

Constraints from F and D supersymmetry breaking in general supergravity theories

Marta Gomez-Reino^{1,*} and Claudio A. Scrucca^{2,**}

¹ Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland.

² Inst. de Th. des Phén. Phys., Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

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We study the conditions under which a generic supergravity model involving chiral and vector multiplets can admit vacua with spontaneously broken supersymmetry and realistic cosmological constant. We find that the existence of such viable vacua implies some constraints involving the curvature tensor of the scalar geometry and the charge and mass matrices of the vector fields, and also that the vector of F and D auxiliary fields defining the Goldstino direction is constrained to lie within a certain domain. We illustrate the relevance of these results through some examples and also discuss the implications of our general results on the dynamics of moduli fields in string models. This contribution is based on [1, 2, 3].

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1 Introduction

Recently, substantial progress has been achieved in the search of non-supersymmetric Minkowski/dS vacua in the context of string/M-theory compactifications. This was mainly related to the understanding of the superpotentials generated by background fluxes [4] and by non-perturbative effects like gaugino condensation [5], which generate a potential for the moduli fields coming from the compactification and have suggested new interesting possibilities for model building, like in particular those proposed in refs. [6, 7]. From a phenomenological point of view, this type of models must however possess some characteristics in order to be viable: supersymmetry must be broken, the cosmological constant should be tiny, and all the moduli fields should be stabilized. In the low energy effective theory all these crucial features are controlled by a single quantity, the four-dimensional scalar potential, which gives information on the dynamics of the moduli fields, on how supersymmetry is broken and on the value of the cosmological constant. The characterization of the conditions under which a supersymmetry-breaking stationary point of the scalar potential satisfies simultaneously the flatness condition (vanishing of the cosmological constant) and the stability condition (the stationary point is indeed a minimum) is therefore very relevant in the search of phenomenologically viable string models. In this note we review the techniques presented in [1, 2, 3] to study the possibility of getting this type of vacua in the context of general supergravity theories in which both chiral and vector multiplets participate to supersymmetry breaking.

2 Viable supersymmetry breaking vacua

The goal of this section is to find conditions for the existence of non-supersymmetric extrema of the scalar potential of general supergravity theories fulfilling two basic properties: i) they are locally stable and ii)

* Corresponding author E-mail: marta.gomez-reino.perez@cern.ch, Phone: +0041227674523, Fax: +0041227673850

** E-mail: claudio.scrucca@epfl.ch.

they lead to a negligible cosmological constant. We will first study this issue for theories with only chiral multiplets and then when also vector multiplets are present.

2.1 Constraints for chiral theories

The Lagrangian of the most general supergravity theory with n chiral superfields is entirely defined by a single arbitrary real function G depending on the corresponding chiral superfields Φ_i and their conjugates $\bar{\Phi}_i$, as well as on its derivatives [8]. The function G can be written in terms of a real Kähler potential K and a holomorphic superpotential W in the following way¹:

$$G(\Phi_i, \bar{\Phi}_i) = K(\Phi_i, \bar{\Phi}_i) + \log W(\Phi_i) + \log \bar{W}(\bar{\Phi}_i). \quad (1)$$

The quantities K and W are however defined only up to Kähler transformations acting as $K \rightarrow K + f + \bar{f}$ and $W \rightarrow e^{-f} W$, f being an arbitrary holomorphic function of the superfields, which leave the function G invariant. The scalar components of the chiral multiplets span an n -dimensional Kähler manifold whose metric is given by $G_{i\bar{j}}$, which can be used to lower and raise indices.

The 4D scalar potential of this theory takes the following simple form:

$$V = e^G \left(G^k G_k - 3 \right). \quad (2)$$

The auxiliary fields of the chiral multiplets are fixed by the Lagrangian through the equations of motion, and are given by $F_i = -e^{G/2} G_i$ where $e^{G/2} = m_{3/2}$ is the mass of the gravitino. Whenever $F_i \neq 0$ at the vacuum, supersymmetry is spontaneously broken and the direction given by the G_i 's defines the direction of the Goldstino eaten by the gravitino in the process of supersymmetry breaking.

