IMPROVING MATRIX ESTIMATION PERTAINING TO DETAILED TRAFFIC INFORMATION AND SOPHISTICATED TRAFFIC STATE

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ABSTRACT

Technical innovation and extensive application of adaptive signal control at intersections have made turning flow information that provide more precise constraints for Origin-Destination matrix (O-D matrix) estimation easily available in great quantity and more accurate than ever. However, the influence of turning flow and duplication of information on the existing matrix estimation models and on the accuracy of O-D matrix estimation has not been broadly investigated. Also, traffic phenomenon in networks becomes complicated and difficult to explain with the increase in number of vehicles, variety of daily activities and sophisticated travel behaviors. As such, general congested traffic state as well as diverse travelers' perception about travel time should be taken into consideration in O-D matrix estimation models. In this paper, the influence of applying finer and duplicated flow information as well as route choice proportion estimates on the performance of the Information minimization (IM) and the modified IM models were examined. It has shown that duplicate information has adverse effect on the accuracy of matrix estimation, whereas additional turning flow information can improve estimation accuracy. Based on the examination results a methodology using the IM model, the stochastic user equilibrium (SUE) assignment and the information screening process, was proposed to optimize the goodness of estimation and enhance the IM model to deal with the traffic situation more realistically. The respective convergence and required computation time were also examined. Furthermore, an empirical route choice study was conducted in order to help determining the size of a route set used in the SUE assignment model.

1. INTRODUCTION

Considerable amount of work has been conducted to obtain a precise origin-destination (O-D) matrix with the application of different traffic information over the years. The main advantage of using traffic information instead of conducting household surveys is time- and cost saving. In addition, O-D matrices could be updated for any desired planning period once the corresponding traffic information is available. The respective models for such O-D matrix estimation mainly include the Entropy Maximization (EM) model, the Least Square Error (LSE) model, the Information Minimization (IM) model, the Bayesian model, the Generalized Least Squares (GLS) model and Path Flow Estimator (PFE) *(1-6)*. Based on the applied theory each model has its characteristics and application conditions. Generally speaking, the EM model will be applied only when the number of total trips is given and consistent link flows exists. The LSE model considers only the difference between the estimated and the measured traffic flows. Existing matrix information will not be taken into account. In comparison with that, matrix information is used in the IM model and traffic counts are used as constraints. The consistent link flows in the EM model is also required in the IM model. This precondition can be however satisfied by applying the method of deriving consistent flows *(7)*. Based on statistical assumptions regarding variance and co-variance, the GLS model considers both the differences between the estimated and the measured flows and between the estimated and the target matrix. The consistence of link flows is not the premise in the GLS model. The Bayesian model is a special case of the GLS model. Except the PFE, O-D estimation is generally a bi-level O-D estimation process, i.e. the upper level is an O-D estimation problem and the lower level is a traffic assignment problem. Through solving the latter, route choice proportions can be determined and used in the O-D estimation problem. The PFE is especially suitable for SUE traffic state according to its objective function. Traffic flows in the PFE are also required to be consistent.

With technical innovation, developed estimation algorithms and data fusion methodologies *(8-11)*, turning flow information, providing more precise constraints for O-D estimation, can be obtained with greater ease and accuracy. However, the influence of turning flow and duplicated information on the existing models and on the accuracy of O-D estimation has not been extensively investigated. Moreover, traffic state with less traffic demand is easier to describe by using an adequate assignment model, such as incremental assignment, where user equilibrium (UE) state is not considered. In contrast, traffic state during rush hours becomes complicated and difficult to explain with regard to the interaction among the increasing number of vehicles, the variety of daily activities and sophisticated travel behaviors. Thus, congested traffic state with user equilibrium should be taken into consideration in O-D estimation models as well as travelers' diverse perception about travel time. As such, how to decide the size of route choice set also becomes a critical issue when conducting matrix estimation, since it decides the quality of both assignment result and O-D estimates. The effect and the disadvantage of introducing turning flow information and redundant information on the accuracy of O-D estimates were investigated in this paper. A methodology to obtain more accurate O-D estimates was proposed. A method to help determining the size of a route set was also developed.

The structure of this paper is organized in the following way. First, an introduction of the adopted O-D matrix estimation model and the respective characteristics is given. Follow by the examination of influence of flow and route choice information on the accuracy of matrix estimation. Then continue with the explanation of the difficulty of considering a UE state in the applied IM model. The proposed methodology and the computational results are presented in Section 3 and Section 4 and finally, the conclusions are presented.

