Electrostatic and magnetic transport of energetic ions in turbulent plasmas

T. Hauff,[1] M. J. Pueschel,[1] T. Dannert,[2] and F. Jenko[1]

[1]*Max-Planck-Institut für Plasmaphysik, Boltzmannstr. 2, D-85748 Garching, Germany*

[2]*Ecole Polytechnique Fédérale de Lausanne, CRPP, CH-1015 Lausanne, Switzerland*

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Analytical and numerical work is used in tandem to address the problem of turbulent transport of energetic ions in magnetized plasmas. It is shown that orbit averaging is not valid under rather generic conditions, and that perpendicular decorrelation effects lead to a slow $1/E$ decay of the electrostatic particle diffusivity of beam ions, while the respective magnetic quantity is even independent of the particle energy E.

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While over the last several years, there has been a lot of progress in our understanding of turbulent transport of particles, momentum, and energy in magnetized plasmas (see, e.g., Ref. [1]), many open questions remain: e.g., is there a significant interaction of turbulent fluctuations with energetic ions? Despite previous experimental investigations along these lines (see, e.g., Refs. [2,3] and references therein), this issue was raised again, in particular, by recent experimental investigations at ASDEX Upgrade which showed a fast radial broadening of the current profile driven by off-axis neutral beam injection in the absence of any measurable magnetohydrodynamic activity [4]. It is also important in light of the fact that future *D-T* based plasma experiments like ITER [5] will have a significant population of fast ions. Moreover, many related astrophysical problems depend on a solid understanding of fast ion dynamics in a turbulent medium [6].

As is well known, fast particle trajectories in toroidal, axisymmetric magnetic fields deviate from the field lines in two ways. First, they perform a gyration about the field lines, and second, grad-B drifts and curvature drifts also induce an oscillation of a fast particle gyrocenter about a magnetic field line in a nearly circular (actually slightly elliptical) fashion, [7] but on a much slower time scale. Past theoretical studies often addressed this topic by alluding to a presumed analogy between orbit averging and gyroaveraging (see, e.g., Refs. [8,9]). According to this point of view, concerning its long-time-scale dynamics, a fast particle only feels reduced, orbit-averaged (and gyroaveraged) potentials. Consequently, even for only moderately energetic particles, one would expect practically no cross-field transport. However, the validity of such an approach is usually not discussed, and as will be shown below, fast ions generally do not fulfill the required conditions.

Inspired by these experimental and theoretical findings, the present work represents a systematic study of the interaction between fast (passive) particles and a turbulent background based on first principles. Revisiting this problem, we find that due to a specific perpendicular decorrelation mechanism, the turbulent particle diffusivity of beam ions decreases only quite slowly in the electrostatic case, inversely proportional to the particle energy, and is even found to be independent of the particle energy in the magnetic case. To allow for a better understanding of the underlying physical effects, we adopt a two-step approach. First, we give an analytical treatment of the scaling of the diffusion coefficient with the particle energy, and second, we perform nonlinear gyrokinetic simulations including fast ions with the GENE code [10,11], a plasma microturbulence code which can be used to efficiently compute gyroradius-scale fluctuations and the resulting transport coefficients in magnetized fusion plasmas. The results of these approaches are found to agree with each other very well and shed new light on a rather old question.

In order to better understand the interaction of fast particles with the background microturbulence in a tokamak, we start by recalling some properties of particle orbits in toroidal, axisymmetric geometry. In the absence of fluctuations, the latter can be calculated analytically. As is well known, the curvature and grad-B drifts lead to oscillatory deviations from magnetic field lines in both perpendicular directions as well as to a continuous drift in the toroidal direction (corresponding to the y direction here). In terms of the pitch angle $\eta = v_\parallel/v$, one finds

$$T_{\mathrm{orbit}} = \frac{2^{1/2}\pi q}{\eta}\left(\frac{E}{T_e}\right)^{-1/2}\frac{R_0}{c_i},$$
$$\Delta r = 2^{3/2}\eta q\left(\frac{E}{T_e}\right)^{1/2}\rho_i, \qquad v_y \approx 2\eta^2\hat{s}\left(\frac{E}{T_e}\right)\frac{\rho_i c_i}{R_0} \tag{1}$$

