Interactions and Dynamics of D-branes

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Abstract

This is an introductory review of the theory of $D-branes$ compactified to four dimensions on the Z_3 orbifold. We compute the partition function and the scattering amplitude of closed string states from a system of two identical D-branes. We then consider two $D-branes$ moving with relative non relativistic velocity.

I. INTRODUCTION

Recent progress in non perturbative string theory has displayed a larger content of objects in the theory other than just strings. Extended objects of higher dimensionality, known as $p-$ *branes*, are necessary for the overall consistency of the theory. Indeed, most non perturbative duality symmetries require the existence of elementary Ramond-Ramond (RR) charges which are not contained in string perturbation theory. Since a $(p+1)$ form couples naturally to a $p-brane$, these extended objects are required by duality symmetries exchanging RR gauge fields with NS-NS gauge fields.

Historically the $p-branes$ first appeared as soliton solutions to the low energy effective action of string theory [1]. Following Polchinski's observation [2] that the $p-branes$ carrying RR charges admit a simple description in terms of open strings with mixed Dirichlet and Neumann boundary conditions, there has been intense activity in the field. In this conformal field theory formulation the $D-branes$ are described in type II superstring theories by imposing Neumann boundary conditions on $p+1$ coordinates X^A, $A=0,...,p$ and Dirichlet boundary conditions on the remaining $9-p$ coordinates X^i, $i=p+1,...,9$. This is a consistent string theory provided p is even in the type IIA theory or odd in the type IIB. One of the main advantages of this formulation is that the properties of the $D-branes$ can be obtained by perturbative calculations in the weak coupling limit. The leading perturbative amplitudes can be computed by evaluating correlation functions on world sheets with holes (disk, annulus). The amplitudes on a disk reproduce the leading perturbative terms in the physics of a single $D-brane$ whereas interactions between two $D-branes$ are described by scattering amplitudes on the cylinder. Thus we can learn something about the structure and properties of these objects by performing elastic and inelastic scattering experiments using a beam of closed strings. For instance the size of the $D-branes$ can be infered from the measurement of the scattering form factors in the elastic scattering of a closed string off a $D-brane$. A change in the internal states of the $D-branes$ can be evaluated from the scattering amplitude of open string states. An inelastic process where a closed string is absorbed by a $D-brane$, exciting its internal state by creating a pair of open strings, or the reverse process of spontaneous emision by excited $D-branes$, can be computed by inserting closed and open string vertex operators on the disk. Amplitudes of this kind have been computed by several authors [3–6]. It is also possible to scatter one $D-brane$ off another and in this case, length scales shorter than the string scale appear [7], and hints of an extra dimension in the theory become visible. Indeed, the mass of the $D-brane$ is determined by a BPS formula and for $0-branes$ this matches their interpretation as Kaluza Klein modes of an eleven dimensional theory, M-theory [8]. The scattering of two $0-branes$ [9,10] reveals a non perturbative length scale of the order of Planck length in M-theory.

The $D-branes$ are also a source for the gravitational field, and as such they belong to the family of generalized black holes. The world sheet description of $D-branes$ proposed in [2] has allowed considerable progress in accounting for the black hole information paradox. Examples of four and five dimensional black holes have been constructed for which the degeneracy of microscopic $D-brane$ states matches the Bekenstein-Hawking entropy [11].

The outline of this talk is the following. We first review the computation of the vacuum amplitude for two static $D0-branes$ in ten spacetime dimensions. Then we consider $D0-branes$ compactified to four spacetime dimensions on the Z_3 orbifold. Here we also take care of mixed Dirichlet-Neumann boundary conditions in the compactified directions -we call them $D0-branes$ in reference to the uncompactified directions since we still get pointlike objects in spacetime. In this way we learn about non perturbative aspects of string theory in four dimensions and on the other hand, since this compactification breaks supersymmetry, the dynamics is more interesting. We construct the twisted boundary states on the orbifold and compute the partition function. We then consider the scattering amplitude of closed string states, in particular we compute the axion production amplitude from an incoming graviton, a process that vanishes on a single $D-brane$ and on two branes in ten dimensions. The leading term of the amplitude at large distance from the branes suggests that the process takes place through the coupling of an axion to the RR sates exchanged between the $0-branes$. Finally we compute the partition function for a system of two moving

$D0 - branes$ and discuss the results.

II. VACUUM AMPLITUDE IN 10 DIMENSIONS

The branes are characterized by their RR charge and their tension and these properties can be obtained from a calculation on the disk or from the amplitude on the cylinder by factorization [2]. The interaction between two $D - branes$, one at $X^i(\sigma, \tau = 0) = 0$ and the other one at $X^i(\sigma, \tau = l) = Y^i$, can be described by the diagram shown in Figure 1. The two $D - branes$ interact via the exchange of closed string states which form a cylinder joining them. Here σ and τ are the world sheet coordinates. The result of the vacuum amplitude calculation indicates that the $D - branes$ are BPS states and as such, they break one half of the supersymmetries.

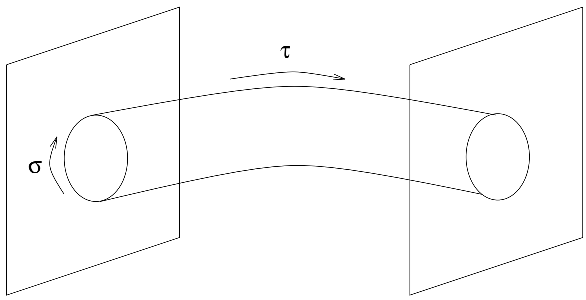

Figure 1

This diagram can be interpreted as a closed string propagating from one brane to the other during an Euclidean time $i\tau = l$, and σ is a periodic coordinate running from 0 to 1. In this formalism, the vacuum amplitude can be written as

$$Z = \int_0^\infty dl < B|e^{-lH}|B > \tag{1}$$

where $|B >$ is the state created by the boundary of the open string connecting the two branes. In order to construct this state let us recall the mode expansion of the bosonic and fermionic fields of the string

