SATISFIABILITY OF ELASTIC DEMAND IN THE SMART GRID

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May 26, 2011

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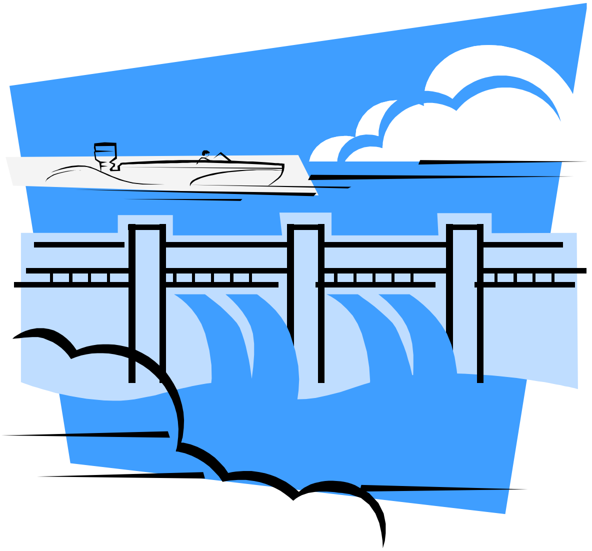
Demand Management

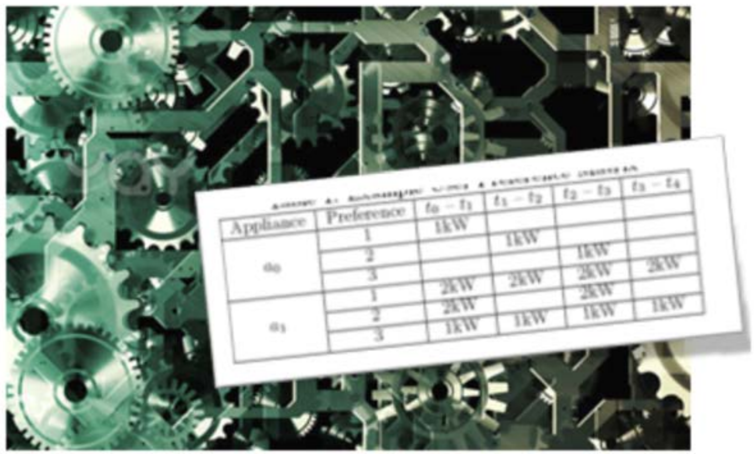


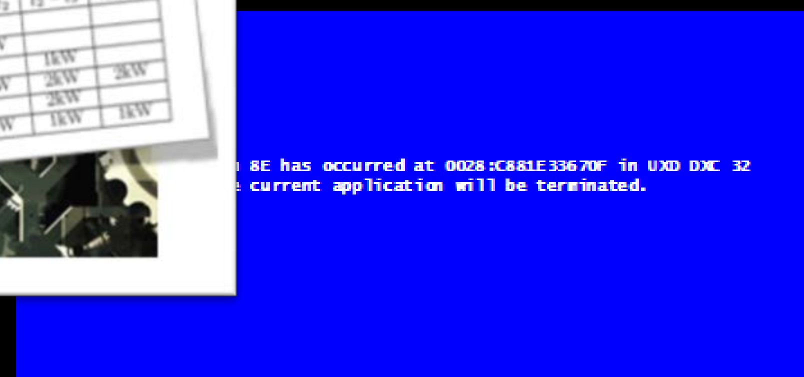
- Variable Supply
- Variable Demand
- Demand management required in future grid

Demand Management must Be Simple, Adaptive and Distributed

- Global, optimal schedules
- But they are
  - inflexible
  - complex

- *Managing End-User Preferences in the Smart Grid*, C. Wang and M. d. Groot, E-energy 2010, Passau, Germany, 2010

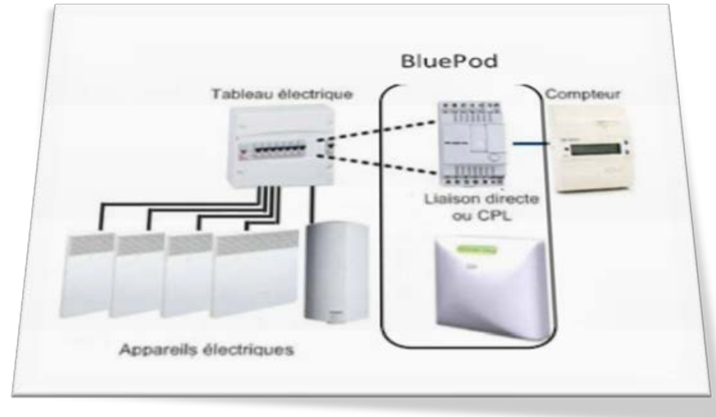




Home automation controller hung yesterday night. Hot water was not replenished overnight.