2. IM AND MODIFIED IM MODELS

2.1 Model specification

The IM model has been developed by Van Zuylen in 1980 *(1)* and based on the information minimization theory, proposed by Brillouin *(12)*. The mathematical form of this model is defined as:

$$T_{ij} = t_{ij} \cdot \prod_a x_a^{p_{ij}^a / g_{ij}} \tag{1}$$

$$g_{ij} = \sum_a p_{ij}^a \tag{1-1}$$

$$\sum_{ij} T_{ij} \cdot p_{ij}^a = v_a^{obs} \tag{1-2}$$

Where

T_{ij} : estimated O-D flow from origin i to destination j

t_{ij} : historical O-D flow or a prior guess of the O-D flow from origin i to destination j

p_{ij}^a : route choice proportion of O-D flow from origin i to destination j passing link a

$x_a^{n+1} = x_a^n \times \frac{v_a^{obs}}{v_a^s}$, adjustment factor for link a in iteration $n+1$ by using the ratio of the observed traffic flow v_a^{obs} and estimated traffic flows v_a^s for link a in iteration n.

With the application of this model, the most likely O-D flows can be estimated through an iterative process until all flow constraints are satisfied, i.e. estimated traffic flows are equal to the respective detected traffic flows. However, it was found out that the IM model will not perform well and respective estimation results will not be stable if the route choice proportions are not completely known and partially duplicated. It is caused by the overweight effect of x_a to the power of p_{ij}^a/g_{ij}. In order to overcome this problem, Van Zuylen *(13)* made a modification by adding a parameter x_0 to adjust the difference between the total number of historical trips and that of the actual trips. The form of the modified IM model is shown in Equation (2). These two parameters, x_0 and x_a, and the most likely O-D matrix can then be solved with the satisfaction of the link flow constraints in an iterative process.

$$T_{ij} = t_{ij} \cdot x_0 \prod_a x_a^{p_{ij}^a} \tag{2}$$

$$\sum_{ij} T_{ij} \cdot p_{ij}^a = v_a^{obs} \tag{1-2}$$

$\mathrm{x}_0 = \frac{1}{\sum_{ij} t_{ij}} \cdot \sum_{ij} T_{ij}$ and $x_0 = \frac{1}{\sum_a \sum_{ij} p_{ij}^a t_{ij}} \cdot \sum_a v_a^{obs}$ are used as the initial value based on available link flows.

2.2 Influence of turning flow and estimated route choice information

Since it is not easy to obtain exact route choice proportions on links, they are usually estimated by assigning a historical matrix onto the investigated network. With estimated route choice proportions on all links, the abovementioned overweight problem in the IM model can be resolved.

With the estimated p_{ij}^a and the turning flow information, we examined the performance of the IM and the modified IM models with a small test network shown in Figure 1. The traffic count information is generated manually by proportionally assigning the assumed real matrix. Based on these traffic counts, the respective route choice proportions will be estimated with the application of the incremental assignment. To focus on the influence of flow information on the O-D estimation, a unit matrix was applied as a start matrix solution.

Synthetic real matrix:

o \ d	1	2	3	4	sum
1	0	100	200	150	450
2	50	0	100	150	300
3	200	150	0	150	300
4	50	100	50	0	200
sum	300	350	350	450	1450