$$\begin{aligned} X^\mu(\sigma,\tau) &= X_0^\mu + ik^\mu\tau + \frac{i}{\sqrt{4\pi}}\sum_n \frac{1}{n}(a_n^\mu e^{2\pi n(\tau+i\sigma)} + \tilde{a}_n^\mu e^{2\pi n(\tau-i\sigma)}) \\ \psi^\mu(\sigma,\tau) &= \sum_m \psi_m^\mu e^{2\pi m(\tau+i\sigma)}; \quad \bar{\psi}(\sigma,\tau) = \sum_m \tilde{\psi}_m^\mu e^{2\pi m(\tau-i\sigma)} \end{aligned} \tag{2}$$

where in the NS (R) sector, the label m is half integer (integer). The oscillators $a^{\mu}_n|0>=\tilde{a}^{\mu}_n|0>=0,\ n>0$ and $\psi^{\mu}_m|0>=\tilde{\psi}^{\mu}_m|0>=0,\ m>0$ satisfy the following (anti)commutation relations

$$[a^{\mu}_n, a^{\nu\dagger}_m] = [\tilde{a}^{\mu}_n, \tilde{a}^{\nu\dagger}_m] = n\delta_{n,-m}\eta^{\mu\nu}$$
$$\{\psi^{\mu}_n, \psi^{\nu}_m\} = \{\psi^{\mu}_n, \psi^{\nu}_m\} = \delta_{n,-m}\eta^{\mu\nu} \tag{3}$$

The $D-branes$ are dynamical objects whose transverse positions are specified by collective coordinates Y^i and whose fluctuations are described by the excitations of open strings attached to them. As indicated above the $D-brane$ construction consists in taking Neumann boundary conditions for $(p+1)$ coordinates

$$\partial_\tau X^A(\sigma,\tau=0) = \partial_\tau X^A(\sigma,\tau=l) = 0 \quad A=0,...,p \tag{4}$$

and Dirichlet boundary conditions for the remaining $9-p$ coordinates

$$X^i(\sigma,\tau=0)=0; \quad X^i(\sigma,\tau=l)=Y^i \quad i=p+1,...,9 \tag{5}$$

In terms of oscillators, these boundary conditions translate into

$$Neumann: \quad (a^A_n + \tilde{a}^A_{-n})|B>=0; \quad (\psi^A_n + i\eta\tilde{\psi}^A_{-n})|B>=0$$
$$Dirichlet: \quad (a^i_n - \tilde{a}^i_{-n})|B>=0; \quad (\psi^i_n + i\eta\tilde{\psi}^i_{-n})|B>=0 \tag{6}$$

where $\eta=\pm$ denotes the spin structure.

The boundary state satisfying these conditions can be written as

$$|B>_B = e^{\sum_{n>0}\frac{1}{n}(-\eta_{AB}a^A_{-n}\tilde{a}^B_{-n}+\delta_{ij}a^i_{-n}\tilde{a}^j_{-n})}|0>$$
$$|B>_{NS,R'} = e^{i\eta\sum_{m>0}(-\eta_{AB}\psi^A_{-m}\tilde{\psi}^B_{-m}+\delta_{ij}\psi^i_{-m}\tilde{\psi}^j_{-m})}|0> \tag{7}$$

where R' indicates that the zero modes are not included (they deserve a separate treatment).

The GSO projection has to be performed with the operator

$$(-1)^F = -(-1)^{\sum_{m>0}\psi_{-m}\cdot\psi_m} \tag{8}$$

which acting on $|B>$ produces the GSO projected boundary state

$$|\mathcal{B}>=\frac{1}{2}(|B,\eta>-|B,-\eta>) \tag{9}$$

In order to compute the partition function (1) we have to take the product of the bosonic and fermionic coordinates as well as the contributions from the $(b-c)$ and $(\beta-\gamma)$ ghosts. The $b-c$ ghost contribution cancels the contribution of the bosonic pair X^0X^1 and the $\beta-\gamma$ contribution is like the inverse of the fermionic pair $\psi^0\psi^1$ for every spin structure. Thus the partition function has the form of a "light-cone" expression.

Recalling the expression for the Hamiltonian in terms of the oscillators in the NS sector

$$\begin{aligned} H &= \frac{k^2}{2} + 2\pi\{\sum_{n\geq 1}(a_{-n}\cdot a_n + \tilde{a}_{-n}\cdot\tilde{a}_n) + \sum_{m=1/2}^{\infty} m(\psi_{-m}\cdot\psi_m + \tilde{\psi}_{-m}\cdot\tilde{\psi}_m) - 1\} \\ &= \frac{k^2}{2} - 2\pi + H_B + H_F \end{aligned} \tag{10}$$

and Fourier transforming the boundary states

$$\begin{aligned} |B, X^i = 0 > &= \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{ik\cdot(X=0)}|B,k> \\ |B, X^i = Y^i > &= \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{ik\cdot Y}|B,k> \end{aligned} \tag{11}$$

after integrating over k, we obtain

$$\begin{aligned} Z_{NS} &= \int_0^{\infty} dl(2\pi l)^{-(\frac{9-p}{2})} e^{-\frac{Y^2}{2l}} e^{2\pi l} < B|e^{-lH_B}|B>_B \times \\ &\qquad \frac{1}{2}\{< B,\eta|e^{-lH_F}|B,\eta>_{NS} - < B,\eta|e^{-lH_F}|B,-\eta>_{NS}\} \\ &= 8\pi^4 V_{p+1} \int_0^{\infty} dl(2\pi l)^{-(\frac{9-p}{2})} e^{-\frac{Y^2}{2l}} \vartheta_1'(0|2il)^{-4}\left[\vartheta_3(0|2il)^4 - \vartheta_4(0|2il)^4\right] \end{aligned} \tag{12}$$

where $V_{p+1} =< 0|0 >$ is the volume of the D-brane.

