Draft Code Proposal

PUNCHING SHEAR

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1. The following provisions relate to the punching of columns through slabs. They shall also be used in cases of punching due to other concentrated loads.

2. The average stress at the top of the column shall not exceed $f_c$.

3. **Bending**

The slab bending resistances in the column region shall not be less than the values given in Figure 1.

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**Figure 1**

| Interior Column | $\tilde{m}_{xR} \geq 0.125 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{yR} \geq 0.125 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{xR}$ and $\tilde{m}_{yR}$ are average bending resistances in the column region over $0.3 \cdot l_y$ or $0.3 \cdot l_x$ |
| $V_d$ = Design Punching Shear |
| $\gamma_R$ = Resistance Factor |
| $l_x$ = Span length in x-direction |
| $l_y$ = Span length in y-direction |

| Edge Column | $\tilde{m}_{xR} \geq 0.250 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{yR} \geq 0.125 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{xR}$ and $\tilde{m}_{yR}$ are bending resistances in the column region |
| $V_d$ = Design Punching Shear |
| $\gamma_R$ = Resistance Factor |
| $l_x$ = Span length in x-direction |
| $l_y$ = Span length in y-direction |

The y-direction bottom reinforcement anchored in the column shall be capable of developing the tensile force $F_R \geq 0.5 \cdot \gamma_R \cdot V_d$.

| Corner Column | $\tilde{m}_{xR} \geq 0.500 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{xR} \geq 0.500 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{yR} \geq 0.500 \cdot \gamma_R \cdot V_d$
| $\tilde{m}_{yR}$ = Bending resistances in the column region |
| $V_d$ = Design Punching Shear |
| $\gamma_R$ = Resistance Factor |

The x-direction and y-direction bottom reinforcement anchored in the column shall be capable of developing the tensile forces $F_{xR} \geq 0.5 \cdot \gamma_R \cdot V_d$ and $F_{yR} \geq 0.5 \cdot \gamma_R \cdot V_d$. 

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The influence of openings shall be considered in the calculation of the resistances and in the arrangement of the reinforcement.

The yield stress shall be used in the calculation of the bending resistances according to Figure 1 when mild reinforcing steel or bonded prestressing steel is used. For slabs prestressed using unbonded tendons, only the effective prestress after deduction of all losses shall be used.

The minimum reinforcement ratio of mild steel reinforcement in the column region of the slab shall be 0.3 %.

**Punching Shear**

Punching shear resistance shall be taken as

\[ V_R = 1.8 \tau_c u d, \]

where

\( \tau_c \) = Design shear strength as given in the following table:

<table>
<thead>
<tr>
<th>( f_{c, \text{min}} ) [MPa]</th>
<th>8.</th>
<th>12.</th>
<th>16.</th>
<th>20.</th>
<th>24.</th>
<th>28.</th>
<th>&gt;32.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_c ) [MPa]</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

\( f_{c, \text{min}} \) = 5% fractile value of concrete cylinder strength

\( d \) = distance from extreme compression fibre to centroid of longitudinal tension reinforcement

\( u \) = perimeter of critical section

The critical section is defined as follows:

**For Interior Columns**

The critical section is defined as the shortest convex closed curve which completely contains the column without approaching closer than \( d/2 \) to the column perimeter.
Since the shear stress is primarily concentrated at the curved corners of the critical section, the length of the straight segments of the critical section shall not exceed 3d.

When it is apparent from the support geometry or from the distribution of slab stresses that the shear stress is concentrated only in restricted areas, the value of $u$ shall be correspondingly reduced.

In general, $u$ shall not be taken greater than 16d.

The perimeter shall be calculated according to Figure 4 for slabs of variable thickness:

The critical section shall be determined according to Figure 4a for column capitals. The increased thickness of drop panels may be considered, with the critical section determined in the normal manner. In this case, however, the slab located outside of the drop panel shall be checked according to the provisions of Section 3.2.52.
For openings located closer than a distance 5d from the edge of the column, the lesser of the values calculated according to Figures 5a and 5b shall be taken for the perimeter of the critical section.

Figure 5

For Edge and Corner Columns

Due to the rotation of the slab, the reaction can act eccentrically at edge and corner columns. An effective reaction zone is defined on the column cross-section corresponding to an average compressive stress $f_c$ (Figure 6). Only that portion of the critical section located in the region of the reaction zone shall be considered. The line n-n shall be taken parallel to the axis of slab rotation.