for $\eta \rightarrow 1$ and

$$T_{\mathrm{orbit}} = \frac{2^{3/2}\pi q}{\sqrt{\epsilon(1-\eta^2)}}\left(\frac{E}{T_e}\right)^{-1/2}\frac{R_0}{c_i},$$
$$\Delta r = \frac{2^{3/2}\eta q}{\epsilon}\left(\frac{E}{T_e}\right)^{1/2}\rho_i, \qquad v_y \approx (1-\eta)^2\left(\frac{E}{T_e}\right)\frac{\rho_i c_i}{R_0} \tag{2}$$

for $\eta \rightarrow 0$ (see, e.g., Ref. [7]). Here, T_{orbit} is the orbit circulation time, Δr the diameter of the deviation from the flux surface in the radial r (or x) direction, and v_y is the particle precession drift in the toroidal y direction. Moreover, q is the safety factor, $\hat{s}$ is the magnetic shear, $\epsilon \equiv r/R_0$ is the inverse aspect ratio of the relevant magnetic surface, R_0 is the major radius, c_i is the ion thermal speed, and ρ_i is the corresponding gyroradius.

We would like to note in this context that while the above expressions, Eqs. (1) and (2), have been derived assuming circular, concentric flux surfaces, they still capture the key features of more complicated geometries. Comparing particle orbits in the simple geometry with orbits in a realistic magnetic field constructed from ASDEX Upgrade data—using the GOURDON code [12]—showed that only moderate differences occur which do not affect the dependence on the field and particle parameters. Thus, the following considerations hold also for shaped plasmas.

First, we want to concentrate on the interaction of test particles with *electrostatic* turbulence. From GENE simulations of ion temperature gradient and trapped electron mode turbulence for parameters similar to those mentioned in Ref. [13], we find as typical scales of these fluctuations in four different cases $\lambda_c \sim 6\rho_i$ (correlation length), $\tau_c \sim 20R_0/c_i$ (correlation time), and $V_E \sim 3\rho_i c_i/R_0$ (average $E \times B$ drift velocity). Moreover, diamagnetic drifts with a velocity $v_{\rm dr}$ of the order of $\rho_i c_i/R_0$ are found. These values shall be taken as representative in the following discussions, although our conclusions will be independent of the precise numbers.

How does the interaction of particle orbits with the background turbulence influence the cross-field turbulent diffusion of energetic ions? For $E/T_e \gtrsim 3$, one has $\Delta r \gtrsim \lambda_c$. So according to the traditional view, fast particles "average out" the turbulent fluctuations and are therefore not affected much by them (see the discussion in [14]). Furthermore, the diamagnetic drift of the background potential and/or the toroidal precession drift of the particles can produce a "drift barrier" which is known to suppress the diffusion quite effectively [15]. However, both of these arguments implicitly presuppose that orbit averaging is valid, and—as we will show next—this is usually not the case for energetic ions.

A prerequisite for the validity of orbit averaging is illustrated in Fig. 1. Given that deviations from a simple circular motion can be caused by both $E \times B$ and toroidal drifts, we find the necessary condition

$$\Xi_{\rm o.a.} \equiv \max\{V_E, |v_{\rm dr} - v_y|\}\frac{T_{\rm orbit}}{\lambda_c} < 1, \qquad T_{\rm orbit} \ll \tau_c. \tag{3}$$

For passing ions with $E/T_e \gtrsim 4$, v_y is the dominating term, and one obtains $\Xi_{\rm o.a.} = v_y T_{\rm orbit}/\lambda_c \sim \hat{s}q(E/T_e)^{1/2}$, which clearly states that orbit averaging is not valid in the high-energy limit. [In deriving this equation, we have neglected a term not depending on $\hat{s}$ which would become relevant for low shear values.] For trapped particles, we find a similar expression which is independent of $\hat{s}$. So we may conclude that for conventional values of q and $\hat{s}$ (in the range of unity), orbit averaging becomes invalid almost as soon as the particle energies clearly exceed the thermal energy of the background plasma. As can be inferred from Fig. 1, this means that (for $\Delta r \gtrsim \lambda_c$) a particle gets decorrelated during its orbit motion, since it does not return into the correlated zone. This orbit decorrelation time is given by