We recall the standard definitions

$$\begin{aligned} \vartheta_1(z|2il) &= \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}(z|2il) = 2f(q^2)q^{1/4}sin\pi z\prod_{n=1}^{\infty}(1-2q^{2n}cos2\pi z+q^{4n}) \\ \vartheta_2(z|2il) &= \vartheta\begin{bmatrix}1/2\\0\end{bmatrix}(z|2il) = 2f(q^2)q^{1/4}cos\pi z\prod_{n=1}^{\infty}(1+2q^{2n}cos2\pi z+q^{4n}) \\ \vartheta_3(z|2il) &= \vartheta\begin{bmatrix}0\\0\end{bmatrix}(z|2il) = f(q^2)\prod_{n=1}^{\infty}(1-2q^{2n-1}cos2\pi z+q^{4n-2}) \\ \vartheta_4(z|2il) &= \vartheta\begin{bmatrix}0\\1/2\end{bmatrix}(z|2il) = f(q^2)\prod_{n=1}^{\infty}(1-2q^{2n-1}cos2\pi z+q^{4n-2})\,. \end{aligned} \tag{13}$$

where $q = e^{i\pi\tau}$ and $f(q^2) = \prod_{n=1}^{\infty}(1 - q^{2n})$.

In a similar way we can proceed in the R-R sector. In this case, the mode expansions are the same as in eq.(2) but m is integer. Here, the vacuum is degenerate due to the zero modes $\psi_0^\mu, \tilde{\psi}_0^\mu$. In order to deal with them, the fermionic coordinates can be identified with the γ-matrices $\gamma^\mu = i\sqrt{2}\psi_0^\mu$ and $\tilde{\gamma}^\mu = i\sqrt{2}\tilde{\psi}_0^\mu$ with $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$. The boundary state of the $0 - branes$ satisfies

$$(\gamma^0 + i\eta\tilde{\gamma}^0)|B, \eta >= 0, \quad (\gamma^{\mu\neq 0} - i\eta\tilde{\gamma}^{\mu\neq 0})|B, \eta >= 0 \tag{14}$$

It is convenient to pair the zero modes containing the time coordinate as $a = (\gamma^0 + \gamma^1)/2$, $a^* = (\gamma^0 - \gamma^1)/2$ and the space coordinates as $b = (-i\gamma^2 + \gamma^3)/2$, $b = (-i\gamma^2 - \gamma^3)/2$ and similarly for $\mu = 4, ..., 9$, such that $\{a, a^*\} = \{b, b^*\} = 1$, and similarly for $\tilde{a}$, $\tilde{b}$. Defining the vacuum state by $a|0 >= b|0 >= 0$, $\tilde{a}|0 >= \tilde{b}^*|0 >= 0$, the RR zero mode contribution to the boundary state is

$$|B, \eta >= \frac{1}{\sqrt{2}} e^{-i\eta(a^*\tilde{a}^* - b_i^*\tilde{b}^i)}|0 > \otimes|\tilde{0} > \tag{15}$$

In this sector, the GSO projection is performed by the operator

$$(-1)^F = \psi_0^{11}(-)^{\sum_{m>0}\psi_{-m}\cdot\psi_m} \tag{16}$$

which yields

$$|\mathcal{B} >_R = \frac{1}{2}(|B, \eta >_R - |B, -\eta >_R) \tag{17}$$

It can be seen that the term in the partition function corresponding to the odd spin structure, i.e. $< B, \eta|e^{-H_F l}|B, -\eta >_R$, vanishes (it is proportional to $\vartheta_1(0)^4 = 0$). Here H_F is the Hamiltonian for the RR fermions. Repeating the calculation above (i.e. Eq.(12) with the appropriate Hamiltonian) it can be seen that

$$Z_R = 8\pi^4 V_{p+1} \int_0^\infty (2\pi l)^{-(\frac{9-p}{2})} e^{-\frac{Y^2}{2l}} \vartheta_1'(0|2il)^{-4}\vartheta_2(0|2il)^4 \tag{18}$$

In order to make the algebraic sum of the NS-NS and RR sectors, we take the same signs which hold for the partition function on the torus. Thus, the full partition function is:

$$Z = 8\pi^4 V_{p+1} \int_0^\infty (2\pi l)^{-(\frac{9-p}{2})} e^{-\frac{Y^2}{2l}} \vartheta_1'(0|2il)^{-4} \left[\vartheta_2(0|2il)^4 - \vartheta_3(0|2il)^4 + \vartheta_4(0|2il)^4\right] . \quad (19)$$

It vanishes due to the abstruse identity, reflecting the fact that there is no net force between BPS states.

III. VACUUM AMPLITUDE ON THE Z_3 ORBIFOLD

Having in mind a more interesting scenario, we now consider a realistic compactification scheme breaking the supersymmetry down to $N = 2$ in 4 dimensions. The presence of the branes will further reduce the supersymmetry to $N = 1$. In order to be able to do an explicit computation we consider compactification on the standard Z_3 orbifold [12]. This is done by compactifying six coordinates ($\mu = 4, ..., 9$) on a 6-torus and identifying points which are equivalent under $g_a = e^{2\pi i z_a}$ rotations on pairs of them, with $z_{4,5} = z_{6,7} = \pm 1/3$ and $z_{8,9} = -z_{4,5} - z_{6,7}$ for the pairs $X^{4,5} = X^4 + iX^5, X^{6,7} = X^6 + iX^7, X^{8,9} = X^8 + iX^9$ respectively. The request of the same Neumann or Dirichlet boundary conditions for both members of a pair is

$$(\beta_n^a \pm \tilde{\beta}_{-n}^a)|B >= 0, \quad (\beta_n^{a*} \pm \tilde{\beta}_{-n}^{a*})|B >= 0 \quad (20)$$

where $\beta_n^{4,5} = a_n^4 + ia_n^5, \beta_n^{4,5*} = a_n^4 - ia_n^5$, etc. and the corresponding boundary state is

$$|B >= \prod_a exp\{\mp \frac{1}{2n} \sum_{n>0} (\beta_{-n}^a \tilde{\beta}_{-n}^{a*} + \beta_{-n}^{a*} \tilde{\beta}_{-n}^a\}|0 > \quad (21)$$

