This paper discusses the implementation of a synthetic electrical admittance for performing active sound absorption at the diaphragm of electroacoustic transducers. The dynamics of electroacoustic transducers such as moving-coil loudspeakers can be readily controlled either by applying a direct feedback on acoustic quantities or by plugging some shunt circuitry at the electrical terminals. Such basic control strategies make possible to vary the acoustic impedance that the transducer diaphragm presents to the medium where sound waves propagate. Depending on the control settings (feedback gain, shunt resistor, etc.), any conventional loudspeaker system may become a versatile electroacoustic resonator capable of absorbing (or of reflecting as much) the incident sound energy in a frequency dependent way. Recently, it has been demonstrated that the feedback-based principle reveals formal analogies with the electrical shunt approach. In the footstep of this novel approach, an original method for designing electrical networks, in a form of a synthetic electrical impedance or admittance used for mimicking the performances of active feedback techniques, has been highlighted. In this paper the design of a synthetic electrical admittance is presented, along with computed results showing the effect of this additional electric network on the dynamic behavior of a moving-coil loudspeaker. As a conclusion, general remarks on the practical implementation using digital filters on a Field Programmable Gate Array (FPGA) platform are provided.

1. Introduction

The dynamics of electroacoustic transducers such as moving-coil loudspeakers can be readily controlled either by applying a direct feedback on acoustic quantities [1, 2, 3, 4], or by plugging some shunt circuitry at the electrical terminals [5, 6, 7, 8]. With the help of such very basic control strategies variable acoustic impedance can be achieved at the transducer diaphragm, thus changing any loudspeaker into a versatile electroacoustic resonator capable of absorbing sound energy. Looking at the electrical side of the loudspeaker, it turns out that a direct feedback on acoustic quantities behaves as an equivalent electrical load which controls both phase and amplitude at the loudspeaker terminals [8]. It merely means that a specific electrical network, also called electronic generator [5], could be theoretically designed for substituting a direct feedback control. The design and implementation of electrical networks mimicking direct feedback acoustic impedance control by means of digital filters on a Field Programmable Gate Array (FPGA) platform is the main motivation for this paper.
The remainder of this paper is organized as follows. First, a brief description of the dynamics of a moving-coil loudspeaker will be detailed, giving emphasis on the equivalent electrical load impedance that is seen at the transducer terminals when controlled. The problem of controlling the dynamics of the loudspeaker will be then tackled through the design of a digital filter. As a conclusion, general remarks on a practical implementation with a conventional moving-coil loudspeaker are provided, together with an overview of expected acoustic performances and some practical considerations about stability issues.

2. Moving-coil loudspeaker description

2.1 Characteristic equations

A moving-coil loudspeaker, mounted in a sealed enclosure, for small displacement and below the first modal frequency of its diaphragm, is commonly described by a characteristic electromechanical coupling equation system [9], that can be expressed in terms of Laplace transforms as:

\[
SP(s) = -\left( sM_ms + R_{ms} + \frac{1}{sC_{mc}} \right)V(s) - BlI(s) \\
E(s) = (sL_e + R_e)I(s) - BLV(s)
\]

(1)

where \( P(s) \) is the sound pressure acting on the loudspeaker diaphragm, \( V(s) \) is the diaphragm velocity (opposed to total particle velocity), \( I(s) \) the driving current and \( E(s) \) is the voltage applied at the electrical terminals (see Fig. 1). For the model parameters, \( S \) is the effective piston area, \( Bl \) is the force factor of the transducer (product of \( B \), the magnetic field amplitude and \( l \), the length of the wire in the voice coil), \( M_{ms} \) and \( R_{ms} \) are the mass and mechanical resistance of the moving part, \( R_e \) and \( L_e \) are the dc resistance and the inductance of the voice coil. Here, \( C_{mc} = \left( 1/C_{ms} + \rho c^2/V_b \right)^{-1} \) is the equivalent mechanical compliance accounting for both the flexible edge suspension and spider of the loudspeaker \( C_{ms} \) and the cabinet, where \( \rho \) and \( c \) are the density and celerity of air and \( V_b \) is the volume of the cabinet. The term \( BlI(s) \) represents the Laplace force induced by the current circulating through the coil and \( BlV(s) \) is the back electromotive force induced by its motion within the magnetic field.

