Thickness Reconstruction of Dielectric Coatings by the use of Higher Order Impedance Boundary Conditions

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Equivalent boundary conditions has been widely used for mathematical modeling in the wave problems [1]. The most common one is called Impedance Boundary Condition(IBC) where tangential components of the electric field are related to those of the magnetic fields. On the other hand, one of the important subject in inverse problem that is very important for automotive and defense industries is the reconstruction of thickness of coatings. In these framework, a new approach is proposed for the determination of variable thickness of coatings is proposed by employing the Higher Order Impedance Boundary Conditions (HIBC) which is defined in terms of thickness and physical properties of the layer [2].

To this aim, we first consider the HIBC modelling of the object which is made of a perfect conductor coated with dielectric layer whose thickness δ is variable and small in terms of wavelength of incident wave. These conditions lead to approximate models that are close to the original one up to an $O(\delta^{k+1})$ error, where k denotes the order of HIBC. We consider TM polarize incident wave which yields the Dirichlet boundary condition, $u + {D_l}^{\delta} \frac{\partial u}{\partial n} = 0$, at the boundary of coating where u is the total field, ${D_l}^{\delta}$ is the l^{th} order IBC. HIBC is derived up to 3rd order for variable thickness as follows,

$$ {D_1}^{\delta} = -\delta(s) \tag{1} $$

$$ {D_2}^{\delta} = -\delta(s)\left(1 - \frac{1}{2}\delta(s)C(s)\right) \tag{2} $$

$$ {D_3}^{\delta} = -\delta(s)\left(1 - \frac{1}{2}\delta(s)C(s) + \frac{1}{3}\delta^{2}(s)C^{2}(s)\right)\frac{1}{6}\delta^{3}(s)\left(\frac{\partial^2}{\partial s^2} + k^{2}\right) - \frac{1}{2}\delta^{3}(s)\left(\frac{\partial^2}{\partial s^2} + k^{2}\right) \tag{3} $$

where C is the curvature and k is the wavenumber of the coating. For the first order IBC, solution is obvious from (1). In order to determine the thickness of coatings with second order, Newton Method is employed for the nonlinear equation (2). The more accurate approximation given in (3) is in the form of nonlinear diferantial equation that needs to be linearized by the use of thickness obtained from (2). Numerical experiments are presented to show the feasibility and the effectiveness of the proposed method as well as to see the validation limits of the approach in terms coating parameters.

REFERENCES

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