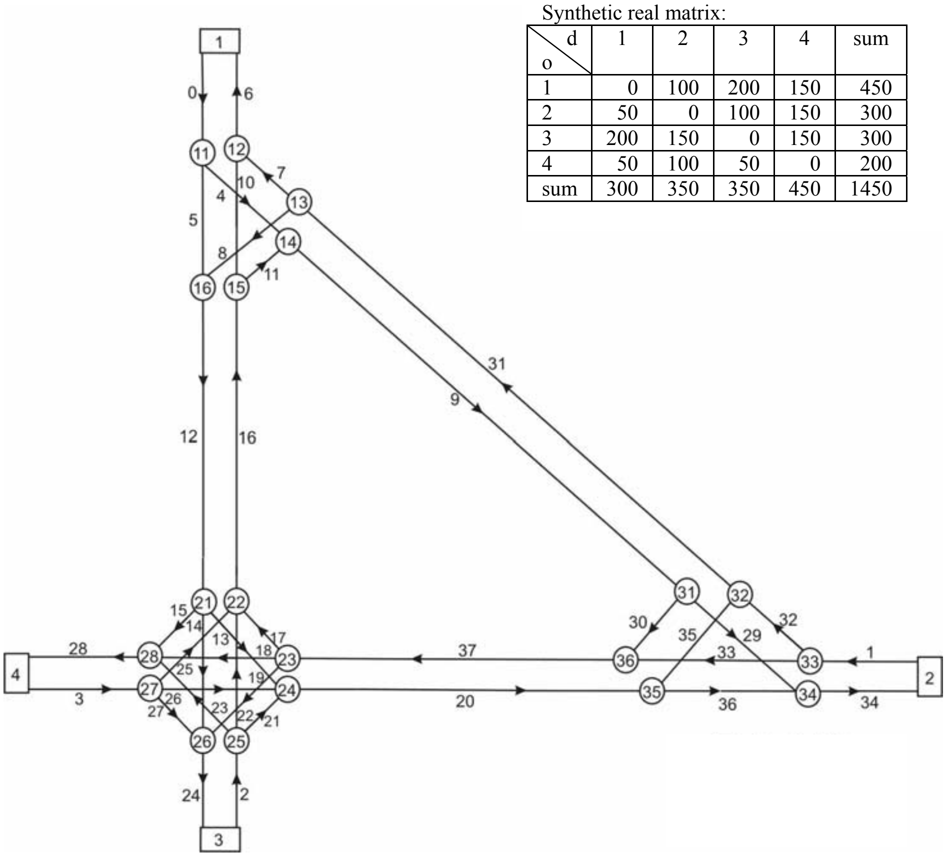


FIGURE 1 Layout of small test network.

To measure the estimation accuracy, three indicators, i.e. correlation coefficient (R), root mean squared error (RMSE) and weighted RMSE (WRMSE), are chosen as shown below.

- Correlation coefficient (R)

$$R = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J}(T_{ij} - \bar{T}) \cdot (T_{ij}^{r} - \bar{T}^{r})}{\sqrt{\sum_{i=1}^{I}\sum_{j=1}^{J}(T_{ij} - \bar{T})^2 \cdot \sum_{i=1}^{I}\sum_{j=1}^{J}(T_{ij}^{r} - \bar{T}^{r})^2}} \tag{3}$$

Where

$\bar{T}$: mean of the estimated O-D flows

$\bar{T}^{r}$: mean of the assumed real O-D flows

$\bar{T}_{ij}^{r}$: assumed real O-D flow from origin i to destination j

- Root mean squared error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{I}\sum_{j=1}^{J}(T_{ij} - T_{ij}^{r})^2}{TN}} \tag{4}$$

Where

TN : the number of O-D pairs with non-zero traffic demand

- Weighted RMSE (WRMSE)

$$WRMSE = \sqrt{\frac{\sum_{i=1}^{I}\sum_{j=1}^{J}T_{ij}^{r} \cdot (\frac{T_{ij} - T_{ij}^{r}}{T_{ij}^{r}})^2}{\sum_{i=1}^{I}\sum_{j=1}^{J}T_{ij}^{r}}} \tag{5}$$

Table 1 shows that the O-D estimates based on the modified IM model in all scenarios are worse than those from the IM model, except when all route choice proportions are precise and available. Scenario S2 and S4 show that the different weightings of each link parameter x_a, which is to the power of p_{ij}^{a} in the modified IM model and to the power of p_{ij}^{a} / g_{ij} in the IM model, result in different estimation accuracies, when p_{ij}^{a} is accurate and turning flow information is lacking. The IM model will deliver more accurate O-D estimates than the modified IM model.

The problem of information redundancy appears when applying estimated route choice proportions on both models (see Scenario S1 and S3 with estimated p_{ij}^{a}). Under this circumstance, the IM model delivers better O-D estimates than the modified IM model. Furthermore, it is noted that IM model can provide better O-D estimates when the duplicate information among links does not exist, as compared to the modified IM model, where it requires relative accurate p_{ij}^a information. Due to the different importance among link flows and turning flows, the duplicated information cannot be removed by just applying numerical methods, such as the Gauss method. A data screening process for keeping only valuable information is thus necessary.

In addition, it also proved that turning flow information is more important and valuable than the link flow information (see Scenario S2 and S3 with exact p_{ij}^a). So, the estimation accuracy can be significantly improved with additional turning flows information when comparing the results in Scenario S2 and S4 either with exact p_{ij}^a or with estimated p_{ij}^a with use of the IM model.

