Gaugino Mass in AdS space

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Abstract

We study supersymmetric QED in AdS_4 with massless matter. At 1-loop the ultraviolet regulator of the theory generates a contribution to the gaugino mass that is naïvely inconsistent with unbroken supersymmetry. We show that this effect, known in flat space as anomaly mediated supersymmetry breaking, is required to cancel an infra-red contribution arising from the boundary conditions in AdS space, which necessarily break chiral symmetry. We also discuss an analogous UV/IR cancellation that is independent of supersymmetry.

1 Introduction

In phenomenologically interesting models the effects of broken supersymmetry in the visible sector are conveniently parameterized, working in an off-shell formulation, by the expectation values of the auxiliary components of some hidden sector supermultiplets. Among the auxiliary fields, the scalar u, belonging to the graviton supermultiplet, $(g_{\mu\nu}, \psi_\mu^\alpha, A_\mu, u)$, stands out as special. Indeed, unlike for auxiliary fields belonging to matter and gauge hidden sector multiplets, the coupling of u is completely fixed (at the leading relevant order) once the masses and self-couplings of the low energy effective theory, prior to supersymmetry breaking, are specified. This property just follows from u being a partner of $g_{\mu\nu}$ whose coupling is equally well specified by the energy momentum tensor of the low energy effective theory. The scenario of 'Anomaly Mediated' (AM) supersymmetry breaking corresponds to the limiting case in which the contribution of u dominates over all of the others [1, 2]. The name 'Anomaly Mediated' is due to the fact that in the MSSM u only couples to the visible fields at the quantum level, via a supersymmetric analogue of the dilatation anomaly of non-supersymmetric field theory.

The purpose of this paper will not be to build phenomenological models based on AM, but rather to investigate some of its more amusing theoretical aspects. In fact, far away from the domain of phenomenology, we shall be working in four dimensional supersymmetric Anti-de-Sitter (AdS) space. We nonetheless believe that our study provides interesting additional insight into the properties of AM, in particular its being UV insensitive, in spite of being UV generated.

To set the stage, it is convenient to derive AM terms via the superconformal approach to supergravity [3]. At tree level, the most general two-derivative Lagrangian may be written as

$$\mathcal{L} = \left[S^\dagger S\, \Omega(\Phi_i^\dagger, e^{q_i V}\Phi_i)\right]_D + \left\{\left[S^3 W(\Phi_i) + f(\Phi_i) W^\alpha W_\alpha)\right]_F + \text{h.c.}\right\}, \tag{1.1}$$

where D and F are superconformally invariant densities, provided that the chiral superfield, S, and the matter fields, Φ_i, have Weyl weights 1 and 0, respectively. Interesting actions are obtained by consistently taking the lowest component of S with non-vanishing expectation value. This breaks the superconformal group down to Poincaré supergravity and turns S into a purely auxiliary field, formally restoring scale invariance, hence the name 'superconformal compensator'. Indeed a suitable superconformal gauge can be chosen where $S = 1 + \theta^2 u$. The couplings of the auxiliary field u are thus fixed by dilations and R-symmetry. In particular a classically scale invariant subsector, like the MSSM, couples to u only at the quantum level. For a massless gauge theory the coupling of S is easily read off by demanding formal scale-

(and R-) invariance of the 1PI action at 1-loop

$$\Gamma = \frac{1}{4}\left[W^\alpha\Big(\frac{1}{g^2(\mu)} + \frac{b}{8\pi^2}\ln(\frac{\sqrt{\Box}}{\mu S})\Big)W_\alpha\right]_F + h.c. \tag{1.2}$$

By expanding in components, one finds a gaugino mass term which is proportional to the β-function

$$m_\lambda = -\frac{bg^2}{16\pi^2}u. \tag{1.3}$$

The dependence of Γ on S is local, compatibly with its being UV generated. However, it belongs to a non-local supergravity invariant 'structure' (involving $\ln\Box$), and this is why it is convenient to use the 1PI action to determine it. This is just the supersymmetric generalization of a dilaton coupling to the trace anomaly, hence the name 'anomaly mediation'.

In models with broken supersymmetry and vanishing cosmological constant, $\langle u\rangle = O(m_{3/2})$, implying a 1-loop contribution of order $(\alpha/4\pi)m_{3/2}$ to gaugino and sfermion masses. However, one may also have $\langle u\rangle \neq 0$, with unbroken supersymmetry on AdS. In that case, the expectation value is given by the superpotential: $\langle u\rangle = W/M_P^2 = 1/L$, where L is the AdS radius. Indeed, at tree level, $\langle u\rangle = 1/L$ generates the mass splittings, of order of the AdS curvature, that are required by supersymmetry in AdS. The rôle of a loop effect like anomaly mediation is less clear in this case, though it ought to be easy to understand, given that the theory still enjoys unbroken supersymmetry.