This is the same as for the torus compactification but in the orbifold case there can be a twist in the σ-direction giving noninteger moding. Moreover the presence of the orbifold opens new possibilities for BPS states. In fact, we can consider D3-branes in TypeIIB theory with Neumann b.c.s for X^i and Dirichlet b.c.s for X^{i+1} ($i = 4, 6, 8$) at both ends $\tau = 0, l$. That is

$$(\beta_n^a + \tilde{\beta}_{-n}^{a*})|B >= 0 \qquad (\beta_n^{a*} + \tilde{\beta}_{-n}^a)|B >= 0 \quad (22)$$

and the corresponding boundary state is

$$|B>=\prod_a exp-\frac{1}{2}\sum_{n\geq 1}(\beta^a_{-n}\tilde{\beta}^a_{-n}+\beta^{a*}_{-n}\tilde{\beta}^{a*}_{-n})|0>. \qquad (23)$$

The physical boundary state, which is required to be Z_3 invariant, is

$$|B_{phys}>=\frac{1}{3}(|B,1>+|B,g>+|B,g^2>) \qquad (24)$$

where we have introduced the "twisted boundary state" satisfying

$$(g^a\beta^a_n+g^{a*}\tilde{\beta}^{a*}_{-n})|B,g>=0, \qquad (25)$$

namely:

$$|B,g>=\prod_a exp\{-\frac{1}{2}\sum_{n\geq 1}((g^a)^2\beta^a_{-n}\tilde{\beta}^a_{-n}+(g^{a*})^2\beta^{a*}_{-n}\tilde{\beta}^{a*}_{-n})\}|0> \qquad (26)$$

Since g^2 is generically an element of Z_3 we will write, in the following, $g\beta\tilde{\beta}$ for $g^2\beta\tilde{\beta}$.

It is now possible to compute the bosonic contribution to the vacuum amplitude. Expressing the Hamiltonian in terms of the new oscillators one gets for a pair of compact coordinates ($g^*_a g'_a=e^{2\pi i z_a}$),

$$<B_a,g_a|e^{-lH}|B_a,g'_a>=\prod_{n=1}^{\infty}\left|\frac{1}{1-g^*_a g'_a e^{-4\pi l n}}\right|^2=\frac{2f(q^2)q^{1/4}sin(\pi z_a)}{\vartheta_1(z_a|2il)}. \qquad (27)$$

Taking into account all the contributions from the compactified directions as well as the spacetime sector and the normal ordering term from the Hamiltonian ($q^{-2/3}$), the oscillator part of the bosonic vacuum amplitude is

$$Z(g,g')_B=\left[2f(q^2)\right]^4\frac{\pi q^{1/3}}{\vartheta'_1(0|2il)}\prod_a\frac{sin(\pi z_a)}{\vartheta_1(z_a|2il)}. \qquad (28)$$

On the orbifold, σ-twisted sectors are also possible. We will be concerned with this sector only in the case when the branes are on an orbifold fixed point, since only then the twisted closed string is shrinkable to zero. Thus we consider this sector only for Dirichlet b.c.s on every compact coordinate (thus for D0-branes in TypeIIA) and the corresponding boundary state is the one of eq. (21). In this sector the pairs of fields in the compactified directions may be diagonalized [12] such that

$$X^{a,b}(\sigma+1) = e^{2\pi i z_a} X^{a,b}(\sigma), \quad X^{*a,b}(\sigma+1) = e^{-2\pi i z_a} X^{*a,b}(\sigma)\,. \tag{29}$$

with $(a,b) = (4,5),(6,7),(8,9)$. This leads to fractional moding of the oscillators. The oscillator part of the bosonic amplitude for a pair of cordinates $X^{a,b}$ becomes

$$< B_a|e^{-lH}|B_a >= \prod_{n=1}^{\infty}(1-e^{-4\pi l(n-\frac{1}{3})})^{-1}(1-e^{-4\pi l(n-\frac{2}{3})})^{-1}\,. \tag{30}$$

Combining with the spacetime part and converting to Jacobi theta functions we get the full bosonic amplitude to be

$$Z_B(\sigma - twisted) = 2\left[f(q^2)\right]^4 \vartheta_1'(0|2il)^{-1}\vartheta_1(-2il/3|2il)^{-3}\,. \tag{31}$$

Let us now consider the fermionic contribution. Again we combine the fermionic coordinates into complex pairs

$$\chi_n^{(4,5)} = \psi_n^4 + i\psi_n^5, \quad \chi^{(4,5)*} = \psi^4 - i\psi^5 \tag{32}$$

and similarly for $\chi^{(6,7)}, \chi^{(8,9)}$. The condition to be satisfied by the twisted boundary state is

$$(g^a\chi_n^a + ig^{a*}\eta\tilde{\chi}_{-n}^{a*})|B_a, g_a, \eta >= 0 \tag{33}$$

for each pair of coordinates. The state satisfying this condition is

$$|B,g,\eta >= exp\left\{\frac{i\eta}{2}\sum_{n>0}(g\chi_{-n}\tilde{\chi}_{-n} + g^*\chi_{-n}^*\tilde{\chi}_{-n}^*\right\}|0> \tag{34}$$

and its contribution to the partition function is given by

$$< B_a, g_a, \eta|e^{-lH}|B_a, g_a', \eta' >= \prod_{n>0}\left|1 \pm e^{2\pi i z_a}q^{2n}\right|^2 \cdot Z_0^c(\pm)\,. \tag{35}$$

where $Z_0^c(\pm) = 1$ for the NS-NS sector, $Z_0^c(+) = 2cos\pi z_a$ and $Z_0^c(-) = 2isin\pi z_a$ for the RR sector.

Taking into account all the compactified directions as well as the spacetime contribution we find in the σ-untwisted sector that the fermionic amplitude is

$$< \mathcal{B}, g|e^{lH}|\mathcal{B}, g' >_F = \frac{1}{q^{1/3}f(q^2)^4} \times$$
$$\left\{\vartheta_2(0|2il)\prod_a\vartheta_2(z_a|2il) - \vartheta_3(0|2il)\prod_a\vartheta_3(z_a|2il) + \vartheta_4(0|2il)\prod_a\vartheta_4(z_a|2il)\right\}\,. \tag{36}$$

In making the algebraic sum of the RR and NS-NS sectors, we take the same signs which hold for the partition function on the torus, thus this expression vanishes due to the Riemann identity since $\sum_a z_a = 0$. The first term is the contribution from the RR sector (even spin structure) whereas the second and the third are from the two NS-NS GSO projections. Since the Z_3 invariant physical boundary state is given by the linear combination of eq. (24), one has still to sum the product of the bosonic (28) and fermionic (36) amplitudes over the three possibilities for g and g' (actually only three possibilities for $g^{-1}g\prime$ are distinct).

