SAMPLING OF SPARSE CHANNELS WITH COMMON SUPPORT

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ABSTRACT

The present paper proposes and studies an algorithm to estimate channels with a sparse common support (SCS). It is a generalization of the classical sampling of signals with Finite Rate of Innovation (FRI) [1] and thus called SCS-FRI. It is applicable to OFDM and Walsh-Hadamard coded (CDMA downlink) communications since SCS-FRI is shown to work not only on contiguous DFT pilots but also uniformly scattered ones. The support estimation performances compare favorably to theoretical lower-bounds, and importantly this translates into a substantial equalization gain at the receiver compared to the widely used spectrum lowpass interpolation method.

Keywords— Channel estimation, MIMO, OFDM, CDMA, Finite Rate of Innovation

1. INTRODUCTION

Modern communication devices have seen their number of antennas cropping up. The rationals behind multiple output systems are linked to the physical properties of the electromagnetic (EM) multipath channel [2]: different antennas witness diferent channel conditions. Under this assumption, it seems natural to estimate each of these channels, and then select the best one or a combination to achieve greater capacity than a single output system.

Most multi-output EM multipath channels have a Sparse Common Support (SCS) property, i.e. the paths' Time of Arrivals (ToA) are the same for every output up to a small error $\leq \varepsilon$. Under this assumption, we will outline a Finite Rate of Innovation (FRI) sampling [3] based algorithm which takes advantage of the SCS property, conveniently called SCS-FRI [4, 5]. Compared to other algorithms which also try to estimate the channels from a subset of Fourier "probes" (such as lowpass interpolation of the channel spectrum), SCS-FRI has four main advantages. First, parametric estimation allows for joint recovery of the support common to the multiple outputs, independently of the paths amplitudes. Second, the number of probes used to sense the channel can be reduced, saving precious bandwidth for data transmition. Third, for an equal number of probes, channel estimation is more robust to noise corruption, yielding a higher equalization gain. Last but not least, the channel estimate is characterized by a very small set of parameters, saving some bandwidth for multiple input systems using non-blind transmit diversity techniques [6], e.g. beamforming. SCS channels in a discrete time setting were previously studied in [7] within the compressed sensing framework.

We will first describe and motivate the SCS channel model and proceed with the description of the SCS-FRI algorithm. This algorithm uses baseband DFT pilots (probes) to solve the aforementioned estimation problem. Within this setup, we will derive Cramér-Rao Bounds (CRB) on the support estimation for both deterministic and Rayleigh fading channels.

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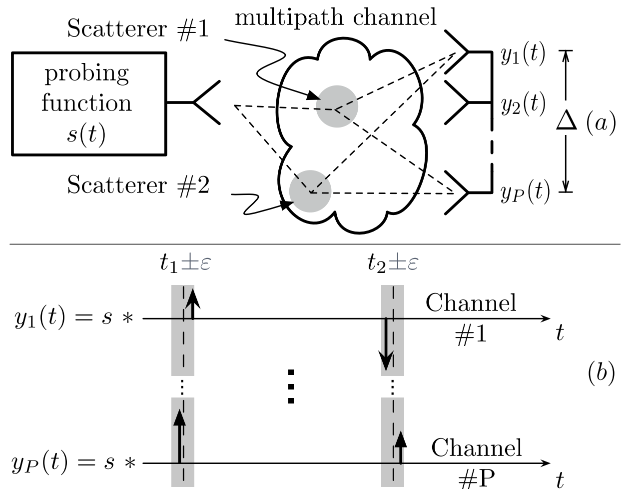

Fig. 1. (a) Transmission over a medium with two scatterers and P receiving antennas. (b) The P channels contain two paths arriving at the same time up to $\pm\varepsilon$. The amplitudes of paths from a scatterer are (possibly correlated) Rayleigh variates [8].

