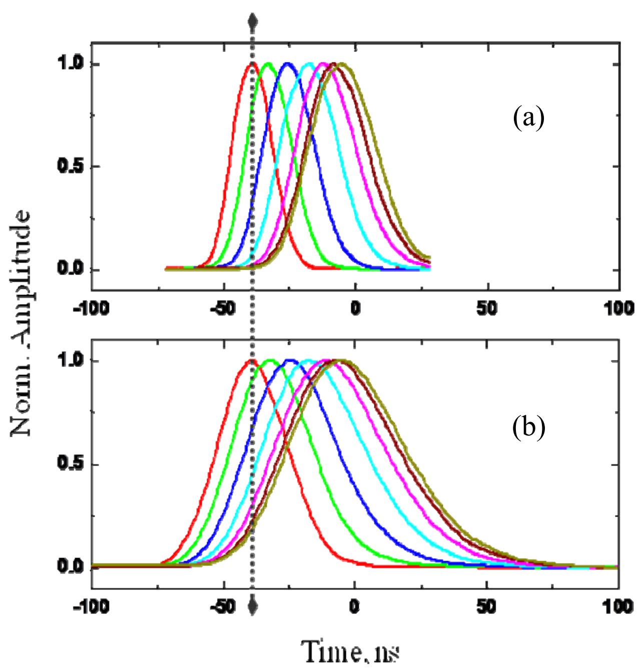


Fig. 4: experimental results (a) with and (b) without NOLM.

Another advantage of this setup concerns the signal-to-noise ratio of the setup. The contrast in dB between signal “on” and “off” states is expected to triple in logarithmic scale since the intensity at the output of the NOLM follows a cubic law with respect to the input power. We observed that the output signal contained a very clean zero background level, which is rather unusual in Brillouin amplifier setups. This zero-background level can be very helpful in real transmission systems to enhance the contrast between the “on” and “off” states in the detection process.

5 CONCLUSIONS

In conclusion, we have studied analytically pulse distortion in linear slow light systems, providing some useful limits. Additionally, we have also shown that the contributions of phase and amplitude broadening can be de-coupled and quantified. It is observed that phase broadening is generally smaller than amplitude broadening in conventional slow light media (lorentzian gain profile) except for very large fractional delays, where it becomes larger. Upon these expressions, we may envisage new strategies to minimize the distortion in the delaying of pulses, depending on the specific application and the required fractional delay

To overcome the broadening limitation, we have also experimentally demonstrated a configuration to realize a pulse delay line with essentially no pulse broadening. The inevitable pulse broadening in the linear slow light medium was completely compensated after transmission through a nonlinear optical loop mirror. Experimental results showed 1.3-bit delays with actually no broadening. It must be noticed that any other type of fast saturable absorber can replace the nonlinear optical loop mirror. As a second advantage, this setup eliminated the background noise of the signal pulse, which is introduced by all the amplifiers of the system. We estimate that there is no practical limitation to cascade the system to achieve large fractional time delays without any broadening effect, and that the bandwidth of the delay line can be enlarged as well (by SBS pump spectral broadening). In the context of extending this setup to higher data rates, the NOLM should provide a large flexibility since the response is instantaneous. With this simple implementation we have illustrated that nonlinear systems can be a very attractive solution to solve issues related to signal distortion in slow light systems.

ACKNOWLEDGMENTS

We acknowledge financial support from the Ministerio de Educacion y Ciencia through projects TEC2009-14423-C02-02, and the support from the Comunidad Autónoma de Madrid through the project FACTOTEM2 S2009/ESP-1781.

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