Conoscopic holography

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A new method for recording holograms using incoherent light is described. The method is based on optical propagation through birefringent crystals. Optical methods for the reconstruction of such a hologram are also presented.

It was recognized early on that holograms, which are naturally implemented by utilizing coherent light, can also be formed with incoherent or partially coherent light. In ordinary holography each object point, i.e., the intensity and the lateral and longitudinal locations of the object, is interferometrically recorded as a Fresnel zone plate (FZP) and stored on photographic film. In any system (including incoherent coded aperture imaging systems[1,2]) in which each object point is coded as a FZP, the image can be reconstructed by optical diffraction and can be viewed as an hologram. Several schemes have been proposed for incoherent recording, such as shadow casting[1] and interferometric systems[3–6] including an amplitude-splitting interferometer based on a double-focus birefringent lens.[6] All incoherent holographic schemes are relatively cumbersome or require complicated mechanical or electronic apparatus. The low signal-to-bias ratio[7] that is inherent in incoherent holography is also a severe limitation.

In this Letter we introduce a simple method for incoherent holographic recording based on light propagation in anisotropic crystals. Light propagation in uniaxial crystals is a phenomenon closely related to interference[2] (an analyzer produces the interference between the ordinary and the extraordinary beams of a previously polarized light beam). The angular dependence of the extraordinary index of refraction causes an angle-dependent change in optical path. This effect is responsible for the formation of the familiar conoscopic figures.[9] The same effect can be used in place of geometrical interference as a basis for incoherent holography. The most advantageous feature of conoscopic incoherent holography is the fact that the two interfering light beams have identical geometrical paths. Furthermore, the natural space invariance of the system permits the equalization of the optical paths of the two beams over the full image frame, and as a consequence the spatial coherence of the source imposes no limitations on the sizes of the object and the hologram. The availability of the entire crystal-optics framework, a well-known and -developed area, can make this method practical in the near future. Finally, it is also possible to make the system achromatic.[10]

The optical arrangement that is used for conoscopic hologram recording is shown in Fig. 1. Let a light wave with wavelength λ propagate in a birefrigent crystal at an angle ϑ relative to the optical axis. If the ordinary and the extraordinary indices of refraction are n_O and n_E, respectively, and their difference is $\Delta n = n_E - n_O$, then two orthogonally polarized waves will propagate, one with an index of refraction n_O (the ordinary ray), and the second, the extraordinary ray, with an index of refraction $n_E(\vartheta)$ given approximately by[9]

$$n_E(\vartheta) \simeq n_O + \Delta n \sin^2 \vartheta. \tag{1}$$

The phase retardation between the extraordinary and the ordinary waves is given by

$$\Delta\varphi = (2\pi/\lambda)(L/\cos\vartheta)\Delta n \sin^2\vartheta \simeq (2\pi L/\lambda)\Delta n \vartheta^2, \quad \vartheta \ll 1. \tag{2}$$

(For simplicity, we use the convention that the light outside the crystal propagates in a medium of refractive index n_O, and we scale the distances accordingly.) Let a circularly polarized point source P, located at object coordinates (x, y, z), radiate light intensity $I(P)$ in a cone of half-angle ϑ_0. The origin of the z axis is taken at the film plate.

The light intensity detected at the film plate through an analyzer at a point R with coordinates $(x', y', 0)$ that is due to the light originating from point P is given by

$$I(R,P) = I(P)\cos^2[\Delta\varphi(P,R)/2] = 0.5I(P) + 0.5I(P)\cos[\Delta\varphi(P,R)]. \tag{3}$$

The total intensity $I(R)$ at point R is given by

$$I(R) = \int_v I(R,P)\mathrm{d}P = \int_v I(P)T(R,P)\mathrm{d}P, \tag{4}$$

where v contains all the points P in the object *volume* from which light can reach R and $T(R, P)$ is the impulse response of the system. Using Eq. (2) and the paraxial approximation

$$\vartheta^2 \simeq [(x - x')^2 + (y - y')^2]/z^2, \tag{5}$$

we obtain

$$I(R,P) = I(P)T(P,R) = I(P) \times (0.5 + 0.5\cos\{2\pi L \Delta n[(x - x')^2 + (y - y')^2]/z^2\lambda\}). \tag{6}$$

This is a FZP, centered at $x' = x$, $y' = y$, plus a constant bias. If we view the formation of the conoscopic figure as interference between the ordinary and extraordinary waves, the image and reference beams have identical geometrical paths but different optical path lengths because of the angular dependence of the refractive index of the extraordinary wave. Thus a hologram of the extraordinary wave is recorded by using the ordinary wave as a reference beam.