We will then show that SCS-FRI is not restricted to baseband DFT pilots, and works equivalently well in the uniformely spaced ("scattered") DFT pilots layout ubiquitous in OFDM based communications. Like other channel estimation techniques based on scattered pilots, the only requirement is for the channel impulse response (CIR) to be relatively short compared to the symbol duration.

Then we investigate the efficient use of other probing sequences. It is shown that they must have the same span as a subset of DFT basis vectors to warrant the use of a DFT pilot based algorithm. Interestingly, the set of Walsh-Hadamard sequences, used in most CDMA based standards, verifies this property with the added benefit of providing uniformely scattered DFT pilots. This enables the use of SCS-FRI on CDMA downlink channels as if they were OFDM coded channels. All these equivalences allow to use the CRB derived in Section 4.

We conclude the study with numerical results showing the efficiency of the SCS-FRI algorithm in a multi-output Rayleigh fading OFDM setup, and the potential equalization gain compared to the industry standard which is lowpass interpolation of the CIR spectrum.

2. PROBLEM FORMULATION

Consider a bandpass channel of bandwidth B. The inverse bandwidth $1/B$ sets a limit on the distance at which two pulses of bandwidth B

Algorithm 1 Block-Cadzow denoising

Require: A block-Toeplitz matrix $\boldsymbol{H}^{(L)}$ and a target rank K.
Ensure: A block-Toeplitz matrix $\boldsymbol{H}^{(L)}$ with rank $\leq K$.

```
repeat
    Reduce H^(L) to rank K by a truncated SVD.
    Make H_p^(L) p = 1 ... P, Toeplitz by averaging diagonals.
until convergence
```

can be reliably resolved in typical operating conditions. For channel estimation, a resolution of a 10^{th} of the inverse bandwidth is reasonable.

Application of this principle to multiple output multipath channels leads to the following approximation: antennas at a distance not exceeding $\Delta \ll c/B$ share the same path ToA up to a delay Δ/c. This principle is shown in Figure 1, where the antenna topology guarantees the support differs at most by $\pm\varepsilon$. We call this realistic channel model the *Sparse Common Support* (SCS) model. For analytic purposes the parameter ε may be set to 0. This particular case is called the *exact SCS* model.

The SCS assumption is quite relevant as many modern communication schemes use only a few megahertz of bandwidth. For example the 3GPP LTE standard [6] uses at most 20 MHz of bandwidth thus $c/20$ MHz $= 15$m, which is one order of magnitude larger than a typical antenna array size. Other ubiquitous standards such as IS-95 [9] or ETSI DVB-T [10] use less than 10 MHz of bandwidth.

Let $\{h_p\}_{p=1\ldots P}$ be a set of P baseband equivalent exact SCS channels with K paths:

$$h_p(t) = \sum_{k=1}^{K} c_{k,p}\varphi(t - t_k), \tag{1}$$

The paths coefficients $c_{k,p}$ are treated as complex random variables and $t_k \in [0\ \tau[$ with τ the delay-spread. $\varphi(t) = \sin(\pi Bt)/(N\sin(\pi tB/N))$ is the Dirichlet kernel of bandwidth B, such that $N \geq 2K+1$ is an odd integer. The channel is probed by a τ-periodic function $s(t)$, and $N = 2M+1$ noisy measurements $y_p[n]$ are acquired over one period:

$$y_p[n] = (s * h_p)(n/B) + q_p[n] \quad n \in \{0, \ldots, N-1\}, \tag{2}$$

where $\boldsymbol{q}_p \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \sigma^2\mathbb{I})$. The DFT domain representation

$$\widehat{y}_p[m] = \widehat{s}[m] \cdot \sum_{k=1}^{K} c_{k,p} W^{mt_k} + \widehat{q}_p[m]\,, \quad m \in \{-M,\ \ldots,\ M\}. \tag{3}$$

where $W = e^{-2\pi j/(NT)}$. The goal is to estimate the support $\{t_k\}_{k=1\ldots K}$ and the paths amplitudes $\{c_{k,p}\}_{k=1\ldots K, p=1\ldots P}$ from the NP DFT samples. Once the support is known, estimation of the path amplitudes is simple linear algebra as seen in (3).