Just as for the bosonic amplitude, when the position of the brane is on the fixed point of the orbifold we also have to include the closed string σ-twisted sectors. Here too the oscillator moding is modified from the usual integer and half-integer in the RR and NS-NS sectors [13]. By grouping the coordinates into pairs we find for each pair of compactified coordinates in the NS-NS sector that

$$< B,\eta|e^{-lH}|B,\eta' >_{NS}= \prod_{n=1}^{\infty}\left[1 \pm e^{-4\pi l(n-\frac{5}{6})}][1 \pm e^{-4\pi l(n-\frac{1}{6})}\right] . \tag{37}$$

For the RR sector (in this twisted sector there are no zero modes) we have

$$< B,\eta|e^{-lH}|B,\eta' >_{R}= \prod_{n=1}^{\infty}\left[1 \pm e^{-4\pi l(n-\frac{1}{3})}][1 \pm e^{-4\pi l(n-\frac{2}{3})}\right] . \tag{38}$$

The net result in the twisted sector is that the full fermionic amplitude, including the spacetime sector and the appropriate normal ordering contributions, can now be written in terms of Jacobi theta functions as

$$\begin{aligned} < \mathcal{B}|e^{-lH}|\mathcal{B}, >_F &= f(q^2)^{-4}\Big\{\vartheta_2(0|2il)\prod_a \vartheta_2(z_a - 2il/3|2il) \\ &-\vartheta_3(0|2il)\prod_a \vartheta_3(z_a - 2il/3|2il) - \vartheta_4(0|2il)\prod_a \vartheta_4(z_a - 2il/3|2il)\Big\} . \end{aligned} \tag{39}$$

Recall that in the twisted sector for the Z_3 orbifold there has to be a relative positive sign between the two NS sectors because of invariance under the modular transformation $\tau \to \tau + 3$. Therefore here again the result is zero due to the Riemann identity.

IV. CONSTRUCTION OF SCATTERING AMPLITUDES

Let us now consider the scattering of NS-NS fields from the system of two $D0-branes$ on the Z_3 orbifold. This is done by inserting vertex operators constructed from world sheet fields with components only in the uncompactified directions in (1), namely

$$A(l;z,w) = \int_0^\infty dl < B|e^{-lH}V_1(z)V_2(w)|B > \tag{40}$$

For massless string states ($p^2 = 0$), the vertex operators are

$$V(z,\bar{z}) = \int d^2z\epsilon_{\mu\nu}(\partial X^\mu(z,\bar{z}) + ip\cdot\psi(z)\psi^\mu(z))(\bar{\partial}X^\nu(z,\bar{z}) + ip\cdot\bar{\psi}(\bar{z})\bar{\psi}^\nu(\bar{z}))e^{ip\cdot X(z,\bar{z})} \tag{41}$$

with $\epsilon_{\mu\nu}$ transverse ($p^\mu\epsilon_{\mu\nu} = p^\nu\epsilon_{\mu\nu} = 0$) and symmetric for gravitons ($\epsilon_{\mu\nu} = \epsilon_{\nu\mu}; \epsilon^\mu_\mu = 0$) and dilatons ($\epsilon_{\mu\nu} = \eta_{\mu\nu} - p_\mu l_\nu - l_\mu p_\nu$, where $p\cdot l = 1$) and antisymmetric for antisymmetric tensor particles. Here $z = \sigma + i\tau$, $\partial \equiv \partial_z$ and $\bar{\partial} \equiv \partial_{\bar{z}}$.

We now explicitly consider the scattering of a graviton and an antisymmetric tensor, with polarization tensors h_{ij} and b_{ij} respectively, off the two 0-branes. The amplitude for this process vanishes in the case of one D-brane [5] (on the disk). It vanishes also on the cylinder in 10 dimensions due to the Riemann identity. However, the orbifold breaks enough supersymmetry to allow a non-zero result and the conclusion is more interesting. The brane-brane interaction and the compactification play a crucial role in this non trivial computation and the process can be regarded as a mechanism to obtain novel features of the physics of D-branes.

Without loss of generality, we take the polarization tensors to be non-vanishing only in the directions perpendicular to the 0-brane, i.e. $\epsilon_{00} = \epsilon_{0i} = \epsilon_{i0} = 0; \epsilon_{ij} \neq 0$. The bosonic and fermionic propagators on the cylinder with the appropriate boundary conditions can be constructed from those of the torus [14]. The details of the computation can be found in reference [13]. Here we discuss the results.

We find that the only non-zero contribution to the particular axion-graviton amplitude under consideration comes from the odd spin structure sector (recall that the boundary conditions for world sheet fermions can be classified according to the spin-structure). In this sector four fermionic zero modes are necessary to obtain a non vanishing result, and these are provided by the fermions in the vertex operators. Interestingly enough, the odd spin-structure corresponds to one term of the GSO projection of the RR world sheet fermions. We also find that the only contribution to the odd spin structure amplitude from the compactified coordinates arises in the twisted sector of the closed string (recall that in an orbifold compactification twisted and untwisted sectors, by an element of the symmetry group, have to be considered).