Figure 6
If the resistance $V_R$ is insufficient considering concrete shear strength alone, then the entire force shall be resisted by punching shear reinforcement. The maximum allowable resistance provided by reinforcement shall in no case exceed

$$V_{R,\text{max}} = 1.5V_R.$$ 

The slab region with punching shear reinforcement shall be designed on the basis of a three-dimensional truss model, using compression diagonals inclined at an angle of less than $45^\circ$ from the slab plane. The design value of the yield stress of the punching shear reinforcement shall not be taken greater than

$$f_y = 460 \text{ MPa},$$

even if higher strength steels are used.

In normal cases, punching shear reinforcement shall consist of vertical stirrups which enclose the top and bottom longitudinal reinforcement or vertical bars which are well anchored at both ends. The radial distance between two neighbouring stirrups or bars shall not exceed $0.5d$. Only those reinforcement arrangements whose effectiveness have been experimentally verified shall be allowed. Bent-up longitudinal bars shall not be used as punching shear reinforcement.

The punching shear reinforcement shall be provided up to that distance from the column edge beyond which the punching load can be resisted by the concrete alone, when the concrete resistance is calculated as follows:

$$V_R = r_{\text{Cud}},$$

where $u =$ length of the shortest closed convex curve which completely encloses the zone reinforced for punching shear.
7 Twisting moments of significant magnitude can occur in the neighbourhood of edge and corner columns. These shall be resisted along the free slab edges by reinforcement connected to the top and bottom slab reinforcement.

Figure 7

8 The beneficial effect of the deviation forces of the tendons effective over the supports may be considered in prestressed slabs. The deviation forces shall be considered as additional resistance in the verification of safety in shear. The prestressing force after deduction of all losses shall be used, with no increase in stress.

The transfer of the deviation forces of tendons outside of the column region to the column shall be investigated based on a three-dimensional truss model. Only those tendons whose distance from the column edge is less than the effective depth \( d \) shall be considered. The transverse shear forces shall be resisted by adequately anchored reinforcement.

9 The design of structural steel shear reinforcement (shearheads) shall be based on a state of equilibrium which satisfies the statical boundary conditions and the yield conditions at all locations. The provisions of Standard SIA 161 (Steel Structures) shall be observed in this regard. In given cases, the suitability of the shearhead shall be verified experimentally.
Influence of the Reinforcement Ratio on the Punching Shear Resistance

The behaviour of slabs in the column region can be described as either ductile or brittle, depending on the reinforcement ratio. These two different types of behaviour can be readily recognized in Figure 8.

![Figure 8](image)

Figure 8  Results of Punching Shear Tests on Slab Segments  
(Tests B-2, B-4, B-9, and B-14 from Reference 1)

In the figure, the load is represented as a function of deformation in the column region for four tests with \( \rho = 0.5\%, 1.0\%, 2.0\%, \) and \( 3.0\% \). For the tests with \( \rho = 0.5\% \) and \( \rho = 1.0\% \), the entire reinforcement yielded, so that the measured ultimate load corresponded to the theoretical "ultimate load in bending". Although the test samples ultimately failed in punching shear after large deformations, this effect can be considered as secondary. The "ultimate load in bending" was however not reached for the tests with \( \rho = 2.0\% \) and \( \rho = 3.0\% \). The deformations at failure were so small that the reinforcement yielded only locally.

The results of other tests on flat slab segments are shown in Figure 9. The vertical axis denotes measured punching shear resistance \( (\tau_R/\tau_C) = (V_R/(ud)) \cdot 1/\tau_C \), while the reinforcement ratio \( \rho \) is given on the horizontal axis.
Figure 9  Results of Punching Shear Tests on Slab Segments at Interior Columns

In this diagram, only the results of those tests where the theoretical ultimate load in bending was not reached are given. It can easily be seen that the influence on the reinforcement ratio on shear resistance is relatively small.

A clear connection between reinforcement ratio and ultimate load can only be established if the tests for which the theoretical ultimate load in bending was reached are also considered.
According to the Draft Standard SIA 162 (Concrete Structures), plastic design may be used for flat slabs. This means that the moment diagram can be freely chosen as long as the equilibrium conditions are satisfied. Figure 10 shows two extreme moment diagrams for an interior span: one with a large bending resistance in the column region, the other with more resistance in the span.