In order to find local non-supersymmetric minima of the potential (2) with small non-negative cosmological constant, one should proceed as follows: First impose the condition that the cosmological constant is negligible and fix $V = 0$. This flatness condition implies that:

$$G^k G_k = 3. \quad (3)$$

Then look for stationary points of the potential where the flatness condition is satisfied. This implies:

$$G_i + G^k \nabla_i G_k = 0, \quad (4)$$

where by $\nabla_i G_k = G_{ik} - \Gamma_{ik}^n G_n$ we denote the covariant derivative with respect to the Kähler metric.

Finally, make sure that the matrix of second derivatives of the potential,

$$m^2 = \begin{pmatrix} m_{i\bar{j}}^2 & m_{ij}^2 \\ m_{i\bar{j}}^2 & m_{ij}^2 \end{pmatrix}, \quad (5)$$

is positive definite. This matrix has two different n -dimensional blocks, $m_{i\bar{j}}^2 = \nabla_i \nabla_{\bar{j}} V$ and $m_{ij}^2 = \nabla_i \nabla_j V$, and after a straightforward computation these are found to be given by the following expressions:

$$\begin{aligned} m_{i\bar{j}}^2 &= e^G \left(G_{i\bar{j}} + \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}p\bar{q}} G^p G^{\bar{q}} \right), \\ m_{ij}^2 &= e^G \left(\nabla_i G_j + \nabla_j G_i + \frac{1}{2} G^k \{ \nabla_i, \nabla_j \} G_k \right), \end{aligned} \quad (6)$$

where $R_{i\bar{j}p\bar{q}}$ denotes the Riemann tensor with respect to the Kähler metric. The conditions under which this $2n$ -dimensional matrix (5) is positive definite are complicated to work out in full generality, the only way being the study of the behaviour of the $2n$ eigenvalues. Nevertheless a necessary condition for this matrix to be positive definite can be encoded in the condition that the quadratic form $m_{i\bar{j}}^2 z^i \bar{z}^{\bar{j}}$ is positive for

¹ We will use the standard notation in which subindices i, \bar{j} mean derivatives with respect to $\Phi^i, \bar{\Phi}^{\bar{j}}$ and Planck units, $M_P = 1$.

any choice of non-null complex vector z^i . Our strategy will be then to look for a special vector z^i which leads to a simple constraint.

In this case there is only one special direction in field space, that is the direction given by $z^i = G^i$. Indeed projecting in that direction we find the following simple expression:

$$m_{i\bar{j}}^2 G^i G^{\bar{j}} = 6 - R_{i\bar{j}p\bar{q}} G^i G^{\bar{j}} G^p G^{\bar{q}}. \quad (7)$$

This quantity must be positive if we want the matrix (5) to be positive definite². Using the rescaled variables $f^i = -\frac{1}{\sqrt{3}}G^i$ the conditions for the existence of non-supersymmetric flat minima can then be written as:

$$\begin{cases} G_{i\bar{j}} f^i f^{\bar{j}} = 1, \\ R_{i\bar{j}p\bar{q}} f^i f^{\bar{j}} f^p f^{\bar{q}} < \frac{2}{3}. \end{cases} \quad (8)$$

The first condition, the flatness condition, fixes the amount of supersymmetry breaking whereas the second condition, the stability condition, requires the existence of directions with Kähler curvature less than $2/3$ and constraints the direction of supersymmetry breaking to be sufficiently aligned with it.

2.2 Constraints for gauge invariant theories

It can happen that the supergravity theory with n chiral multiplets Φ^i we just described has a group of some number m of global symmetries, compatibly with supersymmetry. In this subsection we consider the possibility of gauging such isometries with the introduction of vector multiplets. The corresponding supergravity theory will then include in addition to the n chiral multiplets Φ^i , m vector multiplets V^a .

The two-derivative Lagrangian is specified in this case by a real Kähler function $G(\Phi^k, \bar{\Phi}^{\bar{k}}, V^a)$, determining in particular the scalar geometry, m holomorphic Killing vectors $X_a^i(\Phi^k)$, generating the isometries that are gauged, and an m by m matrix of holomorphic gauge kinetic functions $H_{ab}(\Phi^k)$, defining the gauge couplings³. In this case the minimal coupling between chiral and vector multiplets turn ordinary derivatives into covariant derivatives, and induces a new contribution to the scalar potential coming from the vector auxiliary fields D^a , in addition to the standard one coming from the chiral auxiliary fields F^i . The 4D scalar potential takes the form:

$$V = e^G \left(g^{i\bar{j}} G_i G_{\bar{j}} - 3 \right) + \frac{1}{2} h^{ab} D_a D_b. \quad (9)$$

The auxiliary fields are fixed from the Lagrangian through the equations of motion to be:

$$F_i = -m_{3/2} G_i, \quad (10)$$

$$D_a = -G_a = i X_a^i G_i = -i X_a^{\bar{i}} G_{\bar{i}}, \quad (11)$$

where to get the relations in (11) one should also use gauge invariance of the action.