![Figure 1. Schematic of the moving-coil loudspeaker in closed-box.](image)

2.2 Advantage of electromechanical coupling reversibility

Generally speaking, any electroacoustic transducer can be employed either as a sound transmitter, when it converts electrical energy into acoustical energy, or as a sound receiver when operating in the opposite way [9]. When operating as a sound transmitter for instance, an auxiliary voltage source \( E_s(s) \) is connected at the electrical terminals and the applied voltage can be expressed as:

\[
E(s) = E_s(s) - Z_s(s)I(s)
\]

(2)
Table 1. Visaton® AL 170 small signal parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>dc resistance</td>
<td>$R_e$</td>
<td>5.6</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Voice coil inductance</td>
<td>$L_e$</td>
<td>$0.9 \times 10^{-3}$</td>
<td>H</td>
</tr>
<tr>
<td>Force factor</td>
<td>$Bl$</td>
<td>6.9</td>
<td>N.A $^{-1}$</td>
</tr>
<tr>
<td>Moving mass</td>
<td>$M_{ms}$</td>
<td>0.013</td>
<td>kg</td>
</tr>
<tr>
<td>Mechanical resistance</td>
<td>$R_{ms}$</td>
<td>0.8</td>
<td>N.m $^{-1}.s$</td>
</tr>
<tr>
<td>Mechanical compliance</td>
<td>$C_{ms}$</td>
<td>$1.35 \times 10^{-3}$</td>
<td>m.N $^{-1}$</td>
</tr>
<tr>
<td>Effective area</td>
<td>$S$</td>
<td>0.0133</td>
<td>m$^2$</td>
</tr>
</tbody>
</table>

where $Z_s(s)$ is the internal impedance of the power source. When operating in reverse as a sound receiver, $E_s(s)$ and $Z_s(s)$ can be tailored in view of modifying the transducer dynamics so that it more or less reflects some sound energy in a frequency-dependent way [8].

3. Electroacoustic absorber concept

3.1 Sound absorption capabilities at the loudspeaker diaphragm

Let us consider a small-amplitude acoustic plane wave being refracted on a plane surface such as a wall. Generally speaking, a part of the energy is reflected while the remaining is absorbed by the surface. The amount of energy that is not reflected depends on acoustical properties of the material forming the surface, which are commonly defined by the acoustic impedance $Z_a(s) = \frac{P(s)}{V(s)}$, i.e. by the ratio of the sound pressure at a point of the surface over the normal velocity communicated to this point [9]. In the following discussion it will be more convenient to deal with the acoustic admittance, reciprocal of the impedance, since the boundary surface (here the diaphragm) is not completely rigid but reacts to incident pressure waves. The specific admittance $Y_a$, ratio of the characteristic impedance of air $\rho c$ over the acoustic impedance $Z_a$ can then be introduced so as to deal with a dimensionless parameter as:

$$Y_a(s) = -\rho c \frac{V(s)}{P(s)} \quad (3)$$

Note that the minus sign is required for obeying the used sign convention for the velocity (see Fig. 1). Expressing now the control law as a linear combination of a feedback voltage on both the diaphragm velocity and the sound pressure in the vicinity of the diaphragm, and a source voltage lowering induced by the source electrical resistance $R_s$, the command voltage given by Eq. (2) can be rewritten as:

$$E(s) = \Gamma_v V(s) + \Gamma_p P(s) - R_s I(s) \quad (4)$$

where $\Gamma_v$ and $\Gamma_p$ are proportional feedback gains including sensors sensitivities, in V.(m.s)$^{-1}$ and in V.Pa$^{-1}$ respectively, and $R_s$ is the voltage source internal resistance (or a single shunt resistor without feedback voltage). The general expression for the specific acoustic admittance that the loudspeaker presents to the medium where sound waves propagate can be obtained after substituting the command voltage Eq. (4) in Eq. (1) as:

$$Y_a(s) = \rho c S \frac{s^2 L_e + s \left( R_{eq} + \Gamma_p \frac{Bl}{S} \right)}{s^3 L_e M_{ms} + s^2 (R_{eq} M_{ms} + L_e R_{ms}) + s \left( R_{eq} R_{ms} + \frac{L_e}{C_{mc}} + (Bl)^2 + Bl \Gamma_v \right) + \frac{R_{eq}}{C_{mc}}} \quad (5)$$

where $R_{eq} = R_e + R_s$. 
3.2 Equivalent electrical load impedance at the terminals

Likewise, remarking that both the velocity $V(s)$ and the sound pressure $P(s)$ can be expressed as functions of the electrical current $I(s)$, an equivalent electrical load impedance can be introduced from Eq. (4) as:

$$\Gamma_v V(s) + \Gamma_p P(s) - R_s I(s) = -Z_{el}(s) I(s)$$

By combining Eq. (6) with Eq. (1), this equivalent electrical impedance can be expressed as:

$$Z_{el}(s) = -(sL_e + R_e) - BLs s^2L_e + s(R_e + R_a + BL\Gamma_p)$$
$$\frac{s^2\Gamma_p M_{ms} + s(\Gamma_p R_{ms} - S(Bl + \Gamma_v)) + \frac{\Gamma_p}{C_{mc}}}{s^2}$$

Whatever the control settings, $Z_{el}(s)$ can be viewed as the equivalent electrical load that is seen by the loudspeaker when controlled. As clearly seen in Eq. (7), $Z_{el}(s)$ can be split off into a negative series resistance-inductance $-(sL_e + R_e)$, whose role is to neutralize the electrical dynamics of the loudspeaker, and an electrical impedance which depends on the control settings.