Adaptive Appliances

- Alternative to central control
- DSO provides best effort service with statistical guarantees [Keshav and Rosenberg 2010]



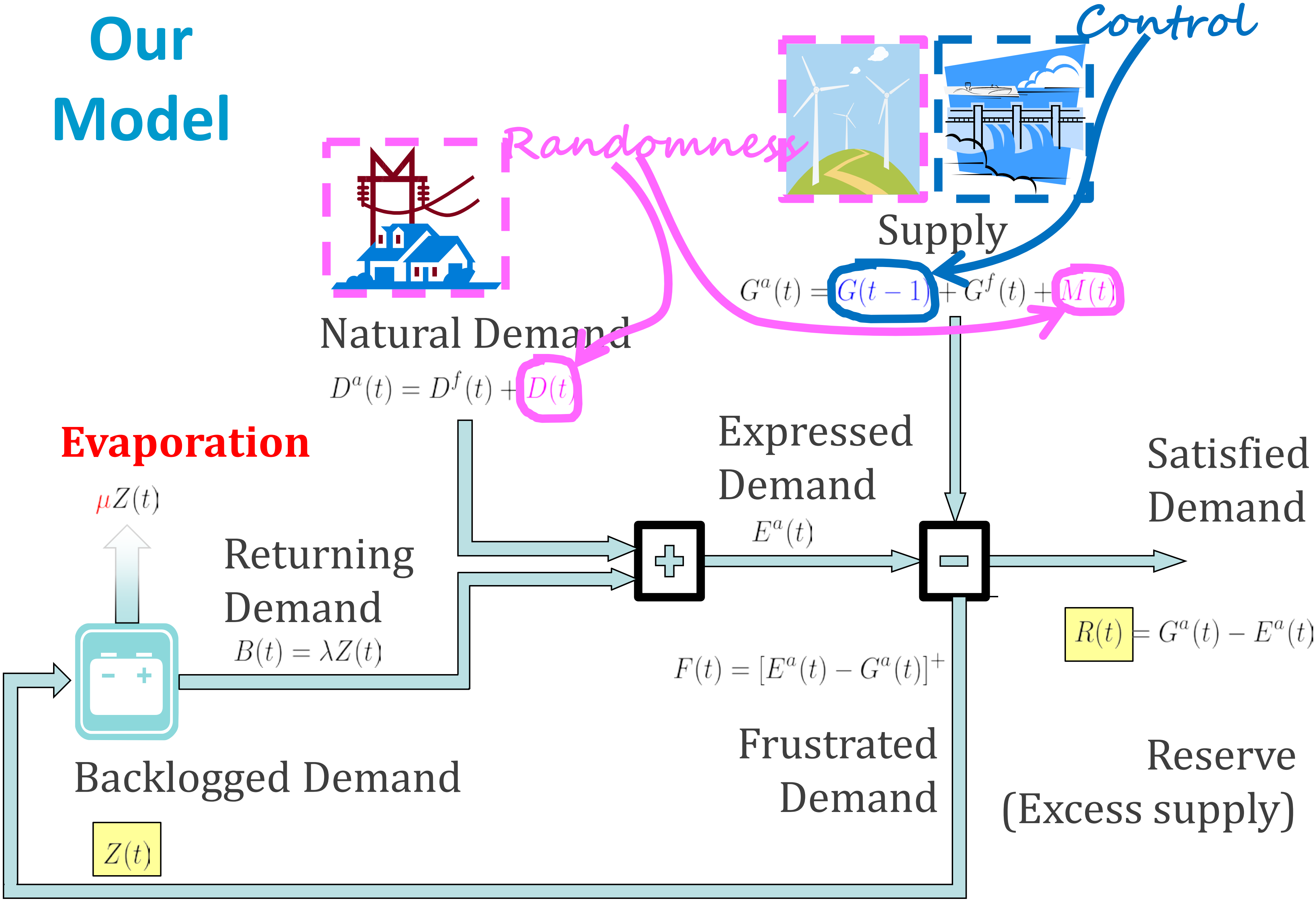
Voltalis Bluepod switches off thermal load for 30 mn