Now in order to find local non-supersymmetric minima of the potential (9) with small non-negative cosmological constant, we will proceed as in the previous subsection. First we will impose the condition that the cosmological constant is negligible and fix $V = 0$. This flatness condition implies that:

$$-3 + G^i G_i + \frac{1}{2} e^{-G} D^a D_a = 0. \quad (12)$$

² Actually, as emphasized in [9], the Goldstino multiplet cannot receive any supersymmetric mass contribution from W , since in the limit of rigid supersymmetry its fermionic component must be massless. This means that, in order to study metastability, it is enough to study the projection of the diagonal block $m_{i\bar{j}}^2$ of the mass matrix along the Goldstino direction G^i , as the rest of the projections can be given a mass with the help of the superpotential.

³ The gauge kinetic function H_{ab} must have an appropriate behavior under gauge transformations, in such a way as to cancel possible gauge anomalies Q_{abc} . Actually, the part $h_{ab} = \text{Re } H_{ab}$ defines a metric for the gauge fields and must be gauge invariant. On the other hand $\text{Im } H_{ab}$ must have a variation that matches the coefficient of Q_{abc} , namely $X_a^i h_{bci} = \frac{i}{2} Q_{abc}$.

The stationarity conditions correspond now to the requirement that $\nabla_i V = 0$, and they are given by:

$$G_i + G^k \nabla_i G_k + e^{-G} \left[D^a \left(\nabla_i - \frac{1}{2} G_i \right) D_a + \frac{1}{2} h_{abi} D^a D^b \right] = 0. \quad (13)$$

The $2n$ -dimensional mass matrix (5) for small fluctuations of the scalar fields around the vacuum has as before two different n -dimensional blocks, which can be computed as $m_{i\bar{j}}^2 = \nabla_i \nabla_{\bar{j}} V$ and $m_{ij}^2 = \nabla_i \nabla_j V$. Using the flatness and stationarity conditions, one finds, after a straightforward computation [10, 11]:

$$m_{i\bar{j}}^2 = e^G \left[g_{i\bar{j}} - R_{i\bar{j}p\bar{q}} G^p G^{\bar{q}} + \nabla_i G_k \nabla_{\bar{j}} G^k \right] - \frac{1}{2} \left(g_{i\bar{j}} - G_i G_{\bar{j}} \right) D^a D_a - 2 D^a G_{(i} \nabla_{\bar{j})} D_a \quad (14)$$

$$+ \left(G_{(i} h_{ab\bar{j})} + h^{cd} h_{aci} h_{bd\bar{j}} \right) D^a D^b - 2 D^a h^{bc} h_{ab(i} \nabla_{\bar{j})} D_c + h^{ab} \nabla_i D_a \nabla_{\bar{j}} D_b + D^a \nabla_i \nabla_{\bar{j}} D_a,$$

$$m_{ij}^2 = e^G \left[2 \nabla_{(i} G_{j)} + G^k \nabla_{(i} \nabla_{j)} G_k \right] - \frac{1}{2} \left(\nabla_{(i} G_{j)} - G_i G_j \right) D^a D_a + h^{ab} \nabla_i D_a \nabla_j D_b \quad (15)$$

$$- 2 D^a G_{(i} \nabla_{j)} D_a - 2 D^a h^{bc} h_{ab(i} \nabla_{j)} D_c + \left(G_{(i} h_{abj)} + h^{cd} h_{aci} h_{bdj} - \frac{1}{2} h_{abij} \right) D^a D^b.$$

We want to analyze now the restrictions imposed by the requirement that the physical squared mass of the scalar fields are all positive. In general the theory displays a spontaneous breakdown of both supersymmetry and gauge symmetries, so in the study of the stability of the vacuum it is necessary to take appropriately into account the spontaneous breaking of gauge symmetries. In that process m of the $2n$ scalars, the would-be Goldstone bosons, are absorbed by the gauge fields and get a positive mass, so we do not need to take them into account for the analysis of the stability. Nevertheless the would-be Goldstone modes correspond to flat directions of the unphysical mass matrix, and get their physical mass through their kinetic mixing with the gauge bosons. This means that positivity of the physical mass matrix implies semi-positivity of the unphysical mass matrix in (14), (15). We can use then the same strategy as before but changing the strictly positive condition to a semi-positive one.