3. SPARSE COMMON SUPPORT FRI (SCS-FRI)

We start from (3). The classical FRI algorithm [3] works on baseband DFT samples, hence we set the probe to be $s[m] = 1_{[-M\ M]}[m]$ the unit box sequence on $[-M\ M]$. In the absence of noise $\widehat{y}_p[m]$, $|m| \leq M$ forms a linearly recurrent sequence of degree K [11], i.e.

$$\widehat{y}_p[m] = f_1\widehat{y}_p[m-1] + \cdots + f_K\widehat{y}_p[m-K].$$

The coefficients $\{f_1, \ldots, f_K\}$ depend only on the ToA, and are thus common to all channels with common support. This simple observation leads to joint estimation of the support by computation of a unique annihilating filter [12, 4] :

Proposition 1. *Let* $\boldsymbol{H}^{(K+1)} = \left[\boldsymbol{H}_1^{(K+1)}; \boldsymbol{H}_2^{(K+1)}; \cdots; \boldsymbol{H}_P^{(K+1)}\right]^T$ *be the block-Toeplitz matrix with blocks size* $(2M-K+1) \times K+1$ *and structure:*

$$\boldsymbol{H}_p^{(K+1)} = \begin{bmatrix} \widehat{y}_{p,K-M} & \widehat{y}_{p,K-M-1} & \cdots & \widehat{y}_{p,-M} \\ \widehat{y}_{p,K-M+1} & \widehat{y}_{p,K-M} & \cdots & \widehat{y}_{p,1-M} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{y}_{p,M} & \widehat{y}_{p,M-1} & \cdots & \widehat{y}_{p,M-K} \end{bmatrix}, \tag{4}$$

In the absence of noise, a set of exact SCS channels with K distinct paths verifies

$$\boldsymbol{H}^{(K+1)}\boldsymbol{f} = \boldsymbol{0}, \tag{5}$$

where $\boldsymbol{f} = [1\ -f_1\ \cdots\ -f_K]^T$ *are the annihilating filter coefficients such that the polynomial* $p_{\boldsymbol{f}}(z) = 1 - \sum_{k=1}^{K} f_k z^k$ *has K roots* $\{e^{-2\pi j t_k/(NT)}\}_{k=1\ldots K}$.

The second property is on the rank of $\boldsymbol{H}^{(L)}$ ($L \geq K$) and is useful to denoise the measurements:

Proposition 2. *For a set of exact SCS channels with K distinct paths and in the absence of noise, the matrix $\boldsymbol{H}^{(L)}$ built according to (5) with column dimension $L \geq K$ verifies:*

$$\textit{rank}\ \boldsymbol{H}^{(L)} = K.$$

For the proofs of Propositions 1 and 2 see [4]. Proposition 2 together with the block-Toeplitz structure of $\boldsymbol{H}^{(L)}$ yields a *Block-Cadzow* denoising algorithm (Algorithm 1)[4]. Fitting all the pieces together result in the SCS-FRI algorithm, listed under Algorithm 2.

Algorithm 2 SCS-FRI channel estimation

Require: An estimate on the number of effective paths K^{est}, $2M+1$ ($M \geq K$) noisy channel DFT coefficients $\widehat{y}_p[m] = \sum_{k=1}^{K} c_{k,p} W_N^{mt_k} + \widehat{q}_p[m]$ for $|m| \leq M$, $p = 1\ldots P$.
Ensure: Support estimate $\{t_k^{\text{est}}\}_{k=1\ldots K^{\text{est}}}$

- Build $\boldsymbol{H}^{(M)}$ according to (4).
- $\boldsymbol{H}^{(M)} \leftarrow$ Block-Cadzow$(\boldsymbol{H}^{(M)}, K^{\text{est}})$.
- Update $\widehat{y}_p[m]$ with the first row and column of the denoised block $\boldsymbol{H}_p^{(M)}$.
- Build $\boldsymbol{H}^{(K^{\text{est}}+1)}$ according to (4).
- Solve the annihilating filter equation (5) to get $\boldsymbol{f}$.
- $\{t_k^{\text{est}}\}_{k=1\ldots K^{\text{est}}} \leftarrow -\frac{NT}{2\pi}\angle$ roots$(\boldsymbol{f})$.
- Estimate $\{c_{k,p}\}$ solving P linear Vandermonde systems (3).