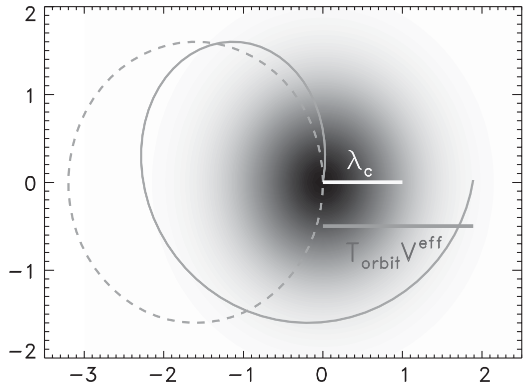

FIG. 1. The dashed line denotes a circle over which the potential is (orbit-)averaged for a particle starting at the origin, while the solid line denotes a real particle trajectory with a large drift velocity $V^{\rm eff}$. After one period, the particle is displaced by $T_{\rm orbit}V^{\rm eff}$ from the origin as well as from the corresponding point on the dashed curve. Therefore, if the particle does not return into the correlated zone [in the background, the autocorrelation function $\langle\phi(0)\phi(\mathbf{x})\rangle$ of an isotropic stochastic potential with correlation length λ_c is plotted], orbit averaging is not valid.

$$\tau^{\rm orbit} = \frac{T_{\rm orbit}\lambda_c}{\pi\Delta r} \approx \frac{\lambda_c}{2\eta^2\rho_i}\left(\frac{E}{T_e}\right)^{-1}\frac{R_0}{c_i} \tag{4}$$

for passing particles (one obtains a similar expression for trapped particles). Since, on the other hand, the decorrelation time due to the parallel motion of the particles can be approximated by $\tau_\parallel \sim T_{\rm orbit}/2$, it is clear that for $E/T_e \gtrsim 1$, the decorrelation is caused by the *perpendicular* orbit motion, not the *parallel* transit motion (as suggested, e.g., in Refs. [16,17]). We would like to point out that the validity of gyroavaging is not disputed, since the temporal scale of gyration is much smaller than that of the orbit motion, so that the respective version of Eq. (3) is always satisfied. The mechanism of gyroaveraging, as described in Ref. ([18]), will indeed be relevant for particles with a significant perpendicular velocity component.

On the basis of these considerations, we can now predict the scaling of the diffusion coefficient with the particle energy for passing ions quite easily. Since $\tau^{\rm orbit} < \lambda_c/V_E$, the decorrelation occurs in the ballistic regime, and the diffusion coefficient can be expressed as $D = V_E^2\tau^{\rm orbit}$. However, from the $D(t)$ curves, we have observed that the saturation occurs already at $t \approx \tau^{\rm orbit}/3$; therefore, we will use the expression $D \approx V_E^2 \tau^{\rm orbit}/3$. Since $\Delta r \propto E^{1/2}$ and $T_{\rm orbit} \propto E^{-1/2}$, we find $D(E) \propto E^{-1}$. Using the full expressions from Eq. (1) instead, one gets

$$D(E) \approx \frac{\hat{V}_E^2 \hat{\lambda}_c}{6\eta^2}\left(\frac{E}{T_e}\right)^{-1} \frac{\rho_i^2 c_i}{R_0}. \tag{5}$$

Here, we have introduced the dimensionless quantities $\hat{V}_E = V_E/(\rho_i c_i/R_0)$ and $\hat{\lambda}_c = \lambda_c/\rho_i$. An analogous treatment of trapped particles yields

$$D(E) \approx \frac{\hat{V}_E^2 \hat{\lambda}_c^2 \sqrt{\epsilon}}{12\sqrt{\pi \eta^2 (1-\eta^2)}}\left(\frac{E}{T_e}\right)^{-3/2} \frac{\rho_i^2 c_i}{R_0}, \tag{6}$$

where we have used Eq. (36) from Ref. [14] to take into account finite Larmor radius effects (which are negligible for $\eta \sim 1$). Both of these high-energy expressions of $D(E)$ yield values which are very large compared to the ones one would get if orbit averaging were valid (mainly due to the existence of drift barriers mentioned above).