In this case there exist two types of special complex directions z^i one could look at. The first is the direction G^i , which is associated with the Goldstino direction in the subspace of chiral multiplet fermions. Projecting into this direction one finds, after a long but straightforward computation:

$$m_{i\bar{j}}^2 G^i G^{\bar{j}} = e^G \left[6 - R_{i\bar{j}p\bar{q}} G^i G^{\bar{j}} G^p G^{\bar{q}} \right] + \left[-2 D^a D_a + h^{cd} h_{aci} h_{bd\bar{j}} G^i G^{\bar{j}} D^a D^b \right] \quad (16)$$

$$+ e^{-G} \left[M_{ab}^2 D^a D^b + \frac{3}{4} Q_{abc} D^a D^b D^c - \frac{1}{2} \left(D^a D_a \right)^2 + \frac{1}{4} h_{ab}^i h_{cdi} D^a D^b D^c D^d \right],$$

where $Q_{abc} = -2i X_a^i h_{bci}$. The condition $m_{i\bar{j}}^2 G^i G^{\bar{j}} \geq 0$ is then the generalization of the condition in (7) for theories involving only chiral multiplets. In terms of the rescaled variables:

$$f_i = \frac{1}{\sqrt{3}} \frac{F_i}{m_{3/2}} = -\frac{1}{\sqrt{3}} G_i, \quad d_a = \frac{1}{\sqrt{6}} \frac{D_a}{m_{3/2}}, \quad (17)$$

the flatness and stability conditions take then the following form:

$$\begin{cases} f^i f_i + d^a d_a = 1, \\ R_{i\bar{j}p\bar{q}} f^i f^{\bar{j}} f^p f^{\bar{q}} \leq \frac{2}{3} + \frac{2}{3} \left(M_{ab}^2 / m_{3/2}^2 - 2 h_{ab} \right) d^a d^b + 2 h^{cd} h_{aci} h_{bd\bar{j}} f^i f^{\bar{j}} d^a d^b \\ \quad - \left(2 h_{ab} h_{cd} - h_{ab}^i h_{cdi} \right) d^a d^b d^c d^d + \sqrt{\frac{3}{2}} \frac{Q_{abc}}{m_{3/2}} d^a d^b d^c. \end{cases} \quad (18)$$

Again we have that the flatness condition fixes the amount of supersymmetry breaking whereas the stability condition constrains its direction. One could also consider the directions X_a^i , which are instead associated with the Goldstone directions in the space of chiral multiplet scalars. Nevertheless the constraint $m_{i\bar{j}}^2 X_a^i X_a^{\bar{j}} \geq 0$ turns out to be more complicated and no useful condition seems to emerge from it.

3 Analysis of the constraints

The analysis of the flatness and stability conditions in the case where both chiral and vector multiplets participate to supersymmetry breaking presents an additional complication with respect to the case where only chiral multiplets are present, due to the fact that the auxiliary fields of the chiral and vector multiplets are not independent of each other. The rescaled auxiliary fields f_i and d_a are actually related in several ways. One first relation (consequence of gauge invariance) can be read from eq. (11) and is given by:

$$d^a = \frac{i X_i^a}{\sqrt{2} m_{3/2}} f^i. \quad (19)$$

This relation is satisfied as a functional relation valid at any point of the scalar field space. It shows that the d_a are actually linear combinations of the f_i . Using now the inequality $|a^i b_i| \leq \sqrt{a^i a_i} \sqrt{b^j b_j}$ one can derive a simple bound on the sizes that the d_a can have relative to the f_i :

$$|d_a| \leq \frac{1}{2} \frac{M_{aa}}{m_{3/2}} \sqrt{f^i f_i}. \quad (20)$$

There is also a second relation between f_i and d_a , that is instead valid only at the stationary points of the potential. It arises by considering a suitable linear combination of the stationarity conditions along the direction X_a^i , in other words, by imposing $X_a^i \nabla_i V = 0$. This relation reads [12, 13] (see also [14]):

$$i \nabla_i X_{a\bar{j}} f^i f^{\bar{j}} - \sqrt{\frac{2}{3}} m_{3/2} (3 f^i f_i - 1) d_a - \frac{M_{ab}^2}{\sqrt{6} m_{3/2}} d^b + Q_{abc} d^b d^c = 0. \quad (21)$$