TABLE 1 Performance of the IM and the Modified IM Models with Small Test Network

Scenario	IM model						modified IM model					
	exact p_{ij}^a			estimated p_{ij}^a			exact p_{ij}^a			estimated p_{ij}^a		
	R	RMSE	WRMSE	R	RMSE	WRMSE	R	RMSE	WRMSE	R	RMSE	WRMSE
S1	1.000	0.000	0.000	0.999	3.456	0.032	1.000	0.000	0.000	0.998	3.759	0.036
S2	0.919	20.488	0.179	0.912	21.813	0.192	0.916	20.842	0.174	0.897	23.710	0.194
S3	1.000	0.000	0.000	0.992	8.117	0.082	1.000	0.000	0.000	0.988	8.764	0.087
S4	0.960	14.538	0.134	0.942	17.755	0.160	0.950	16.223	0.144	0.900	23.357	0.191

S1: all flows detected; S2: only link flows detected; S3: only turning flows detected; S4: S2 with detected turning flow for link 18

2.3 Difficulty coping with User Equilibrium (UE)

In order to obtain required route choice proportions, the prerequisite for applying the IM model is that the investigated traffic state should be un-congested, i.e. route choice proportions are independent of O-D matrix. Nevertheless, in the real world, congested traffic situation tends to appear regularly with increase in traffic demand hence such prerequisite becomes unreasonable. In order to handle the real traffic situation, some studies have been conducted by applying IM and EM models to congested traffic states and the Wardrop's UE principle was used as an additional constraint *(14-15)*. Research results indicate that the existence of a feasible solution cannot be guaranteed when applying either the IM or the EM model with the UE-principle based constraint because the investigated traffic network may not be in a perfect UE condition.

3. METHODOLOGY

3.1 Framework

With reference to the above mentioned issues, a methodology for optimizing O-D estimation for realistic route choices is proposed and shown in Figure 2. The proposed methodology corresponds to a bi-level estimation process. The IM model and the SUE assignment model are taken into consideration in the estimation process. A historical matrix, network data and the travelers' behaviors about route choice are given as input. Based on the given data related to route choice behaviors, the number of k efficient paths will be determined by a given average acceptable travel time deviation. The proposed method to determine the respective travel time deviation is based on empirical data und will be explained in Section 3.2. After that, the SUE assignment process will be performed by applying the c-logit model, proposed by Cascetta et al. *(16)*, in order to avoid overestimating the flow on overlapping routes. The more realistic and accurate route choice proportions can be calculated based on the assignment result.

In addition, an information screening process is proposed and carried out. After eliminating redundant information, valuable flow information will be maintained for the next process called "matrix estimation". The estimated matrix will be used as input in the k-path enumeration and the SUE assignment processes until the defined convergence criterion regarding the stability of the estimated matrix with a SUE state is reached.

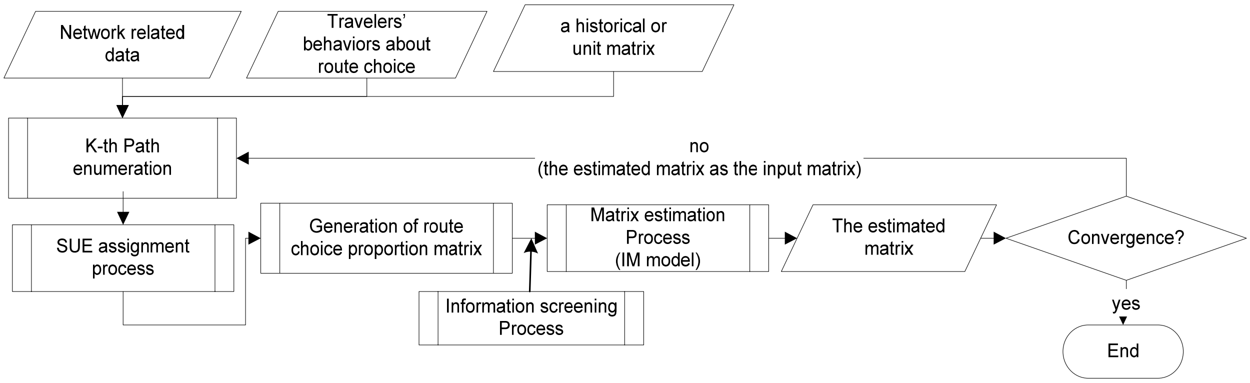


FIGURE 2 Framework of the proposed methodology.

3.2 Determination of path sets

The number of alternative paths connecting a typical O-D pair in a fully developed urban network is extremely large. Consequently, it is very time-consuming to enumerate all these paths for all O-D pairs. In fact, it is not necessary to do so, since used paths are relatively limited as compared to available paths. The decision-making process of route choice is affected by both quantitative and non-quantitative factors, e.g. travel time, travel distance, road safety, comfort, environmental security and individual values. A significant amount of studies have been conducted to analyze the generation of path choice sets subject to behavioral rules. It was found that although idiosyncratic situations such as cultural norms and values may be different and result in different travel patterns *(17-18)*, the main universal decision parameter is travel time and human cognitive mechanism of travel decision-making. Therefore an upper bound with regard to acceptable travel time should exist. The above mentioned decision factors result in the deviation between the travel time of a used route and that of the most efficient route, i.e. the route with the minimal travel time. Such travel time deviation and the respective upper bound can be identified by conducting empirical studies.