Starting from this observation, if we are able to impose an appropriate equivalent load impedance at the loudspeaker terminals, it is expected that the loudspeaker dynamics would behave as if it was controlled using a direct feedback on acoustic quantities. In other words, the proposed formulation shows that it should be theoretically possible to mimic the behavior of a loudspeaker under control by direct feedback using some suitably designed electrical networks, mainly due to the moving-coil drive unit which is a bidirectional device. As an illustration, Fig. 2 shows the behavior of a Visaton® AL 170 low-midrange loudspeaker under control, whose specifications are given in Table 1, on both the acoustical side (a), and the electrical side (b). As shown in Fig. 2, slight discrepancies on the frequency response are observed between the computed and measured data, thus highlighting the vagueness of the small signal parameters of the loudspeaker that are commonly used for modeling.

Figure 2. Bode diagram of the measured and computed specific acoustic admittance (a) and its corresponding equivalent electrical impedance (b). The control settings for the case A (direct feedback control) are $\Gamma_v = 40 \text{ V.(m/s)}^{-1}$ and $\Gamma_p = 0.1 \text{ V.Pa}^{-1}$, and for the case B (shunt electrical control) $R_s = 4.7 \Omega$. 
4. Digital synthesis of the equivalent electrical load

4.1 Digital filter synthesis

Owing to the vagueness of the small signal electrical parameters of the loudspeaker, an analog implementation of the required electrical load is a arduous task. Especially since the first constraint to be respected is the neutralization of the electrical dynamics of the transducer. Therefore, we turned to the synthesis of the electrical load using digital filters design in the hope of having much more flexibility, accuracy and selectivity than via analog circuits. Let us consider the general form of the equivalent electrical load impedance given by Eq. (7). For causality reason \( Z_{el}(s) \) cannot be directly implemented since the order of the numerator exceeds that of the denominator. Therefore, the inverse of Eq. (7) must be computed so as to deal with a proper transfer function, which is now homogenous to an electrical admittance. The corresponding difference equation that will be implemented on a digital controller takes the form of an Infinite Impulse Response (IIR) filter, which is given by:

\[
y_{el}[n] = \sum_{k=0}^{2} b_k x_{el}[n - k] - \sum_{k=1}^{3} a_k y_{el}[n - k]
\]  

(8)

where \( b_k \) and \( a_k \) are the filter coefficients and \( N = 2 \) and \( M = 3 \) the number of zeros and poles, respectively. As indicated in Eq. (8), the output \( y_{el}[n] \) depends on both current and previous inputs \( x_{el}[n - k] \) as well as the previous outputs \( y_{el}[n - k] \). More details concerning the implementation of IIR filter can be found in [10].

4.2 Real-time FPGA hardware implementation

The practical implementation of the requested IIR filter is performed on an embedded real-time FPGA target CompactRIO\textsuperscript{®}. An FPGA platform is a processor that allows implantation of logic functions or truth tables via circuits that are reprogrammable. It also has on-chip input and output blocks to allow easy interfacing with external devices [10]. The first step of the digital filter design is to compute a floating-point IIR filter that meets the requested specifications for the desired synthetic admittance. So as to accommodate the finite-precision constraints of the FPGA platform, a conversion of the filter coefficients to fixed-point representation must be performed while still trying to meet the specifications of the filter. The final step is generating code of the modeled fixed-point filter for the FPGA hardware target using the Xilinx Code Generator. More details concerning the implementation of IIR filter on an embedded real-time FPGA target can be found in [10].