4. CRAMÉR-RAO BOUNDS WITH BASEBAND DFT PROBES

Based on the samples collected in (2), we may bound the expected RMSE on the ToA estimation obtained by any unbiased estimator. For single path exact SCS channels with ToA t_1 and amplitude c_1, it is possible to derive a simple scalar formula (Theorem 1) for this Cramér-Rao bound. This formula is also a good approximation of the CRB in a multipath environment where paths are well-separated, empirically defined as distant by more than $2(2M+1)/(NT)$ (see Figure 2.b).

Theorem 1. *The expected minimal uncertainties on the estimation of the parameters in the SCS-FRI scenario with P signals are given by*

$$\mathbb{E}\left[(\frac{\Delta t_1}{NT})^2\right] \geq \frac{3(2M+1)}{4\pi^2 NM(M+1)}\,\mathbb{E}\left[\mathsf{ESNR}^{-1}\right]$$
$$\mathbb{E}\left[(\frac{\Delta c_\ell}{c_\ell})^2\right] \geq \frac{2M+1}{N}\,\mathbb{E}\left[\mathsf{PSNR}_\ell^{-1}\right] \qquad \ell = 1, \ldots, P$$

where $\mathsf{ESNR} = \frac{1}{2\sigma^2}\sum_{\ell=1}^{P} c_\ell c_\ell^*$ *denotes the effective signal to noise ratio and* $\mathsf{PSNR}_\ell = c_\ell c_\ell^*/(2\sigma^2)$.

Proof. See [4] for a proof on real-valued signals and noise. Application to complex signals is straightforward. □

An exact formula for multipath channels can be found in [13]:

Proposition 3. *[13] Let $\mathbf{\Phi}$ and $\mathbf{\Phi}'$ be $N \times K$ matrices such that*

$$\Phi_{n,k} = \varphi((n-1)T - t_k)\ , \quad \Phi'_{n,k} = \varphi\prime((n-1)T - t_k),$$

$n \in \{1, \ldots, N\},\ k \in \{1, \ldots, K\}$*. Given the stochastic matrix*

$$\boldsymbol{C} = \sum_{p=1}^{P} \boldsymbol{c}_p \boldsymbol{c}_p^*,$$

with $\boldsymbol{c}_p = [c_{1,p} \cdots c_{K,p}]^T$, the Fisher information matrix $\boldsymbol{J}$ conditionned on the path amplitudes is given by

$$\boldsymbol{J} = 2\sigma^{-2} \mathbf{\Phi}'^* P_{ker \mathbf{\Phi}} \mathbf{\Phi}' \odot \boldsymbol{C}. \tag{6}$$

such that $P_{ker\mathbf{\Phi}} = \mathbb{I} - \mathbf{\Phi}\mathbf{\Phi}^\dagger$ is the projection into the nullspace of $\mathbf{\Phi}$ and "$\odot$" denotes the entrywise matrix product.

See [13] for the proof. The Cramér-Rao bounds for the estimation of the normalized ToAs are on the diagonal of the expectation of $\boldsymbol{J}^{-1}$.