The purpose of this note is to explain the rôle played by anomaly mediation in supersymmetric AdS. This issue was briefly considered in [4], in the context of a general discussion in which the short distance origin of AM was emphasized. However our explanation for the rôle of AM in AdS space differs from the one proposed in [4]. We will argue that the existence of AM is a necessary consequence of supersymmetry, given the large-distance properties of AdS space, in particular the presence of a (conformal) boundary. In this sense, our work represents yet another way of deriving AM masses, purely via consideration of IR saturated quantities. The outline is as follows. In section 2, we review supersymmetry in AdS and supersymmetric QED therein. In section 3, we compute the 1-loop contributions to the gaugino self-energy in SQED with massless matter, and discuss the implications for the gaugino mass. In section 4, we present conclusions. The case of SQED with massive matter is relegated to the appendix.

2 Supersymmetry in AdS Space

In this section, we briefly review some basic features of supersymmetry in four-dimensional AdS space which will be relevant for the following discussion. For more details, see [5, 6] and refs. therein.

The isometry group of AdS$_4$ is $SO(2,3)$, whose unitary, infinite-dimensional representations are denoted by $D(E,s)$, where E and s represent respectively the energy and spin of the lowest energy state in the representation. The Lagrangian mass parameter of the corresponding fields (in units of $1/L$) are functions of E and s. For instance, for the simplest cases of $s=0,\frac{1}{2}$, we have

$$D(E,0) \qquad \longrightarrow \qquad m_0^2 = \frac{E(E-3)}{L^2}, \tag{2.1}$$

$$D(E,\frac{1}{2}) \qquad \longrightarrow \qquad m_{\frac{1}{2}}^2 = \frac{(E-3/2)^2}{L^2}\,. \tag{2.2}$$

Just as in flat space, the simplest irreducible representations of the super-group $Osp(1,4)$ correspond to chiral and vector supermultiplets. A chiral supermultiplet decomposes into the following representations of $SO(2,3)$:

$$D\left(E_0,0\right) \oplus D\left(E_0+\frac{1}{2},\frac{1}{2}\right) \oplus D\left(E_0+1,0\right), \qquad E_0 \geq \frac{1}{2}. \tag{2.3}$$

Note that the supersymmetry generators raise and lower E by a half-integer. Then, according to eqs. (2.1),(2.2), the mass terms for fermions and scalars within the same supermultiplet are not, in general, the same. These splittings are mandated by $Osp(1,4)$ and originate within the lagrangian from two sources. One source is the non-vanishing Ricci scalar and the other source is $\langle u\rangle = 1/L$. Notice, finally, that in the special case of the conformally-coupled supermultiplet, with $E_0=1$, the two scalars have the same mass, even though they belong to different representations: namely $D(1,0)$ and $D(2,0)$.

Turning now to the massless vector supermultiplet, the $SO(2,3)$ representation content is

$$D\left(\frac{3}{2},\frac{1}{2}\right) \oplus D\left(2,1\right). \tag{2.4}$$

This multiplet is both conformally coupled and 'short', corresponding to its being related to a gauge invariant lagrangian. A massive vector multiplet, on the other hand, is characterized by $E_0 > 3/2$, and decomposes as

$$D\left(E_0,\frac{1}{2}\right) \oplus D\left(E_0+\frac{1}{2},0\right) \oplus D\left(E_0+\frac{1}{2},1\right) \oplus \left(E_0+1,\frac{1}{2}\right). \tag{2.5}$$

This is a long multiplet that can be viewed as arising from a Higgs mechanism. Indeed, it has the same state multiplicity as the direct sum of the massless vector supermultiplet and the Goldstone supermultiplet, whose content is $D\left(2,0\right) \oplus D\left(\frac{5}{2},\frac{1}{2}\right) \oplus \left(3,0\right)$. Since it corresponds to multiplet shortening, the masslessness condition must be stable in perturbation theory. In particular, the gaugino mass, for an unbroken gauge symmetry, must be zero to all orders.