Our Problem Statement

- Is elastic demand feasible ?
- We leave out (for now) the details of signals and algorithms

- **Problem Statement**
  Is there a control mechanism that can stabilize demand ?
- Instability can be generated by
  - Delays in demand
  - Increase in demand due to delay
- A very course (but fundamental) first step

Our Model

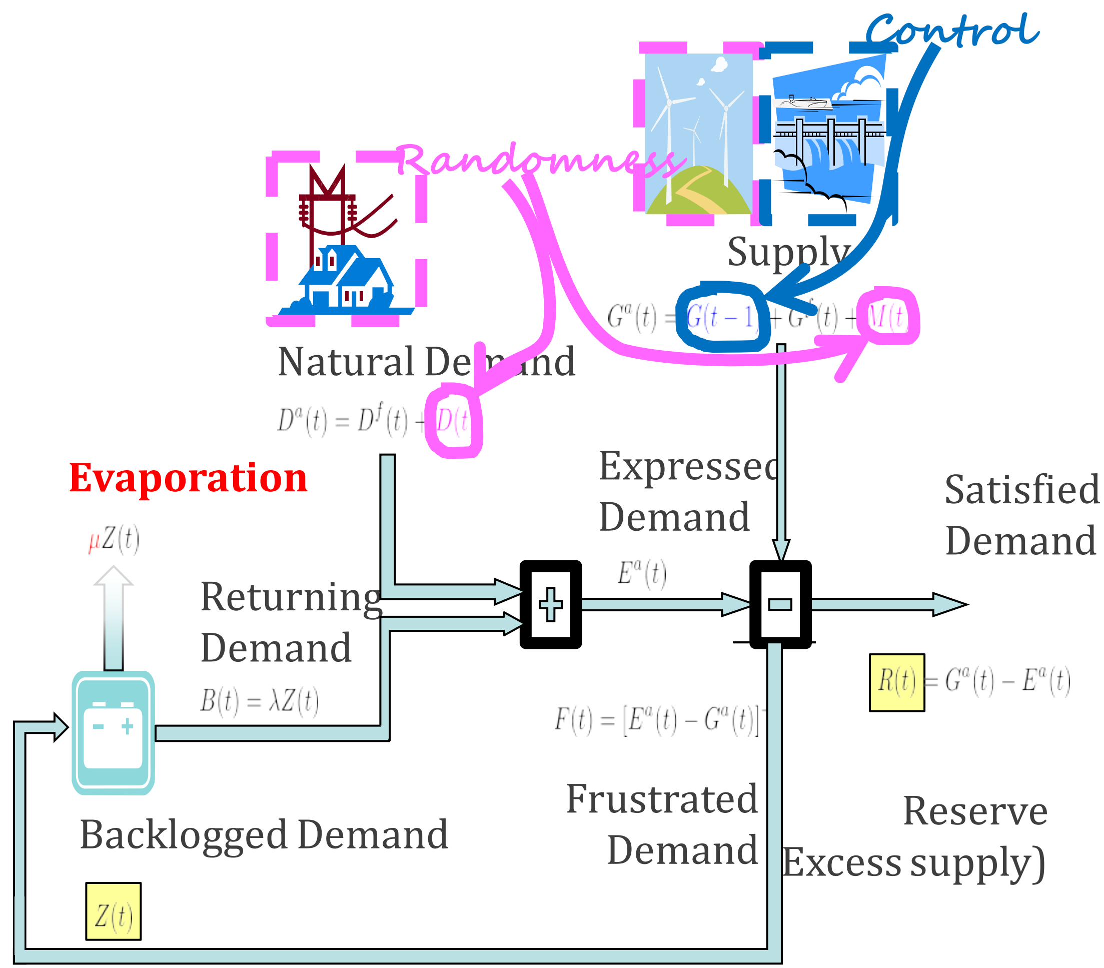


Model inspired by [Meyn et al 2010]