The branes cannot transfer energy but they can transfer momenta. Poles in the momentum transfer arise when the vertices corresponding to the graviton and the axion come together on the cylinder. We refer to this process as the pinching limit. This pole signals the propagation of a massless closed string state, which couples to the point where the axion and graviton vertices come together. It could be interpreted as a virtual axion, propagating out of the two branes' system, which is eventually made real by absorbing the incoming graviton. The residue of the pole can also be singular rather than constant. This further singularity signals the propagation of massless closed string states between the branes, whose proper time is the length of the cylinder. The region in which the length of the cylinder diverges can be viewed as an infrared limit of long time propagation, the field theory limit. It corresponds to the exchange of the lowest closed string states, showing the suppression of massive closed string states at large distances. We study the amplitude in the pinching limit and field theory limit, taking together all the possible sources of singularities in order to find the leading behavior for small momentum transfer, corresponding to large distances. The structure of the resulting expression in this double limit seems to suggest that the process takes place through the exchange of an axion which couples to the RR states being exchanged between the D-particles. Indeed taking $z \to w$ and $l \to \infty$ in (40) we find

$$A \to a\ q \cdot h \cdot q\ \frac{1}{\sqrt{\vec{q}^2}} \cdot\ (constant), \tag{42}$$

where $q^2 = (k+p)^2 = 2k \cdot p;\ p^0 = k_0$ and $k_i^2 = p_i^2 = k_0^2$, k and p being the momenta of the graviton and the axion respectively. Recall that q^2 is purely spacelike. We have written the amplitude in terms of the axion a introduced as

$$-p_0 b_{lm} = \frac{a}{2}\epsilon_{lms} p_s \tag{43}$$

Since the limit $l \to \infty$ corresponds to the exchange of the lowest closed string states between the branes, the pinching limit graviton-axion amplitude has the correct momentum structure to be interpreted as the graph in figure 2, i.e the graviton and axion interact with the 0-branes through the exchange of an intermediate axion which couples to the lowest states being exchanged between the branes. In fact, the pinching limit, as usual, selects the one particle exchange in the momentum transfer channel and the obvious candidate for this particle is the axion, whose coupling to the graviton through the energy momentum tensor corresponds to the right vertex in the diagram and to the structure $aq{\cdot}h{\cdot}q$. Thus in the vertex at the left one sees the coupling of the axion with the lowest RR states exchanged between the branes. The propagators of these RR states carry only three momenta and no energy. The fact that the amplitude does not contain a pole like $1/\vec{q}^2$, but only $1/\sqrt{\vec{q}^2}$, suggests that the axion-RR-RR vertex is proportional to two powers of momenta. In fact, since the RR propagators behave as $1/\vec{k}^2$ and $1/(\vec{q}-\vec{k})^2$, integration over the $\vec{k}$, by dimensional reasons, will give $\sim \vec{q}^2/\sqrt{\vec{q}^2}$, which multiplied by the axion propagator $1/\vec{q}^2$ and the right vertex $aq \cdot h \cdot q$ reproduces our result for the amplitude.

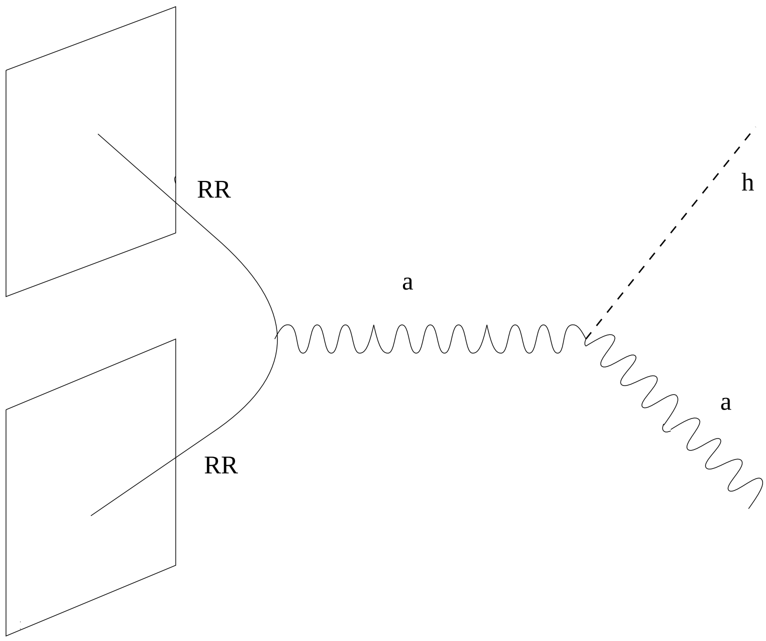

Figure 2

This interpretation of our result (let us stress that our result was obtained independently of the above field theory construction) may pose some problems. In fact it looks at variance with known rules from $N = 2$ supersymmetric gauge theories, which are supposed to hold in the "bulk", away from the branes, since the axion should be in a hypermultiplet and the RR states in a vector multiplet. It would seem that the well-known breaking of $N = 2$ down to $N = 1$ due to the branes sort of propagates away from them. This effect could be related to the fact that the RR states coming off the branes are necessarily off-shell since they carry zero energy, whereas one can verify that the four point amplitude $RR - h \to RR - a$ vanishes (and thus also its momentum transfer pole vanishes) for on-shell RR states in agreement with the above rules.

One notices also from eq. (42) that, if we make a three dimensional Fourier transform of the $1/\sqrt{\vec{q}^2}$ behaviour of the amplitude, we find a $1/r^2$ distribution for the static axion field at large distances from the source. This is to be contrasted with the normal behaviour of a scalar field which goes like $1/r$, and can be interpreted as due to the fact that the axion is coupled to a halo around the pointlike sources rather than to the sources directly.

V. MOVING D-BRANES ON THE Z_3 ORBIFOLD

The non-relativistic dynamics of Dirichlet branes [15–17], plays an essential role in the understanding of string theory at scales shorter than the Planck length [9,7]. It also provides some evidence [16] for the existence of an underlying eleven dimensional theory [8]. The perturbative duality between brane motion and electromagnetic open string backgrounds [18] allows to make transparent in one lenguage statements that are obscure in the other. The dynamics of a slowly varying electric field is known to be governed by the Born-Infeld action. Under the exchange $E \leftrightarrow v$ this becomes the action for a relativistic point particle.