Having discussed and understood the electrostatic transport of energetic ions, we are now in a good position to address its magnetic counterpart. As is well known, the quantity $v_B \equiv v_\parallel(\tilde{B}_r/B_0)$, which represents the projection of the parallel velocity into the radial direction along a fluctuating field line, takes over the role of the radial component of the $E \times B$ drift velocity in the context of magnetic transport. Here, B_0 is the unperturbed magnetic field and $\tilde{B}_r$ is its radial perturbation. Thus, not unexpectedly, it will turn out that many of the previous findings and insights carry over to this case in a more or less straightforward manner.

The nonlinear electromagnetic GENE simulations of ion temperature gradient turbulence for Cyclone Base Case parameters [19] presented in Ref. [20] show that the magnetic fluctuation level tends to scale linearly with the plasma beta. More specifically, one finds the relation $\tilde{B}_r/B_0 \sim C\beta[\%](\rho_i/R_0)$ with $C \sim 0.6$. Consequently, one obtains the estimate

$$V_B \sim \frac{\tilde{B}_r}{B_0}\sqrt{\frac{E}{T_e}} c_i \sim C\beta[\%]\sqrt{\frac{E}{T_e}}\frac{\rho_i c_i}{R_0} \tag{7}$$

for the typical value V_B of the magnitude of v_B. Furthermore, the correlation length of the radial magnetic field perturbations is found to be comparable to (but somewhat smaller than) the electrostatic one, i.e., $\lambda_B \approx 2.5\rho_i$. For magnetic transport, the validity condition for orbit averaging is identical to the electrostatic one, except that V_E is replaced by V_B. So, in general, the magnetic values $\Xi_{\rm o.a.}$ are comparable to the electrostatic ones, and orbit averaging is invalid for $E/T_e \gg 1$. Applying then the same reasoning that lead to Eq. (5) and making the ansatz $D_B \approx V_B^2 \tau^{\rm orbit}/3$, we obtain the expression

$$D_B(E) \approx \frac{(C\beta[\%])^2 \hat{\lambda}_B}{6\eta^2}\frac{\rho_i^2 c_i}{R_0}. \tag{8}$$

So, e.g., for $C = 0.6$ and $\beta[\%] = 0.6$, one gets $D_B \sim 0.05\rho_i^2 c_i/R_0$ which is a reasonably large number. It is important to note in this context that the magnetic transport is independent of the particle energy. The reason for this behavior is that the $1/E$ dependence caused by the perpendicular decorrelation is balanced by the increase of the magnetic drift velocity. For trapped particles, finite Larmor radius effects have to be taken into account, and one obtains $D_B(E) \propto E^{-1/2}$. Thus, the magnetic expressions deviate even more profoundly from the expectations based on the validity of orbit averaging.

In order to test these analytical predictions, we have performed electromagnetic simulations with the gyrokinetic turbulence code GENE [10,11]. GENE is physically comprehensive and well benchmarked, and it can be run either as a local or global code. For simplicity, our present simulations have been performed in a local flux-tube environment with $\hat{s} - \alpha$ geometry (circular flux surfaces). This is a common approximation, and recent numerical investigations show that the resulting turbulence characteristics exhibit moderate quantitative, but no qualitative differences compared to simulations in more realistic geometries [13,21]. We were employing Cyclone Base Case parameters [19] and $\beta = 0.6\%$ (like in the simulations presented in Ref. [20]). Here, we have added an additional passive particle species, however, characterized by an isotropic Maxwellian distribution function with $T/T_e = 50$. During the saturated turbulent phase, the energy dependent particle transport—normalized with respect to the equilibrium distribution at the respective position in velocity space—was written out. The results are shown in Fig. 2. They happen to be in good agreement with the above theoretical considerations. In particular, the magnetic transport is found to be independent of the particle energy for larger energies, and at a level reasonably close to the one predicted by Eq. (8).