The Control Problem

- **Control variable:** G(t-1) production bought one second ago in real time market
- Controller sees only supply $G^a(t)$ and expressed demand $E^a(t)$
- **Our Problem:** keep backlog Z(t) stable
- Ramp-up and ramp-down constraints

  $\xi \leq G(t) - G(t-1) \leq \zeta$



Threshold Based Policies

$$G^f(t) = D^f(t) + r_0$$

Forecast supply is adjusted to forecast demand

$$R(t) = G^a(t) - E^a(t) + r_0$$

R(t) := reserve = excess of demand over supply

Threshold policy:

if R(t) < r* increase supply as much as possible (considering ramp up constraint)

else set R(t)=r*

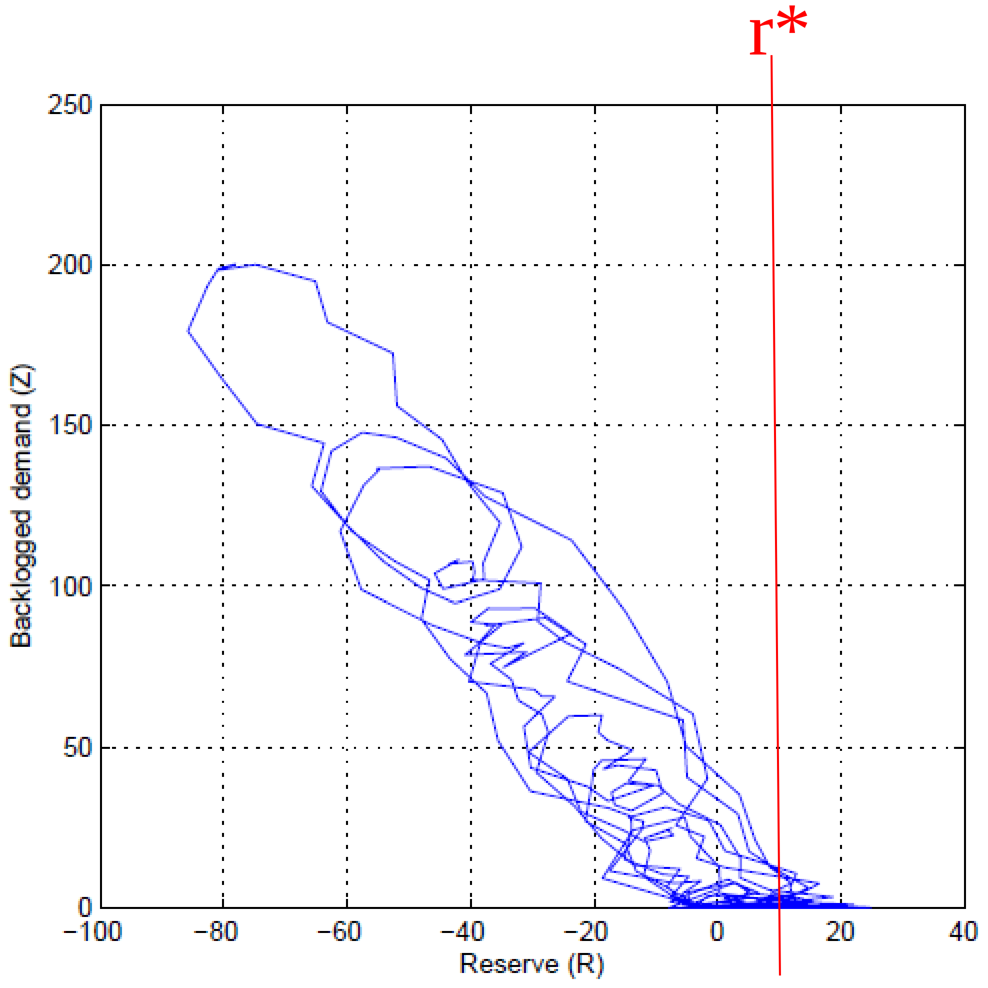
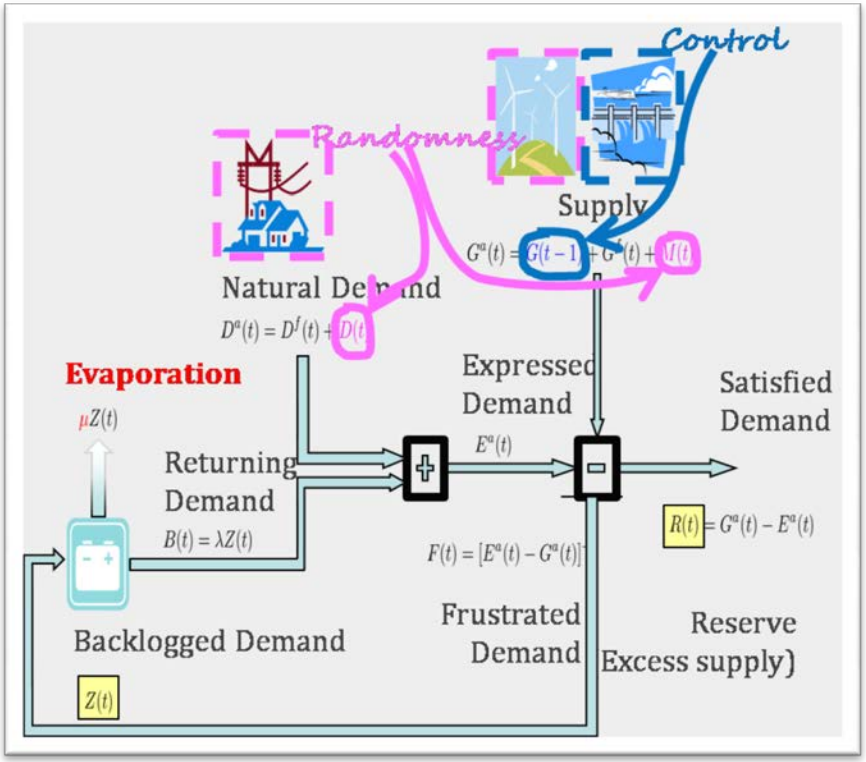