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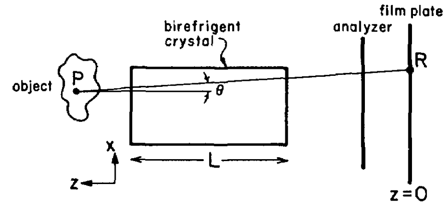


Fig. 1. Propagation of light in a birefringent crystal.

The number of fringes F in the FZP is

$$F = L\Delta n \sin^2 \vartheta_0/\lambda \simeq (L\Delta n A^2)/\lambda z_0^2, \qquad (7)$$

where $2A$ is the size of the FZP and z_0 is the mean distance from the object to the film. We now rewrite Eq. (6) as

$$I(R, P) = I(P)(0.5 + 0.5 \cos\{2\pi[(x - x')^2 + (y - y')^2]/\lambda_{EQ}z\}), \qquad (8)$$

with the following definition:

$$\lambda_{EQ} = \lambda z/\Delta n L \simeq A^2/F z_0. \qquad (9)$$

For $z = L$, λ_{EQ} is the wavelength in a medium of index of refraction Δn. The dependence of the equivalent wavelength on the distance is due to the fact that the anisotropic medium is finite in the z direction.

Conoscopic holograms of two elementary objects recorded in the arrangement of Fig. 1, using a calcite crystal with dimensions 20 mm × 20 mm × 20 mm (x–y–z) and a sodium lamp, are shown in Fig. 2. The hologram of a single point source, shown in Fig. 2(a), is a single FZP. The hologram of two points, Fig. 2(b), shows vertical interference fringes superimposed upon the zone plates.

One method for reconstructing a recorded conoscopic hologram optically is to place the hologram at the plane $z = 0$ in the system of Fig. 1, illuminating it from the right and passing the light back through the system. The light intensity at a point P″ at the left-hand side of the crystal viewed through an analyzer is

$$I(P'') = \int_{s'} I(R)T(P'', R)\mathrm{d}S'. \qquad (10)$$

The surface S′ includes all the points R on the film from which light rays can reach the point P″, or, reversing the optical system, this is the surface reached by light from P″ when the hologram is constructed. Using Eq. (4),

$$I(P'') = \int_{s'}\int_{v} I(P)T(P, R)T(P'', R)\mathrm{d}V\mathrm{d}S'. \qquad (11)$$

From the orthogonality of Fresnel functions for infinite V and S′ we will have $I(P'') = I(P)$, and we will exactly reconstruct the original image. The finite extent of S′ will determine the resolution. The lateral resolution of a FZP is equal to the diffraction limit of a lens with the same aperture.[11] The lateral resolution is

$$\Delta x = (1.22\lambda_{EQ}z_0)/2A = 1.22\lambda z_0^2/\Delta n A L = (1.22A^2 z_0)/2AFz_0 = 0.61A/F. \qquad (12)$$