5. SCS-FRI WITH DIFFERENT PROBING FUNCTIONS: APPLICATION TO OFDM AND CDMA DOWNLINK

5.1. Probing with uniformly scattered DFT pilots (OFDM)

In most communications, the delay-spread τ is much smaller than the symbol length NT. This assumption alone (no multipath structure involved) implies the CIR is characterized by a subset $\{\widehat{h}_p[mD + m_0]\}_{m\in\mathbb{Z},\ 0\leq mD+m_0<N}$ of its N-points DFT coefficients, with $D \leq \lfloor \tau/\Delta \rfloor$. In (3), it corresponds to the probing sequence $\widehat{s}[m] = 1_{m\equiv m_0 \bmod D}$, where $1.$ is the indicator function.

This observation is at the heart of the very popular "scattered" pilot layout used in OFDM communications. The standard method is to perform lowpass interpolation of the CIR spectrum from the pilots.

For FRI, extension to scattered pilots is straightforward [14]:

$$\widehat{y}_p[mD + m_0] = \sum_{k=1}^{K} W_N^{m_0 t_k} c_{k,p} W_N^{mDt_k} + \widehat{q}_p[mD], \tag{7}$$

which can be seen as a dilation by D of the ToA and a phase shift of the path amplitudes. The bound on the delay-spread guarantees $Dt_k \bmod NT = Dt_k$, and so the ToA are retrieved unambiguously dividing by D. Once the ToA are known, the phase shift is invertible.

This dilation of time property together with the AWGN nature of the noise mean estimation with uniformly scattered DFT pilots is equivalent to estimation with baseband DFT pilots and a time dilation by D. Thus the bounds computed in Section 4 apply.

5.2. Probing without DFT-domain coding (CDMA downlink)

We have just shown that SCS-FRI relies on baseband or uniformly scattered DFT samples to estimate the channel. A natural way to extend this is to look for channel coding in an arbitrary domain which will lead to this suitable situation. In our setup, coding in another domain than the DFT domain is motivated by the application of SCS-FRI to a broader class of communication standards, and the gain of free "scrambling" of the data.

Let $\boldsymbol{W}$ be the DFT matrix and $\mathbf{\Pi}$ be the orthogonal projection of the DFT spectrum onto the pilot subspace $\mathcal{P}_{\text{DFT}}$. The goal is to obtain a target DFT pilot vector $\boldsymbol{p} \in \mathcal{P}_{\text{DFT}}$ sending given input data $\boldsymbol{x}_d \in \mathcal{D}$ and a probing vector $\boldsymbol{x}_p \in \mathcal{P}$:

$$\boldsymbol{p} = \mathbf{\Pi} \boldsymbol{W}^* (\boldsymbol{x}_d + \boldsymbol{x}_p). \tag{8}$$

The freedom used to set the pilot sequence $\boldsymbol{p}$ to a given value in the DFT domain comes from the probe $\boldsymbol{x}_p$. Hence the receiver does not know $\boldsymbol{x}_p$ a priori. Since we chose to work with linear subspaces, $\boldsymbol{x}_d$ can be recovered if and only if $\mathcal{D} \perp \mathcal{P}$. Probes and data therefore partition the signal space. Equation (8) surely has a solution if rank $\mathbf{\Pi} = \dim\{\mathbf{\Pi}\boldsymbol{W}\boldsymbol{x}_p\ :\ \boldsymbol{x}_p \in \mathcal{P}\}$. However it is in general unpractical as the energy of $\boldsymbol{x}_p$ may exceed the energy of $\boldsymbol{p}$. There is an isometry between probes and DFT pilots for any input data if and only if $\mathcal{P} \subseteq \mathcal{P}_{\text{DFT}}$. To guarantee existence of a solution, $\mathcal{P} = \mathcal{P}_{\text{DFT}}$.

This condition strictly restricts the codes we can use to code the data and probes, but on the bright side, the widely used Walsh-Hadamard code verifies it for DFT pilots scattered by a power of 2:

Proposition 4. *Let $\boldsymbol{W}_n$ and $\mathbf{\Omega}_n$ be respectively the 2^n-points DFT and WHT matrix obtained by Sylvester's construction:*

$$\mathbf{\Omega}_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{\Omega}_{i+1} = \mathbf{\Omega}_1 \otimes \mathbf{\Omega}_i.$$

Then, for $l \in \{1, \ldots, n-1\}$ the set of $\mathbf{\Omega}_n$'s columns with indices in $\left\{2^l + i\right\}_{i=1\ldots 2^l}$ and the set of $\boldsymbol{W}_n$'s columns with indices in $\left\{(i - 1/2)\cdot 2^{n-l} + 1\right\}_{i=1\ldots 2^l}$ span the same subspace.