Figure 2. 500 iterations of the Markov process (13)-(14) for $\zeta = 1, r^* = 10, \sigma = 5, \lambda = 0.3, \mu = 0.1$

Findings

- If evaporation μ is positive, system is stable (ergodic, positive recurrent Markov chain) for any threshold r*
- If evaporation is negative, system unstable for any threshold r*
- Delay does not play a role in stability
- Nor do ramp-up / ramp down constraints or size of reserve



Method of Proof

- Based on Markov chain theory on general state spaces [Meyn and Tweedie]
- Stability for $\mu>0$ by Lyapunov (quadratic)
- Instability for $\mu<0$ by logarithmic test function

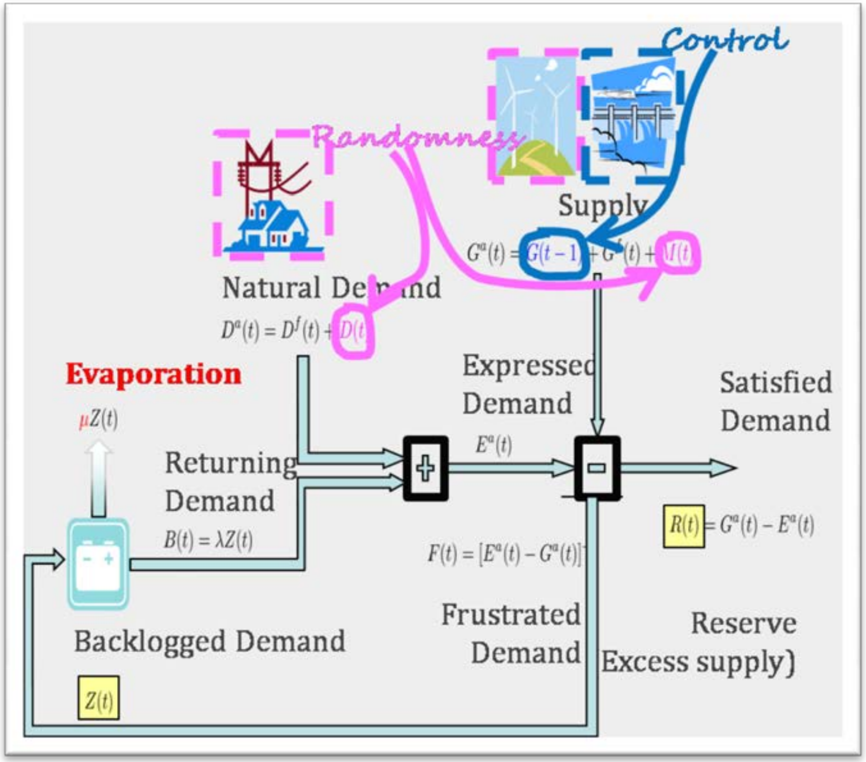
Evaporation

- Evaporation = dropped fraction of delayed demand
- *Negative* evaporation means:

  delaying a demand makes the *returning demand* larger than the original one

- Could this happen ?

  Does letting your house cool down now implies spending more heat later ? (vs keeping constant temperature)



- Do not confuse with the sum of returning demand + current demand, which is always larger than current demand)