![Case 1 and Case 2 diagrams](image)

**Figure 10** Possible Distributions of Moments for an Interior Span

Since the curvatures in the column region are always greater than those in the midspan region, the ultimate load in bending of the slab will be reached for small deformations in the column region in Case 1. In Case 2, however, it is reached only after large plastic rotations at the support, with correspondingly large redistributions of the internal forces. This difference can be seen in Figure 11, where the load is given as a function of deformation in the column region.

![Load-Deformation Diagram](image)

**Figure 11** Load-Deformation Diagram for the Two Cases Illustrated in Figure 10
Since the ductility is limited in the column region, only a portion of the ultimate load in flexure is reached at failure. Solutions such as Case 2 are therefore not good. Minimum resistance, such as prescribed in Figure 1, must be provided in the column region. These limits are purely resistance limits, and not minimum reinforcement ratios. These checks must be carried out in the context of design for flexure. Bending resistances must be calculated based on the yield stress for mild steel reinforcement and bonded prestressed reinforcement. The stress in unbonded prestressing is, however, limited by $\sigma_{Pm}$. Because of the limited ductility in the column region, the increase in stress in unbonded tendons will be negligible. If the introduction of prestressing forces into the concrete structure is hindered by other stiff structural elements, then the effect of prestressing must only be considered to a limited extent in the calculation of bending resistance. This applies not only for the column region, but also for the entire structure, so that in these cases the computation of bending resistance must also proceed exactly as for design for flexure according to Figure 1.

3 Shear Critical Section

As shown in Figure 9, the influence of reinforcement ratio on punching shear resistance is very small. The equation for calculating the resistance in punching shear is therefore very simple:

$$V_R = 1.8 \tau_{cud}.$$  

The critical section is defined according to the CEB/FIP Model Code for normal cases of interior columns (Reference 2).

Relatively small column dimensions were used in the tests analyzed in Figure 9. As shown qualitatively in Figure 12, a smaller nominal shear resistance must be considered for larger columns. The nominal shear resistance of a beam can be considered in extreme cases of very large columns. Since experimental results are lacking for the transition region, the perimeter of the critical section, $u$, is limited to 16d.
Figure 12 Qualitative Distribution of Nominal Shear Resistance as a Function of u/d

If punching shear reinforcement is provided in the column region of the slab, this condition will not normally be satisfied outside of the reinforced zone. In this case, a reduced resistance must be considered:

\[ V_R = \tau_c u d, \]

instead of

\[ V_R = 1.8 \tau_c u d. \]

The shear stress is often not constant along the edge of the column. Figure 13 shows the measured vertical strains in a rectangular column (Ratio length:width = 3:1). It is also apparent from this figure that the shear stresses are concentrated in the corner regions. In the middle region, however, the slab is unstressed in shear. In this case, therefore, the critical section will consist of the normally-defined section in the stressed portion plus portions of length 1.5d to account for the lateral spreading of the force in the slab (Figure 3). The strength prescribed in the 1978 CEB/FIP Model Code for the middle region is not considered, since it can only be developed after large deformations have occurred, i.e., only after the column has already punched through the slab in the corner region.
Figure 13  Distribution of Concrete Strains Along a Column  
(Tests R-1 from Reference 3)

Similar principles also apply for edge and corner columns. The reaction zone is determined from equilibrium considerations and under qualitative consideration of compatibility. The critical section will thus consist of the section in the region of the reaction zone plus two components of length $d$ to account for the lateral spreading of the force in the slab. The calculation of $u$ for edge and corner columns is given in Figure 6 and can be compared to $u$ calculated according to Figure 14 from Reference 4.

Figure 14  Critical Section for Edge and Corner Columns According to Reference 4

The comparison with the results of tests with edge and corner columns, in which the specified punching shear resistance as a function of eccentricity in the column reaction is presented, is given in Figures 15 and 16. For small eccentricities, the agreement is satisfactory.
Figure 15  Specified Punching Shear Resistance of Edge Columns as a Function of the Eccentricity of the Column Reaction

Figure 16  Specified Punching Shear Resistance of Corner Columns as a Function of the Eccentricity of the Column Reaction
Resultate von Durchstanzversuchen an Flachdeckenausschnitten bei Innenstützen
REFERENCES