Based on the empirical data from the work of Borgmann *(19)* the upper bound of the acceptable travel time can be identified. The empirical data regarding home-work based route choice and possible route alternatives were collected by interviewing 53 staffs at the Leibniz University of Hanover, Germany. The results show that the number of the possible route alternatives is principally 3-4 routes, which is consistent to the research result from Bovy and Stern *(17)*. It also shows that the travel time deviation among considered route alternatives, the shortest route and the most efficient route depends on the network structure and is quite significant (see Figure 3 as example).

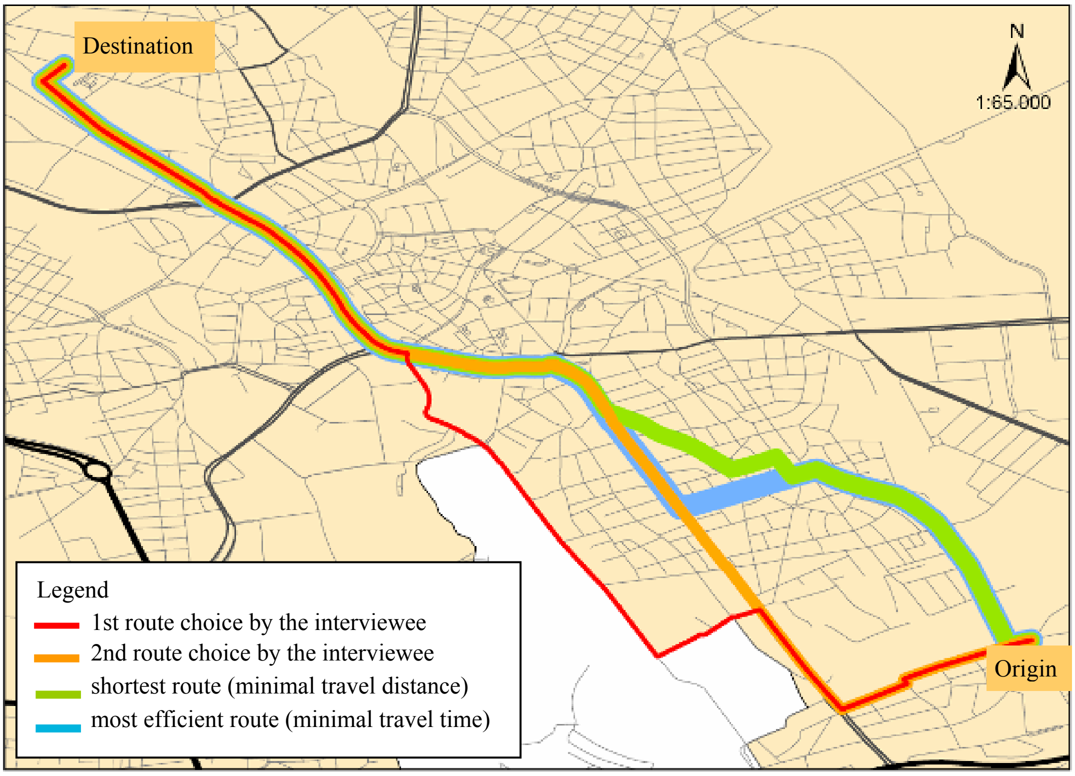


FIGURE 3 Example of the considered route alternatives by an interviewee.

To calculate travel time deviations, the travel times of the respective shortest and the most efficient routes as well as route alternatives are calculated according to the link speed and distance data in the electronic map City-Select.Europe-v7 *(20)*, which is produced by the electronic map software MapSource *(21)*. After the exclusion of extreme values, the total number of effective interviews is 50 and the maximal number of route alternatives among test persons is 3. The result in Figure 4 indicates the travel time deviation between the first chosen route and the most efficient route is less than 5%, 10% and 20% for 60%, 72% and 87% test persons respectively. For the second chosen route, the corresponding travel time deviation is less than 5% for 37% test persons, whereas the respective travel time deviation for 83% test persons is less than 20%. For 70% test persons, the travel time deviation between the third chosen route and the efficient route is less than 25%. And for 95% test persons the travel time deviation between their possible route alternatives and their most efficient routes is less than 30%. As such, 30% is suggested as the upper bound of travel time deviation for determining the respective k shortest paths. Based on this derived upper bound of the acceptable travel time deviation, path sets of the O-D pairs, whose traffic demand exists, will be updated with reference to estimated link travel times at each iteration in the applied assignment model.