Because of the stronger z dependence, the longitudinal resolution will be twice the resolution of a coherent FZP, or

$$\Delta z = 0.5\Delta x(z_0/A) = 0.61(z_0/2F). \qquad (13)$$

The required film resolution l_1 is the same as for a point-reference Fourier process[11] and is given by (a factor of 2 is needed because of the stronger z dependence)

$$l_1 = 2 \times 61/\Delta x = (2F/A). \qquad (14)$$

If we define m, the ratio of the size of the film G to the FZP size A, the space–bandwidth product (SBP) will be

$$\mathrm{SBP} = 1.64 l_1 G = 1.64(2F/A)mA = 3.38\, mF. \qquad (15)$$

The wavelength resolution (the needed monochromaticity) will be

$$\Delta\lambda/\lambda = (1/F). \qquad (16)$$

Another method for reconstructing a conoscopic hologram is based on diffraction. The hologram is recorded on a photographic film as described above and is illuminated by a coherent spherical wave as shown in Fig. 3. The amplitude of the illumination field at plane $z = 0$ is given by

$$a_R = \exp(j\varphi_R) = \exp[j(2\pi/\lambda)(x'^2 + y'^2)/2z_M]. \qquad (17)$$

We define μ, the wavelength ratio at $z = z_0$, by

$$\mu = \lambda_{EQ}(z_0)/\lambda. \qquad (18)$$

For a hologram of a single point, the complex optical wave front A_G immediately following the FZP of Eq. (8) at the record plane $z = 0$ is

$$A_G(R) = \exp(j\varphi_R) \int_s I(P)T(P, R)\mathrm{d}S = \int_s I(P)\exp(j\varphi)\mathrm{d}S + \text{bias}, \qquad (19)$$

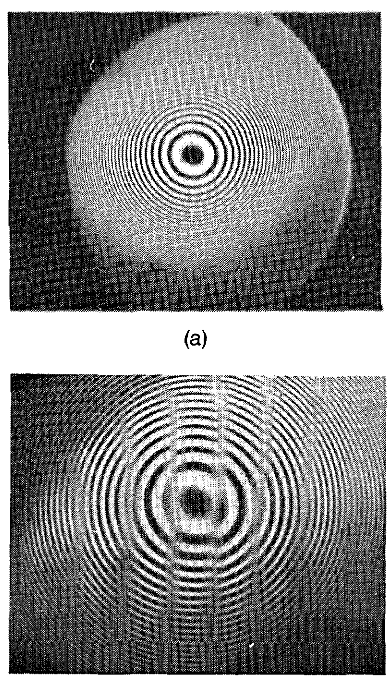


Fig. 2. Experimental demonstration of the formation of conoscopic holograms. (a) Hologram of a single point source. (b) Hologram of two point sources.

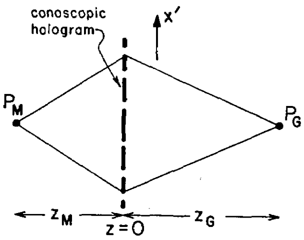
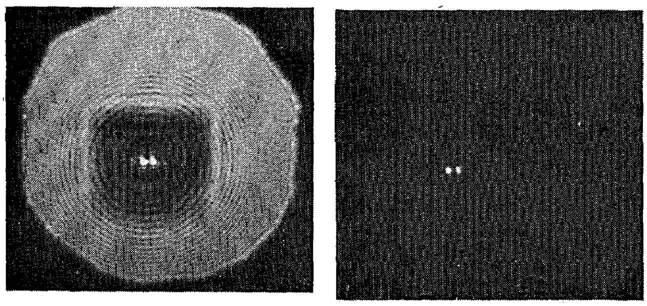


Fig. 3. Coherent reconstruction.



(a) (b)

Fig. 4. Experimental demonstration of coherent reconstruction. (a) Reconstruction of the hologram shown in Fig. 2(b). (b) Reconstruction of the same hologram through a high-pass spatial filter for contrast improvement.

where φ is given by

$$\begin{aligned}\varphi &= (2\pi/\lambda)\{[(x'-x)^2 + (y'-y)^2]z_0/z^2\mu \\ &\quad + (x'^2 + y'^2)/2z_M\}. \\ &= (x'^2 + y'^2)/2z_G + (x'x_G + y'y_G)/z_G \\ &\quad + \text{phase term} \qquad (20)\end{aligned}$$