4.3 Voltage-to-current conversion

Once designed, the filter is downloaded on the embedded FPGA target to be used by the real-time controller which will process the signal arriving at the analog input module (National Instrument\textsuperscript{®} NI-9215) after analog-to-digital conversion. The filtered signal is then delivered by an analog output modules (NI-9263), after digital-to-analog conversion. As the designed electrical load we want to impose to the loudspeaker terminals should be consistent to an electrical admittance, and that the IIR filter delivers an output voltage, a voltage-to-current converter must be involved. A schematic representation of the voltage-controlled-current-source is depicted in Fig. 4. When the loudspeaker diaphragm is subjected to an exogenous sound pressure, the movement of the coil within the magnetic field will create a voltage at the terminals. This induced voltage is used as input for the IIR filter. In turn, the filtered output voltage will be converted into an electrical current which will generate a force to compensate the disturbing sound pressure, and hence to control the dynamics of the diaphragm.
5. Results and discussions

5.1 Experimental setup

In order to assess experimentally the performances of the whole system a closed-box ($V_b = 10 \, l$) Visaton \textsuperscript{©} AL-170 low-midrange loudspeaker, whose specifications are listed in Tab. 1 is used as an electroacoustic absorber. As depicted in Fig. 3, the electroacoustic absorber is placed at one end of an impedance tube (length $L = 3.4 \, m$, internal diameter $\phi = 150 \, mm$), the other extremity being ended by a horn-shape termination so as to exhibit anechoic conditions. Note also that the source loudspeaker is side-mounted and two microphones 1/2” (Norsonic \textsuperscript{©} type 1225 cartridges mounted on Norsonic \textsuperscript{©} type 1201 amplifiers) are located at positions $x_1 = 0.35 \, m$ et $x_2 = 0.46 \, m$ from the diaphragm position. The specific acoustic admittance is processed through a B&K \textsuperscript{©} Pulse multichannel analyzer, after ISO 10534-2 standard \cite{11}. Similarly, the input voltage $e$ and the electrical current $i$ circulating through the coil can be simultaneously processed for assessing the equivalent electrical load $Z_{el}$ that is seen by the loudspeaker when controlled.

5.2 Preliminary results

To date, no stable configurations for mimicking a direct feedback control using the digital controller have been achieved. Only some preliminary results have been obtained for a digital implementation corresponding to a basic shunt control configuration (see Fig. 4). As shown in Fig. 5, the whole closed-loop system (config. C, i.e. loudspeaker + voltage-to-current converter + FPGA controller) behaves as if the loudspeaker was shunted by a pure positive resistance of value 4.7 $\Omega$ (config. A). Note that substituting the digital FPGA controller by an analog voltage divider (config. B) is also equivalent.

![Figure 3. Experimental setup for the assessment of the electroacoustic absorber performances](image)

![Figure 4. Experimental disposal for assessing various configurations equivalent to a shunt control with $R_s = 4.7 \, \Omega$. Config. A: passive shunt resistor of value 4.7 $\Omega$ directly connected at the loudspeaker terminals, config. B: voltage divider including two resistances of 12.7 $\Omega$ and 47 $\Omega$ connected to the voltage to current converter, and Config C: digital filter with gain of value 0.0217 V/V connected to the voltage to current converter.](image)
Because the loudspeaker we are interested in controlling is in the feedback loop of the voltage-to-current conversion circuit (see Fig. 4) it will have a significant effect on stability. If this load was always purely resistive the analysis would be simple and the proposed basic circuit would work. However, the voice-coil involved in the electromechanical conversion is a complex load to drive owing to its inductive behavior. Therefore, modifications of the conversion circuit must be envisaged in view of stabilizing it with the loudspeaker. It is planned to apply the rate of closure technique which refers to how the response of the feedback and that of the amplifier open loop gain intersect. If the slope of the combined intersection is not over 20 dB per decade, the circuit will be stable. The current situation can be readily depicted in Fig. 6 for a OPA548T amplifier with a 1 Ω current sense resistor and a loudspeaker whose specifications are listed in Tab. 1. The intersection of the responses exhibits a combined slope of 40 dB per decade, responsible for the ringing and outright oscillation that can be observed when connecting a synthetic complex admittance using the FPGA controller. For tackling this issue, an alternate feedback path can be introduced in the voltage-to-current converter. An illustration of the expected compensating response is depicted in dashed line in Fig. 6 (b) when the alternate feedback path takes the form of a series resistor and capacitor.
6. Conclusion

A synthetic electric network has been analytically identified for various configurations of active impedance control, the design of which can be specified in a relative simple manner. No satisfactory experimental result for mimicking the behavior of direct feedback impedance control has yet been achieved at this stage. It turns out that a stable behavior of the whole system using a digital real-time controller and a voltage-to-current converter is difficult to be realized, mainly due to the inductive behavior of the voice-coil to be driven. A solution to overcome this stability issue based on rate of closure technique will be tested in a future work. However, such a sensor-less approach offers a promising direction for controlling the dynamic behavior of electroacoustic transducers.

7. Acknoledgements

This work was supported by the Swiss National Science Foundation under research grant 200021-116977.

REFERENCES