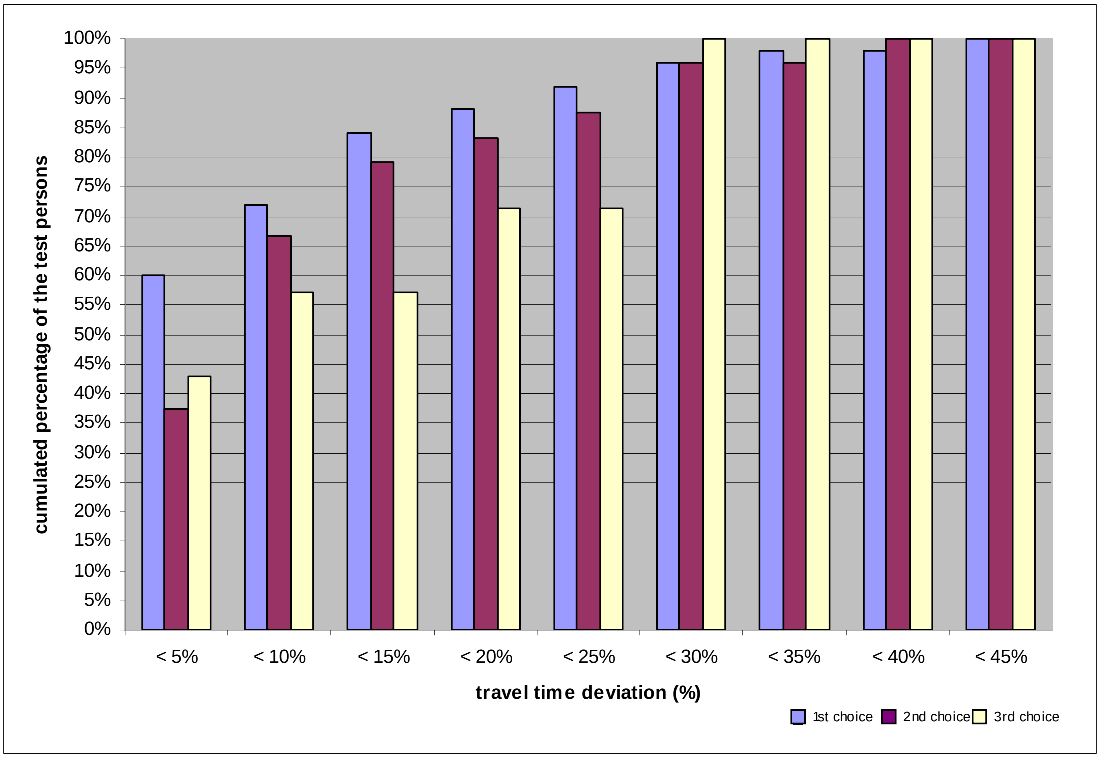
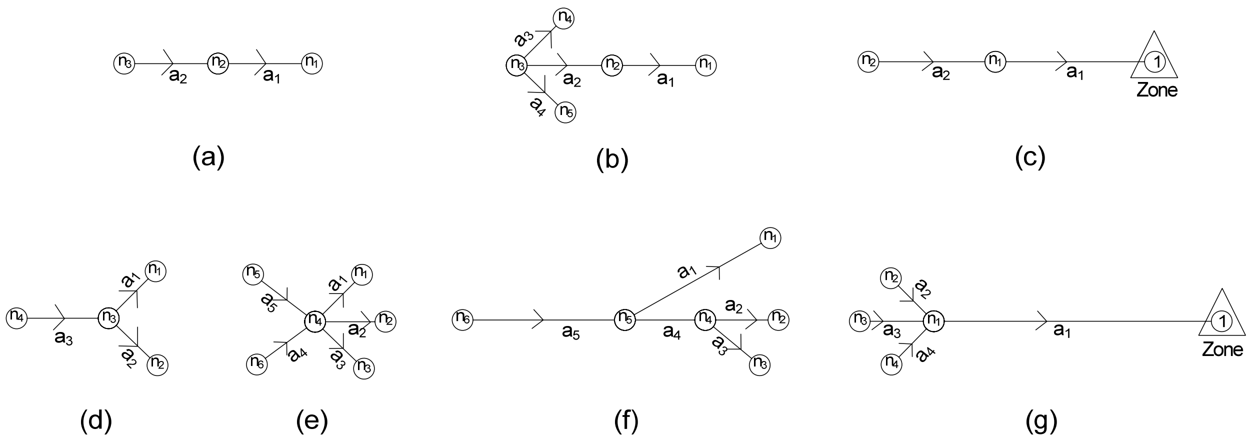


FIGURE 4 Travel time deviations between route alternatives and the most efficient routes.