In this section we use the boundary state technique introduced in the previous sections to compute the interaction between two moving identical D-branes in a TypeIIA or IIB superstring theory compactified down to four dimensions on the Z_3 orbifold. We will always consider particle-like D-branes, that is the time coordinate satisfies Neumann boundary conditions, $\partial_\tau X^0(\tau = 0, \sigma) = \partial_\tau X^0(\tau = l, \sigma) = 0$, whereas the three uncompactified space coordinates satisfy Dirichlet boundary conditions, $X^i(\tau = 0,\ \sigma) = 0, X^i(\tau = l, \sigma) = Y^i$. The boundary conditions are implemented by suitable boundary states.

To evaluate the interaction between moving branes we calculate the amplitude

$$\mathcal{A} = \int_0^\infty dl < B, V, X^i = Y^i | e^{-lH} | B, V = 0, X^i = 0 > , \tag{44}$$

where we have taken one of the branes to be at rest whereas the other is moving with velocity V. The boundary states in position space are given in terms of the momentum states as

$$\begin{aligned} |B, V = 0, X^i = 0 > \ &= \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(X=0)} |B, V = 0, k > \\ |B, V, X^i = Y^i > \ &= \int \frac{d^3q}{(2\pi)^3} e^{iq\cdot Y} |B, V, q > \ . \end{aligned} \tag{45}$$

We consider the case of two branes, one of them moving along one of the uncompactified space directions, say the X^1 direction. We make pairs of fields, $X^A = X^0 + X^1$ and $X^B = X^0 - X^1$, and pair the X^2 and X^3 into the complex fields $X^2 \pm iX^3$. The net effect is as if the $b - c$ ghosts cancel the contribution of the pair of coordinates orthogonal to the boost, X^2

and X^3. Similarly the $\beta - \gamma$ ghost contribution cancels the contribution from the fermionic pair ψ^3 and ψ^4 for each spin structure. The compact directions are treated identically as in the previous section.

In order to get the spacetime contribution to the boundary state of the D-brane moving with constant velocity V, let us consider the boost [19] $|B_v> = e^{iv^j J^0_j}|B>$, where $V = tanh\, v$, ($v = |v^j|$), is the velocity and $J^{\mu\nu}$ is the Lorentz generator.

The full amplitude is a product of the amplitudes for the bosonic and fermionic coordinates. We first consider the bosonic coordinates. With the standard commutation relations for the bosonic oscillators (3), the oscillators (α_n, β_n), for the X^A and X^B fields, respectively, are now defined as $\alpha_n = a^0_n + a^1_n, \beta_n = a^0_n - a^1_n$, etc., with the commutation relations $[\alpha_n, \beta_{-m}] = [\tilde{\alpha}_n, \tilde{\beta}_{-m}] = -2n\delta_{nm}$, and the other commutators being zero.

The Neumann boundary conditions for the time and Dirichlet for the space coordinates translate into

$$(\alpha_n + \tilde{\beta}_{-n})|B>= 0, \quad (\beta_n + \tilde{\alpha}_{-n})|B>= 0\,. \tag{46}$$

Here $|B>$ is the unboosted bosonic spacetime part of the boundary state. Under a Lorentz boost in the 1 direction the oscillators transform as

$$\alpha_n \to e^{-v}\alpha_n \quad \beta_n \to e^{v}\beta_n\,. \tag{47}$$

and similarly for the $\tilde{\alpha}_n, \tilde{\beta}_n$. The bosonic spacetime part of the boundary state of the moving brane is then

$$|B_v>= exp\sum_{n>0}\{\frac{1}{2n}(e^{-2v}\alpha_{-n}\tilde{\alpha}_{-n} + e^{2v}\beta_{-n}\tilde{\beta}_{-n}) + a^T_{-n}.\tilde{a}^T_{-n}\}|0> \;. \tag{48}$$

where a^T_n denote the oscillators of the directions orthogonal to the motion of the brane. The momentum content of the boosted state will be $q^0 = sinh(v)q^1$, $\vec{q} = (cosh(v)q^1, q_\perp)$ Therefore from momentum conservation in eq. (44) we will get $q^1 = k^1 = 0$, $q_\perp = k_\perp$ and thus we get the amplitude at fixed impact parameter $Y_\perp$ as

$$\mathcal{A} = \int d^3k_\perp e^{ik_\perp Y_\perp}\int_0^\infty dl M(l, k_\perp)\,. \tag{49}$$

In the following we write $M = Z_B Z_F$ where $Z_{B,F}$ are the bosonic, fermionic contributions. We start by computing the matrix element representing the bosonic spacetime coordinates contribution to the amplitude

$$Z_B =< B_v|e^{-lH}|B > \tag{50}$$

where H is the usual closed string Hamiltonian.

The oscillator part of Z_B is computed to be [19]

$$\prod_{n=1}^{\infty} \frac{1}{(1-e^{-2v}e^{-4\pi ln})(1-e^{2v}e^{-4\pi ln})} = \frac{2f(q^2)q^{1/4} isinhv}{\vartheta_1(\frac{iv}{\pi}|2il)} \tag{51}$$

Now we consider the fermionic modes' contribution. Again here we combine the fermionic coordinates ψ^0 and ψ^1 into a pair of coordinates $\psi^A = \psi^0 + \psi^1$ and $\psi^B = \psi^0 - \psi^1$. The oscillators satisfy the anti-commutation relations $\{\psi^A_m, \psi^B_n\} = \{\tilde{\psi}^A_m, \tilde{\psi}^B_n\} = -2\delta_{mn}$ with appropriate half-integer or integer moding for the NS-NS and RR cases. The other directions are combined into complex pairs $(2,3)$,$(4,5)$,$(6,7)$, $(8,9)$ as before. With the appropriate normalisation the spacetime modes' b.c.s for the D-brane at rest are given by (Neumann for time and Dirichlet for space)

$$(\psi^0_n + i\eta\tilde{\psi}^0_{-n})|B,\eta >= 0, \quad (\psi^i_n - i\eta\tilde{\psi}^i_{-n})|B,\eta >= 0\,. \tag{52}$$

For the longitudinal coordinates these can be rewritten as $(\psi^A_n + i\eta\tilde{\psi}^B_{-n})|B >_F = 0$,$(\psi^B_n + i\eta\tilde{\psi}^A_{-n})|B >_F = 0$