These relations show that whenever the f_i auxiliary fields vanish also the d_a auxiliary fields should vanish. Therefore we can say that the f_i 's represent the basic qualitative seed for supersymmetry breaking whereas the d_a 's provide additional quantitative effects. Along this section we will address the problem of working out more concretely the implications of these constraints. In order to do so we will concentrate on the case in which the gauge kinetic function is constant and diagonal: $h_{ab} = g_a^{-2} \delta_{ab}$. In this case we can rescale the vector fields in such a way as to include a factor g_a for each vector index a . In this way, no explicit dependence on g_a is left in the formulas and the metric becomes just δ_{ab} . Using this the flatness and stability conditions take the following simple form:

$$\begin{cases} f^i f_i + \sum_a d_a^2 = 1, \\ R_{i\bar{j}p\bar{q}} f^i f^{\bar{j}} f^p f^{\bar{q}} \leq \frac{2}{3} + \frac{4}{3} \sum_a (2 m_a^2 - 1) d_a^2 - 2 \sum_{a,b} d_a^2 d_b^2, \end{cases} \quad (22)$$

where we have defined the quantity $m_a = M_a / (2 m_{3/2})$ measuring the hierarchies between scales. Denoting $v_a^i = \sqrt{2} X_a^i / M_a$ and $T_{ai\bar{j}} = i \nabla_i X_{a\bar{j}} / M_a$ the relations between f^i and d^a read:

$$d_a = i m_a v_a^i f_i \implies |d_a| \leq m_a \sqrt{f^i f_i}, \quad (23)$$

$$d_a = \sqrt{\frac{3}{2}} \frac{m_a T_{ai\bar{j}} f^i f^{\bar{j}}}{m_a^2 - 1/2 + 3/2 f^i f_i}. \quad (24)$$

3.1 Interplay between F and D breaking effects

In this subsection we will study the interplay between the F and D supersymmetry breaking effects. In order to do so it is useful to introduce the variables $\hat{f}^i = f^i / \sqrt{1 - \sum_a d_a^2}$. Using these variables the conditions for flatness and stability can be rewritten as:

$$\begin{cases} \hat{f}^i \hat{f}_i = 1, \\ R_{i\bar{j}p\bar{q}} \hat{f}^i \hat{f}^{\bar{j}} \hat{f}^p \hat{f}^{\bar{q}} \leq \frac{2}{3} K(d_a^2, m_a^2), \end{cases} \quad (25)$$

where the function $K(d_a^2, m_a^2)$ is given by:

$$K(d_a^2, m_a^2) = 1 + 4 \frac{\sum_a m_a^2 d_a^2 - (\sum_a d_a^2)^2}{(1 - \sum_b d_b^2)^2}. \quad (26)$$

In the limit in which the rescaled vector auxiliary fields are small ($d_a \ll 1$) we have that $\hat{f}^i \simeq f^i$ and therefore these variables \hat{f}^i are the right variables to study the effect of vector multiplets with respect to the case where only chiral multiplets are present. Note that in such a limit the relation (24) between F and D auxiliary fields can be written at first order as $d_a \simeq \sqrt{3/2} m_a / (1 + m_a^2) T_{a i \bar{j}} \hat{f}^i \hat{f}^{\bar{j}}$. Using this we get:

$$K \simeq 1 + 6 \sum_a \xi_a^2(m) T_{a i \bar{j}} T_{a p \bar{q}} \hat{f}^i \hat{f}^{\bar{j}} \hat{f}^p \hat{f}^{\bar{q}}, \quad \xi_a(m) = \frac{m_a^2}{1 + m_a^2}, \quad (27)$$

and we can write the flatness and stability conditions as:

$$\begin{cases} \hat{f}^i \hat{f}_i = 1, \\ \hat{R}_{i \bar{j} p \bar{q}} \hat{f}^i \hat{f}^{\bar{j}} \hat{f}^p \hat{f}^{\bar{q}} \leq \frac{2}{3}, \end{cases} \quad (28)$$