3.3 Information screening process

Based on the study *(22)* conducted, valuable information obtained from the information screening process is defined as available information, which is relatively valuable compared to other available information. The less valuable information, which is normally the duplicates, will be removed. Two link groups are classified, i.e. "dummy links" and "turning links with their upstream/downstream links". Seven structure types are defined according to network and intersection structures:

- dummy links:
  - a link connecting only with another link (at road segments) (Figure 5(a))
  - a turning link connecting only with another turning link (at intersections) (Figure 5(b))
  - a link connecting only to another link, connected with a destination zone (at road segments) (Figure 5(c))
- turning links with their upstream/downstream links (at intersections):
  - an upstream link with two or more outgoing turning links (Figure 5(d))
  - two upstream links with multiple outgoing turning links (Figure 5(e))
  - one upstream link with turning links and additional left or right links (Figure 5(f))
  - two or more incoming turning links with one downstream link connecting to a destination zone (Figure 5(g))



FIGRUE 5 Example of the considered network structure types.

The corresponding elimination rules are also developed. The main principle is that each node with its connecting links will be first checked to identify the respective structure type. For each structure type, turning flow information and respective route choice proportions remained as first priority, when the respective flow information is available. The duplicate information on upstream links or turning links will then be deducted. To avoid the loss of information, the link group “dummy links” will be checked first then followed by the link group “turning links and their upstream links”. Finally, the remained valuable flow and route choice information are applied in the IM model according to Equation (1-2).

4. TEST RESULTS

Besides the small test network shown in Figure 1, a larger test network, shown in Figure 6, is used to verify the performance of the proposed methodology. This test network “List” is a part of the network in List in Hanover, Germany. It contains 22 origins/destinations, about 400 links as well as 58000 vehicles. Among the links there are 79 links equipped with detectors represented by dashed lines in Figure 6.

For purpose of comparing programs based on the same methodology but used the modified IM model without the information screening progress, were also implemented as a reference.

The detected flow data were generated based on the assumed real matrix, which was used for evaluating the goodness of the O-D estimation. With the same consideration in the test as with the small test network, a unit matrix was applied as a start matrix solution in the proposed estimation process, so that the influence of flow information, i.e. redundant information and turning flows, on the O-D estimation can be more precisely examined.

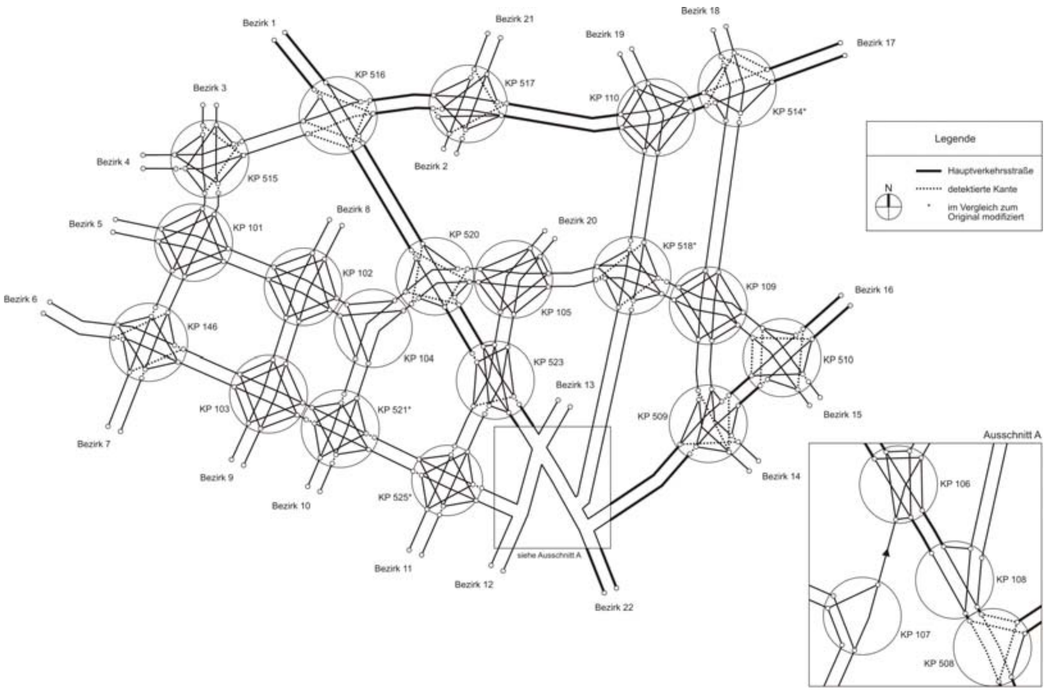


FIGURE 6 Layout of test network "List".