Now to construct the moving boundary state we note that under the boost in the X^1 direction the fields ψ^A and ψ^B transform like the bosonic coordinates $\psi^A \to e^{-v}\psi^A$,$\psi^B \to e^v\psi^B$. Thus the fermionic spacetime part of the boundary state of the moving brane is found to be

$$|B_v,\eta >= exp\sum_{m>0}\{\frac{i\eta}{2}(e^{-2v}\psi^A_{-m}\tilde{\psi}^A_{-m} + e^{2v}\psi^B_{-m}\tilde{\psi}^B_{-m}) - i\eta\psi^T_{-m}\tilde{\psi}^T_{-m}\}\,. \tag{53}$$

The zero modes of the four uncompactified coordinates ψ^μ, with $\mu = 0,1,2,3$ for the R-R state can be identified as before with the γ-matrices. Under the boost we have the transformations $a \rightarrow e^{-v}a, a^* \rightarrow e^{v}a^*$ leading to the boosted boundary state $|B_v, \eta>=$ $e^{v\gamma^0\gamma^1/2}|B, v=0>$, giving for the RR zero mode part of the moving boundary state

$$|B_v, \eta>= \frac{e^{-v}}{\sqrt{2}} e^{-i\eta(e^{2v}a^*\tilde{a}^* - b^*\tilde{b})}|0> \otimes |\tilde{0}> \ . \tag{54}$$

The GSO projected state in both the NS-NS and RR sectors is

$$|\mathcal{B}>_{NS,R}= \frac{1}{2}\{|B, \eta>_{NS,R} - |B, -\eta>_{NS,R}\} \,. \tag{55}$$

We now compute the matrix element $<\mathcal{B}_v|e^{-lH}|\mathcal{B}>_F$ (with the appropriate Hamiltonians for the NS-NS and RR sectors [13]) for the spacetime fermions. Thus the spacetime part of the fermionic amplitude turns out to be

$$<B_v, \eta|e^{-lH}|B, \eta'>= \prod_{n>0}(1 \pm e^{2v}q^{2n})(1 \pm e^{-2v}q^{2n}) \cdot Z_0(\pm) \,. \tag{56}$$

where n is an integer or half integer for NS-NS or RR, $\eta\eta' = \pm$ for the two possible cases of the GSO projection and $Z_0(\pm) = 1$ for NS-NS, $Z_0(+) = 2cosh(v)$ and $Z_0(-) = 0$ for RR. The $\psi^{2,3}$ contribution is cancelled by the $\beta - \gamma$ ghosts, but in the RR $\eta\eta' = -1$ (odd spin structure) case there remains the zero mode of this pair giving zero.

The internal part of the amplitude is exactly as discussed in section III. Putting everything together, taking into account all the compactified directions as well as the spacetime contribution and the normal ordering terms for both the NS-NS sector and the RR sector we find, for the twisted boundary state (σ-untwisted sector), that the fermionic amplitude is, in terms of Jacobi theta functions,

$$\begin{aligned} &<\mathcal{B}_v, g|e^{lH}|\mathcal{B}, g'>_F= \frac{1}{q^{1/3}f(q^2)^4} \times \\ &\left\{\vartheta_2(\frac{iv}{\pi}|2il)\prod_a \vartheta_2(z_a|2il) - \vartheta_3(\frac{iv}{\pi}|2il)\prod_a \vartheta_3(z_a|2il) + \vartheta_4(\frac{iv}{\pi}|2il)\prod_a \vartheta_4(z_a|2il)\right\} \,. \end{aligned} \tag{57}$$

Recall from section III that this expresion for the internal part of the vacuum amplitude corresponds to the boundary state satisfying different b.c. for each member of a pair of coordinates, thus a $D3-brane$ in Type IIB theory. In making the algebraic sum of the RR and NS-NS sectors, we take the same signs which hold for the partition function on the torus. Since the Z_3 invariant physical boundary state is given by the linear combination of eq. (24), one has still to sum the product of the bosonic (51) and fermionic (57) amplitudes over the three possibilities for g and g'.

On using the Riemann identity it is easy to see that the amplitude (57) behaves for small velocities as V^2, if $g \neq g'$ and as V^4 if $g = g'$. We note that (57) vanishes at $v = 0$ for all the twists $(1, g, g^2)$ individually.

Just as for the amplitude in the static case, when the position of the brane is on the fixed point of the orbifold here we also have to include the closed string σ-twisted sectors. Recall that we consider this sector for Dirichlet b.c.s on every compact coordinate, i.e. for $D0-$ *branes* in Type IIA. The net result in the twisted sector is that the full fermionic amplitude, including the spacetime sector and the appropriate normal ordering contributions, can now be written in terms of Jacobi theta functions as

$$
\begin{aligned}
&< \mathcal{B}_v|e^{-lH}|\mathcal{B}, >_F = f(q^2)^{-4} \Big\{ \vartheta_2(\frac{iv}{\pi}|2il)\vartheta_2(-2il/3|2il)^3 \\
&-\vartheta_3(\frac{iv}{\pi}|2il)\vartheta_3(-2il/3|2il)^3 - \vartheta_4(\frac{iv}{\pi}|2il)\vartheta_4(-2il/3|2il)^3 \Big\} \,. \qquad (58)
\end{aligned}
$$

Recall that in the twisted sector for the Z_3 orbifold there has to be a relative positive sign between the two NS sectors because of invariance under the modular transformation $\tau \rightarrow \tau + 3$. At low velocities this amplitude goes like V^2.

Let us compare our results with the V^4 dependance of the potential in the uncompactified case [15]. The absence of the zeroth order term in the amplitudes (57) and (58) is due to the cancellation of gravitational attraction and RR repulsion for static $D-branes$. The absence of V^2 terms in 10 dimensions corresponds in dual lenguage to the fact that the Maxwell term is not renormalized for maximally supersymmetric theories. In compactifications that break half the supersymmetries, the Maxwell term is generically renormalized, consistently with the V^2 dependance that we get for the amplitude in the Z_3 orbifold.

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