where $\hat{R}_{i \bar{j} p \bar{q}} = R_{i \bar{j} p \bar{q}} - 4 \sum_a \xi_a^2(m) T_{a i (\bar{j}} T_{a p \bar{q})}$. This means that the net effect in this case is to change the curvature felt by the chiral multiplets. Note as well that in the case in which the mass of the vectors is large this is not necessarily a small effect and can compete with the curvature effects due to the chiral multiplets. Actually for heavy vector fields one can check that integrating out the vector fields modifies the Kähler potential of the chiral multiplets in a way that accounts for this shift in the Kähler curvature.

For larger values of d_a one can instead find an upper bound to K (see [3] for details):

$$K \leq 1 + 6 \sum_a \xi_a^2(m) T_{a i \bar{j}} T_{a p \bar{q}} \hat{f}^i \hat{f}^{\bar{j}} \hat{f}^p \hat{f}^{\bar{q}}, \quad \xi_a(m) = \frac{m_a^2 (1 + \sum_b m_b^2)}{1 + m_a^2 + (m_a^2 - \frac{1}{2}) \sum_b m_b^2}. \quad (29)$$

So in this general case we get as well that the effect of vector multiplets can be encoded into an effective curvature $\hat{R}_{i \bar{j} p \bar{q}} = R_{i \bar{j} p \bar{q}} - 4 \sum_a \xi_a^2(m) T_{a i (\bar{j}} T_{a p \bar{q})}$.

In this section we have derived the implications of the flatness and stability conditions taking into account the fact that f^i and d^a are not independent variables. The strategy that we have followed is to use the relation (19) to write d_a in terms of f^i . A second possibility would be to use instead the relation (21) to write d^a in term of f^i and a third one would be to impose only the bound (20) to restrict the values of the d^a in terms of the values of f^i . It is clear that switching from the relation (19) to the relation (21) and finally to the bound (20) represents a gradual simplification of the formulas, which is also accompanied by a loss of information. As a consequence, these different types of strategies will be tractable over an increasingly larger domain of parameters, but this will be accompanied by a gradual weakening of the implied constraints. A detailed derivation of the implications of the flatness and stability conditions when the relations (21) and (20) are used can be found in [3].

4 Some examples: moduli fields in string models

In this section we will apply our results to the typical situations arising for the moduli sector of string models. The Kähler potential and superpotential governing the dynamics of these moduli fields typically have the general structure:

$$K = - \sum_i n_i \ln(\Phi_i + \bar{\Phi}_i) + \dots, \quad (30)$$

where by the dots we denote corrections that are subleading in the derivative and loop expansions defining the effective theory. The Kähler metric computed from (30) becomes diagonal and the whole Kahler manifold factorizes into the product of n one-dimensional Kahler submanifolds. Also the only non-vanishing

components of the Riemann tensor are the n totally diagonal components $R_{i\bar{j}p\bar{q}} = R_i g_{i\bar{i}}^2 \delta_{i\bar{j}p\bar{q}}$ where $R_i = 2/n_i$. Recall now that when only chiral fields participate to supersymmetry breaking the flatness and stability conditions take the form (8), so in this particular case they just read:

$$\sum_i |f^i|^2 = 1, \quad \sum_i R_i |f^i|^4 < \frac{2}{3}. \quad (31)$$

These relations represent a quadratic inequality in the variable $|f^i|^2$ subject to a linear constraint. This system of equations can be easily solved to get the condition $\sum_i R_i^{-1} > \frac{3}{2}$, which translates into:

$$\sum_i n_i > 3. \quad (32)$$

Also eqs. (31) constrain the values that the auxiliary fields $|f_i|$ can take.

When a single modulus dominates the dynamics the condition (32) implies $n > 3$ (this result was already found in [15] in a less direct way). For the universal dilaton S we have $n_S = 1$ and therefore it does not fulfill the necessary condition (32). This shows in a very clear way that just the dilaton modulus cannot lead to a viable situation [16] unless subleading corrections to its Kähler potential become large [17, 18]. We can therefore conclude that the scenario proposed in ref. [19], in which the dilaton dominates supersymmetry breaking, can never be realized in a controllable way. On the other hand, the overall Kähler modulus T has $n_T = 3$, and violates only marginally the necessary condition. In this case, subleading corrections to the Kähler potential are crucial. Recently some interesting cases where subleading corrections can help in achieving a satisfactory scenario based only on the T field have been identified for example in [20, 21].