The results in Table 2 show that the proposed methodology produces better and more robust results, even when redundant information existed. It also shows that redundant information may results in worse O-D estimates. Furthermore, the positive influence of additional turning flow information on the estimation accuracy is proven true again, when no noise from redundant information exists. When duplicated information appears, more information may result in worse O-D estimates (see S1 and S2 with the modified IM model in test network "List"). The estimated results of some scenarios with the small test network in Table 2 are worse than those in Table 1, since the SUE assignment model is applied here instead of the incremental assignment and that has resulted in different generated route sets. Such results highlight the importance of an adequate assignment model in matrix estimation.

TABLE 2 Performance of the Modified IM model and the Proposed Methodology

Test network	Scenario	modified IM model			proposed methodology		
		R	RMSE	WRMSE	R	RMSE	WRMSE
small test network[1]	S1	0.997	5.037	0.041	0.998	3.420	0.027
	S2	0.876	25.212	0.233	0.873	24.305	0.230
	S3	0.998	4.200	0.035	0.998	3.420	0.027
	S4	0.885	24.353	0.227	0.894	23.504	0.217
test network "List"[2]	S1	0.916	271.017	1.738	0.996	61.524	0.599
	S2	0.872	334.695	1.598	0.990	97.558	0.857
	S3	0.919	265.839	1.641	0.996	61.524	0.599
	S4	0.912	276.548	1.863	0.849	80.721	1.333

[1]S1: all flows detected; S2: only link flows detected; S3: only turning flows detected; S4: S2 with detected turning flow for link 18

[2] S1: all flows detected; S2: only link flows detected; S3: only turning flows detected; S4: in- and outbound flows and flows on 79 links detected. The 79 links are shown as dashed lines in Figure 6.

Furthermore the respective convergence and computation time are examined as well. Both the IM model based and the modified IM model based estimation processes with the use of SUE assignment model are convergent. The convergent speeds of both processes are similar. Significant improvement of the estimation accuracy is done during the first 6 iterations. For the computation time, Table 3 indicates that the proposed methodology required less computation time than the modified IM model without the information screening process in the small test network, except Scenario 2. The respective computation time difference is not great because of the small size of the network. In the larger test network “List”, the required computation time of the proposed methodology is much lesser than that based on the modified IM model. It is because the number of restraints, which have to be satisfied in the matrix estimation, is significantly reduced with use of the information screening progress. This information screening process does not require much computation time.

TABLE 3 Computation Time of the Modified IM model and the Proposed Methodology

Scenario	Small network		Test network „List“	
	modified IM Model (s)	proposed methodology (s)	modified IM Model (s)	proposed methodology (s)
S1	1.978	1.697	1077.659	62.258
S2	19.940	26.212	1316.662	143.483
S3	1.931	1.744	1051.741	66.628
S4	37.481	26.349	856.982	152.014

5. CONCLUSIONS

With the help of a simple numerical example, an overview on the performance of the IM and the modified IM models pertaining to duplicated information and route choice accuracy is given. The results indicate that the performance of the modified IM model is sensitive to inaccurate route choice information and the quality of the O-D estimates gets significantly worse with estimated route choice proportions. To overcome this problem a new approach using the IM model is proposed. This approach combines the IM model with an information screening process to keep the valuable information and, at the same time, takes a more realistic assignment models into consideration. Moreover, for adopting the SUE assignment in the proposed methodology the method to determine a path set, based on empirical route choice data, is proposed. A number of 3-4 route alternatives for each O-D pair and a 30% upper bound of the travel time deviation in comparison to the most efficient route are suggested. Therefore, the respective k shortest paths can be determined and applied in the assignment model. The results show that the proposed methodology performs better than the one based on the modified IM model. Also, additional turning flow information can significantly improve estimation accuracy provided there is no noise form redundant information. With the application of the proposed information screening process, more turning flow information can be derived from given link and turning flow information. Estimation accuracy can thus be further improved. If redundant information exists, more information may result in worse O-D estimates.

Finally, in the proposed methodology, the respective convergence is examined and confirmed. The use of the information screening process does not result in significant increase in the computation time. On the contrary, the respective computation time is greatly reduced, since the number of considered constraints in the matrix estimation model is reduced by eliminating redundant information. Based on the results of the two test works, it is expected that the whole estimation process is more efficient and delivers more precise estimation results.

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