For a proof see [8]. Proposition 4 tells us 2^l contiguous Walsh-Hadamard codewords can yield 2^l uniformly spread DFT pilots separated by $D = 2^{n-l}$ samples.

This result has a nice interpretation in the context of generalized Fourier transforms, the 2^n-WHT being itself the Fourier transform on the finite group $(\mathbb{Z}/2\mathbb{Z})^n$ instead of $\mathbb{Z}/2^n\mathbb{Z}$ for the classical 2^n-points DFT [8, 15]. A similar result holds for DFT on other finite groups [8].

6. NUMERICAL RESULTS

We test the SCS-FRI algorithm in an OFDM setup. The system has a bandwidth $B = 20$ MHz centered at $B = 2.6$GHz. Each frame is composed of 511 samples with a sampling step $T = 50$ns. Each frame contains 31 DFT pilots scattered by a factor $D = 16$. For all simulation, the delay-spread τ is less than 1.6μs.

For all simulations, the SNR is measured as the ratio between the total signal energy and the total noise energy. Total energy means the energy collected over all antennas.

In simulations **1** and **2** the channel has two paths. All antennas share the same support (exact SCS model). The path coefficients are chosen as a realization of a complex Gaussian random vector. For the sake of comparison, the first path amplitudes $\{c_{1,p}\}$ are normalized such that $\sum_p c_{1,p} c_{1,p}^* = 1$. For the second path $\sum_p c_{2,p} c_{2,p}^* = 1/2$, i.e. the second path has half the total energy of the first path.

Simulation **1** (Figure 2.a) shows the performances for the estimation of t_1 and t_2 with 6 independent antennas. The paths are separated by $2T$. It is shown that at this distance the CRB computed for each path independently accoriding to Theorem 1 provides a decent approximation of the true bound.

Simulation **2** (Figure 2.b) shows the power of joint support recovery. The receiver has either 2 or 6 antennas. Since the SNR is measured globally, for a given abscissa in Figure 2.b, the system with 6 of them has a lower SNR per antenna compared to the system with only 2.

Simulation **3** (Figure 3) simulates a rayleigh fading channel. The ToA of each path is independently perturbed by lapse uniformly distributed on $]-1\ 1]$ns, which is larger than the travel time between antennas at light speed. The receiver is surrounded by 4 scatterers and possess 3 antennas correlated according to a physically motivated model [8] based on [16]. The estimation provided by SCS-FRI is used to synthesize the 3 channels and the result is compared to lowpass interpolation of the channel spectrum (MATLAB's `interp()`), which is a widely used technique for channel estimation from scattered DFT pilots.