Because of the focusing property of a zone plate, an image point P_G is formed at coordinates (x_G, y_G, z_G) given by

$$\begin{aligned}&(2z_G)^{-1} = (\mu z^2/z_0)^{-1} + (2z_M)^{-1}, \\ &x_G = x(z_G z_0/\mu z^2), \qquad y_G = y(z_G z_0/\mu z^2). \qquad (21)\end{aligned}$$

Although the image is reconstructed (each object point is focused), the z dependence of the image-point coordinates will generally introduce distortions, similar to those obtained when acoustic or microwave holograms are optically reconstructed. For a relatively small depth of field the distortion is negligible. Setting $z_M = \mu z_0/2$, we obtain

$$(2z_G)^{-1} = (\mu z^2/z_0)^{-1} + (\mu z_0)^{-1} = \mu(z_0/z^2 + z_0). \qquad (22)$$

For $\Delta z \ll z_0$ we have $z_0/z^2 + 1/z_0 \doteq 2/z$, and Eq. (22) becomes

$$z_G = 2\mu z, \qquad x_G = 2\mu x, \qquad y_G = 2\mu y. \qquad (24)$$

For $\Delta z = 0.1\, z_0$, the longitudinal and lateral distortions will be 5 and 10%, respectively.

The coherent reconstruction of the conoscopic hologram shown in Fig. 2(b) was formed with a He–Ne laser in the optical arrangement shown in Fig. 3. The result is shown in Fig. 4(a). A clear reconstruction of the two points that comprise the object is evident. Also visible in Fig. 4(a) is the background light that is due to the undiffracted light transmitted through the hologram. The ratio of the intensity of a reconstructed object point to the intensity of the background is on the average equal to the ratio of the space–bandwidth products of the recorded FZP and the original object. For the example shown, the space–bandwidth product of the object is only 2, whereas the space–bandwidth product of the FZP used was approximately 100; thus very high contrast can be obtained, as shown in Fig. 4(a). However, for extended objects the contrast is drastically reduced. In this case, the undiffracted light, which contains only small spatial frequencies, can be spatially filtered in the Fourier plane of a lens that is included in the reconstruction setup. The photograph shown in Fig. 4(b) is the reconstruction of the same hologram with a dc block included, showing a dramatic improvement of the contrast compared with the reconstruction in Fig. 4(a). This bias-removal technique is applicable only to coherent reconstruction. The bias that is also present in the conoscopic reconstruction cannot be removed by spatial filtering, but since it is uniform and diffuse, it may be less objectionable to the observer of the reconstruction.

A basic difference between conoscopic and conventional holography is the fact that in conoscopic holography the signal and reference beams have the same geometrical paths but different optical pathlengths. The optical path length in a homogeneous medium is defined as the product of the index of refraction n and the geometrical length L. All other holographic systems detect changes in L, whereas this system is based on detection of changes in n. An extra flexibility is provided by the relative angle between the optical axis of the crystal and the direction of the optical axis of the system, permitting different functional dependences. The only constraint is that the reference and signal optical paths must be equal along the direction of optical axis of the system, providing several different useful two-crystal configurations.

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References

1. L. Mertz, *Transformations in Optics* (Wiley, New York, 1965).
2. G. L. Rogers, *Noncoherent Optical Processing* (Wiley, New York, 1977).
3. G. Cochran, J. Opt. Soc. Am. **56,** 1513 (1966).
4. H. R. Worthington, J. Opt. Soc. Am. **56,** 1397 (1966).
5. G. W. Stroke and R. C. Restrick III, Appl. Phys. Lett. **7,** 229 (1965).
6. A. W. Lohmann, J. Opt. Soc. Am. **55,** 1555 (1965).
7. R. J. Collier, C. B. Burckhardt, and L. H. Lin, *Optical Holography* (Academic, New York, 1971), Chap. 20.
8. R. W. Ditchburn, *Light* (Wiley Interscience, New York, 1963), Chap. XVI.
9. M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1975), Chap. XIV.
10. E. N. Leith and B. J. Chang, Appl. Opt. **12,** 1957 (1973).
11. J. B. De Velis and G. O. Reynolds, *Theory and Applications of Holography* (Addison-Wesley, Reading, Mass., 1967), Chap. 5.