In this context, we would like to mention that the $1/E$ decrease observed for electrostatic transport of beam ions was first reported in Ref. [13], but without any explanation with respect to the underlying physics. In Ref. [17], the authors also claimed to observe a $1/E$ decrease, but no graph was presented to support this statement, and an explanation different from ours was given—based on the validity of orbit averaging—which we believe to be incorrect. Meanwhile, the results presented here for the case of magnetic transport are completely new, and the same applies to the physical explanation in both the electrostatic and the magnetic case. The findings concerning magnetic transport are probably the most central ones as the following discussion shows.

It is of great interest to calculate the overall transport coefficients resulting from the derived scalings of the

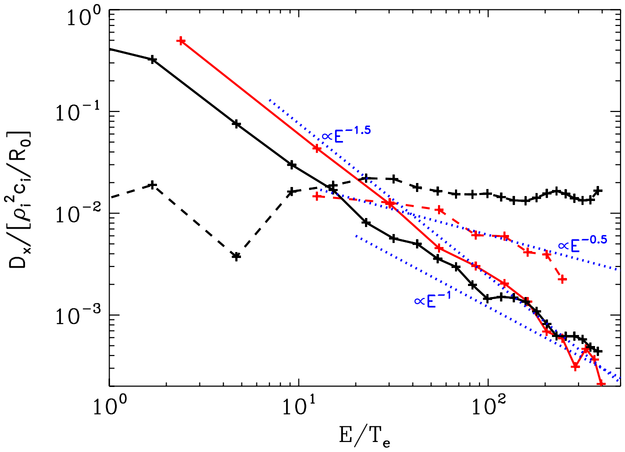

FIG. 2 (color online). Electrostatic (solid lines) and magnetic (dashed lines) particle diffusivities of fast ions for large (black lines) and small (red lines) pitch angles as obtained from GENE simulations. The results agree well with the theoretical expectations (see text) which are shown for comparison.

energetic ion diffusivities. This is done next, focusing on beam ions. Noting that for $E/T_e \gg 1$, the pitch angle dependence of the slowing-down distribution can be integrated out and the low-energy corrections due to the critical velocity can be ignored, one can write $\Gamma_p \propto \int D(E) f_0(E) E^{1/2} dE$, $\Gamma_m \propto \int D(E) f_0(E) E dE$, and $\Gamma_e \propto \int D(E) f_0(E) E^{3/2} dE$ for the particle, momentum, and heat fluxes, respectively, with $f_0(E) \propto E^{-3/2}$. Consequently, the electrostatic heat flux displays $\ln(E_b/T_e)$ corrections due to the $1/E$ tail (while the electrostatic particle and momentum fluxes are not affected much), and the magnetic fluxes scale like $\Gamma_p \propto \ln(E_b/T_e)$, $\Gamma_m \propto \sqrt{E_b/T_e}$, and $\Gamma_e \propto E_b/T_e$, where E_b is the beam energy. Thus, although the turbulent current diffusion due to electrostatic turbulence is likely to be too small to affect the experiments (unless the beam density is quite large), magnetic fluctuations of sufficiently large amplitude could play a role in this context.

To summarize, employing a combination of theoretical considerations and gyrokinetic simulations with GENE, we have investigated the cross-field transport of energetic ions induced by the ambient turbulence—addressing, in particular, decorrelation mechanisms and resulting scalings with the particle energy E. Both approaches agree with each other qualitatively and even semiquantitatively. In particular, they show that the electrostatic transport of beam-type ions (with a pitch angle close to unity) exhibits a slow $1/E$ decay at high energies, while the respective magnetic transport is even independent of E. These findings have their origin in the violation of the orbit averaging condition and can be explained in terms of a perpendicular decorrelation mechanism described in this Letter. The resulting overall transport coefficients exhibit substantial corrections, and consequently, beam ion diffusion by turbulent magnetic fluctuations should be considered a candidate for explaining the experimentally observed fast radial broadening of the current profile driven by off-axis neutral beam injection in the absence of any measurable magnetohydrodynamic activity. Beyond that, in light of the fact that future D-T based plasma experiments like ITER will have a significant population of fast ions, it would certainly be interesting to test these predictions in small-scale laboratory experiments which allow for a more detailed analysis of energetic ion dynamics. Applications of the present work to astrophysics will be addressed elsewhere.

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