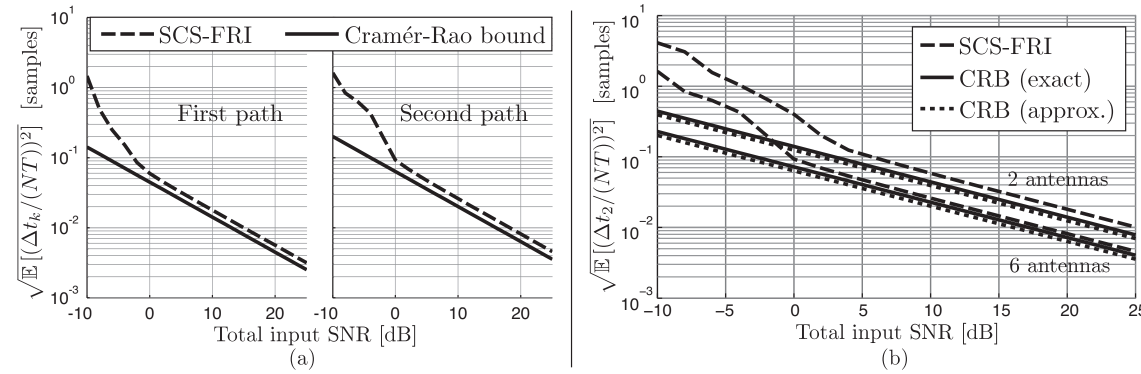

Fig. 2. Part (a) shows the support estimation error based on 6 exact SCS channels. Each channel has two path separated by a time $2T$. The estimation remains close to the lower-bound down to 0dB. Part (b) focus on the second path. Compared to Part (a), the total received signal energy is spread over 2 channels instead of 6. In the 6 channels setup, SCS-FRI performs near optimally at lower SNR values compared to the 2 channels setup.

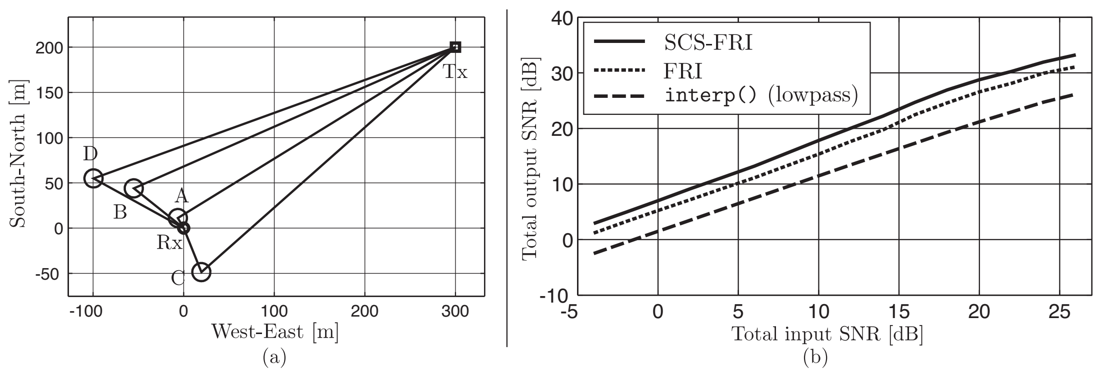

Fig. 3. Part (a) shows the physical layout of the channel: it contains four 20 meter wide scatterers labeled A, B, C and D. The receiver (Rx) has 3 antennas equispaced on a circle of radius 10cm. Part (b) shows performing independent estimation for each antenna with FRI provides a SNR equalization gain of 4-5dB compared to lowpass interpolation. Taking advantage of the common support property with SCS-FRI provides an additional 2-3dB equalization gain.

From these results, we conclude that SCS-FRI takes a near-optimal advantage of spatial diversity given baseband or uniformly scattered DFT samples. The results obtained in Simulation **3** and shown in Figure 3 reveals the additional gain obtained with parametric approaches compared to the standard non-parametric spectrum interpolation. The key property used by parametric methods is sparsity of the support. Usage of the common support property in addition to sparsity provides a substantial equalization gain increment. Note that our results take into account effects like fading and spatial correlation as well as small delays of the support between antennas to account for the time of propagation.

7. CONCLUSIONS AND FUTURE WORK

The proposed SCS-FRI algorithm was shown to recover accurately and robustly the support of SCS channels corrupted by AWGN and possibly undergoing a Rayleigh fading regime. It was shown to work for sampling schemes yielding baseband DFT coefficients as well as schemes yielding uniformly scattered DFT coefficients. The latter can be achieved for some useful non DFT coded channels. It was shown to be the case of Walsh-Hadamard coded channels such as the downlink of many CDMA standards. In this case, it opens these communication standards to the world of Fourier based estimation and equalization (and not only estimation based on correlation in the time-domain).

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