In this case where the dynamics is dominated by just one field the Kähler potential of (30) corresponds to a constant curvature manifold with $R = 2/n$ and it has a global symmetry associated to the Killing vector $X = i\xi$, which can be gauged as long as the superpotential is also gauge invariant. By doing so the potential would get a D -term contribution that should be taken into account in the analysis of stability, as was explained in the previous section. In such a situation the flatness condition in (22) can be solved by introducing an angle δ and parametrizing the rescaled auxiliary fields as $f = \cos \delta$ and $d = \sin \delta$. In terms of this angle the stability condition implies:

$$n > \frac{3}{1 + 4 \tan^6 \delta}. \quad (33)$$

From this expression, it is clear that it is always possible to satisfy the stability condition for a large enough value of $\tan \delta$. Note in particular that eq. (33) implies that when n is substantially less than 3, which is the critical value for stability in the absence of gauging, the contribution to supersymmetry breaking coming from the D auxiliary field must be comparable to the one coming from the F auxiliary field.

A final comment is in order regarding the issue of implementing the idea of uplifting with an uplifting sector that breaks supersymmetry in a soft way. It is clear that such a sector will have to contain some light degrees of freedom, providing also some non-vanishing F and/or D auxiliary field. Models realizing an F -term uplifting are easy to construct. A basic precursor of such models was first constructed in [22]. More recently, a variety of other examples have been constructed, where the extra chiral multiplets have an O' Raifeartaigh like dynamics that is either genuinely postulated from the beginning [23] or effectively derived from the dual description of a strongly coupled theory [24] admitting a metastable supersymmetry breaking vacuum as in [25]. Actually, a very simple and general class of such models can be constructed by using as uplifting sector any kind of sector breaking supersymmetry at a scale much lower than the Planck scale [1]. Models realizing a D -term uplifting, on the other hand, are difficult to achieve. The natural idea of relying on some Fayet-Iliopoulos term [26] does not work, due to the already mentioned fact that such terms must generically be field-dependent in supergravity, so that the induced D is actually proportional to the available charged F 's. It is then clear that there is an obstruction in getting D much bigger than the F 's (see also [27]). Most importantly, if the only charged chiral multiplet in the model is the one of the would-be supersymmetric sector (which is supposed to have vanishing F) then also D must vanish,

implying that a vector multiplet cannot act alone as an uplifting sector [28, 29]. This difference between F -term and D -term uplifting is, as was emphasized in the previous section, due to the basic fact that chiral multiplets can dominate supersymmetry breaking whereas vector multiplets cannot.

Finally we would like to mention that the flatness and stability conditions simplify not only for factorizable Kähler manifolds but also for some other classes of scalar manifolds that present a simple structure for the Riemann tensor. This is the case for example for Kähler potentials generating a scalar manifold of the form G/H which arise for example in orbifold string models [2, 3], and also for no-scale supergravities and Calabi-Yau string models [9].

5 Conclusions

In this note we have reviewed the constraints that can be put on gauge invariant supergravity models from the requirement of the existence of a flat and metastable vacuum, following the results of [1, 2, 3]. We have shown that in a general $\mathcal{N} = 1$ supergravity theory with chiral and vector multiplets there are strong necessary conditions for the existence of phenomenologically viable vacua. Our results can be summarized as follows. These necessary conditions severely constrain the geometry of the scalar manifold as well as the direction of supersymmetry breaking and the size of the auxiliary fields. When supersymmetry breaking is dominated by the chiral multiplets the conditions restrict the Kähler curvature, whereas when also vector multiplets participate to supersymmetry breaking the net effect is to alleviate the constraints through a lower effective curvature. This is mainly due to the fact that the D -type auxiliary fields give a positive definite contribution to the scalar potential, on the contrary of the F -type auxiliary fields, which give an indefinite sign contribution. Nevertheless one should also take into account the fact that the local symmetries associated to the vector multiplets also restrict the allowed superpotentials. These results should be useful in discriminating more efficiently potentially viable models among those emerging, for instance, as low-energy effective descriptions of string models.

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