Evaporation: Heating Appliances

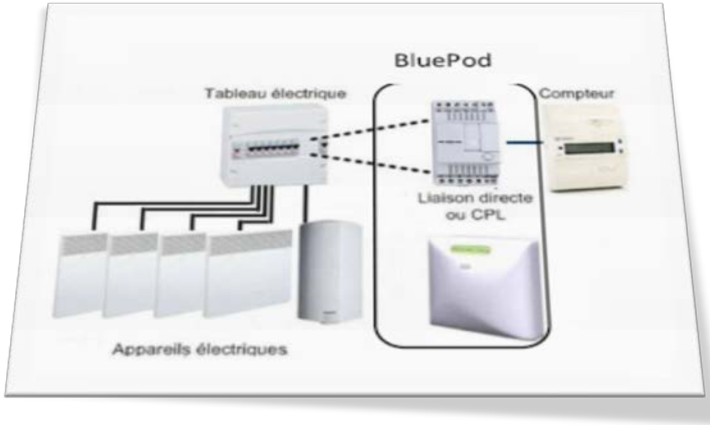
- Assume the house model of [MacKay 2009]

heat provided to building

$$d(t)\epsilon = \boxed{K}(T(t) - \theta(t)) + \boxed{C}(T(t) - T(t-1))$$

leakiness outside inertia

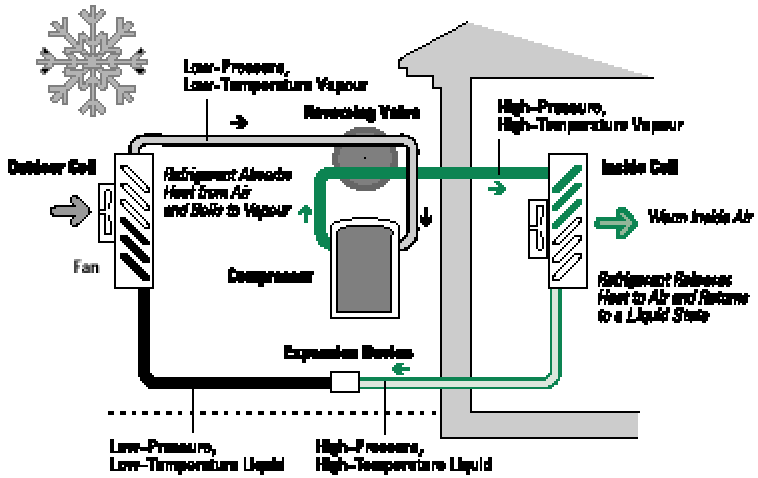
then delayed heating is less heating



- If heat = energy, then evaporation is positive.

  This is why Voltalis bluepod is accepted by users

- If heat = heat pump, coefficient of performance may be variable
  Delayed heating with air heat pump may have negative evaporation



Batteries

- Thermal loss is non linear, delayed loading causes negative evaporation

  (charging at higher intensity)

Conclusions

- A first model of adaptive appliances with volatile demand and supply
- Suggests that negative evaporation makes system unstable,
  - thus detailed analysis is required to avoid it
- Model can be used to quantify more detailed quantities
  - E.g. amount of backlog