2.1 AdS SUSY QED

The presence of the anomaly mediated contribution to the mass (1.3) is, naïvely, at odds with the previous observation that the gaugino should be massless. To clarify the rôle of AM, we shall focus on the simplest non-trivial example, that is the mass of the gaugino in supersymmetric QED. Our theory consists of $\mathcal{N}=1$ supergravity with a vector superfield V, and two chiral superfields $\Phi_\pm$, with opposite charges ± 1. The Kähler and superpotential functions are given by (throughout the paper we use the conventions of Wess and Bagger [8])

$$\begin{aligned} \Omega \equiv -3M_P^2 e^{-K/3M_P^2} &= -3M_P^2 + \Phi_+^\dagger e^{gV}\Phi_+ + \Phi_-^\dagger e^{-gV}\Phi_- + O(\Phi^4), \\ W &= \frac{M_p^2}{L} + m\Phi_+\Phi_-, \qquad (2.6) \\ f &= 1 + O(\Phi_+\Phi_-). \qquad (2.7) \end{aligned}$$

Since we shall be working in the neighbourhood of $\Phi_\pm = 0$, we neglect the higher order terms indicated by $O(\dots)$. The constant term in the superpotential gives rise to the AdS_4 background and to the expectation value of the compensator,

$$\langle S \rangle = 1 + \frac{1}{L}\theta^2 \,. \qquad (2.8)$$

We will find it technically convenient to work in the Poincaré patch, with metric

$$ds^2 = \frac{L^2}{z^2}\left(dx^\mu dx_\mu + dz^2\right). \qquad (2.9)$$

The co-ordinates x^μ ($\mu = 0,1,2$) and z cover only one of an infinite set of similar Poincaré patches of the full AdS space. However Poincaré co-ordinates cover the whole euclidean AdS (EAdS), which can be obtained just by the substitution $t \to i\tau$ (see for instance the discussion in ref. [7]). This last property indicates that, if properly interpreted, computations on the Poincaré patch yield informations about the properties of QFT on full AdS. Assuming L to be positive, in these co-ordinates the four unbroken supersymmetries are parameterized by the Killing spinors

$$\xi = z^{\frac{1}{2}}[\epsilon_0 - i\sigma^3\bar\epsilon_0] + z^{-1/2}x_\mu\sigma^\mu[\epsilon_0 + i\sigma^3\bar\epsilon_0], \qquad (2.10)$$

where ϵ_0 is a two-component constant spinor. Notice that the Killing spinors naturally decompose into two real spinors of $SO(1,2)$. The first of these corresponds to the standard $\mathcal{N}=1$ in 2+1 dimensions, while the other corresponds to the conformal supersymmetry. In fact, for our purposes it will suffice to consider the flat supersymmetries, as the others are implied by the AdS isometries.

By taking the limit $M_P \to \infty$ with L fixed, we decouple gravity and focus on quantum effects that are purely due to SQED on AdS$_4$. The relevant Lagrangian is, therefore,

$$\begin{aligned}\mathcal{L}/\sqrt{g} = & \, [\text{kinetic} + \text{gauge D terms}] - m(\psi_+\psi_- + \bar{\psi}_+\bar{\psi}_-) \\ & - (m^2 - \frac{2}{L^2})(|\phi_+|^2 + |\phi_-|^2) + \frac{m}{L}(\phi_+\phi_- + \phi_+^*\phi_-^*) \\ & + ig\sqrt{2}\lambda(\psi_+\phi_+^* - \psi_-\phi_-^*) - ig\sqrt{2}\bar{\lambda}(\bar{\psi}_+\phi_+ - \bar{\psi}_-\phi_-),\end{aligned} \tag{2.11}$$

where, without loss of generality, we have taken m to be real. One sees that the scalars acquire non-holomorphic mass terms, originating from the non-vanishing Ricci scalar, and holomorphic (B-type) masses, arising from the compensator F-term. (The fermionic mass and interaction terms, by contrast, retain the same form as in flat space.) The scalar mass eigenstates and their masses are given by

$$\phi_{1,2} = \frac{1}{\sqrt{2}}(\phi_+ \mp \phi_-^*), \tag{2.12}$$

$$m_{1,2}^2 = \frac{1}{L^2}\left(-2 \pm mL + (mL)^2\right). \tag{2.13}$$

Eqs. (1.3),(2.8) imply the presence of an AM contribution to the gaugino mass, given by

$$\Delta_{UV}\mathcal{L} = -\frac{g^2}{16\pi^2 L}\lambda\lambda + h.c. \equiv -\frac{1}{2}m_{UV}\lambda\lambda + h.c.\,. \tag{2.14}$$

As explained above and emphasized in [4], a gaugino mass would be incompatible with supersymmetry in AdS$_4$. Indeed, for $m \neq 0$, there is an additional contribution to m_λ, corresponding to a finite threshold effect at the scale m, where matter is integrated out. This is due to the presence of both a fermion mass and an R-breaking B-type mass for the scalars. By the well known property of AM in flat space, we can directly conclude that, at least for $mL \gg 1$, the threshold effect cancels eq. (2.14), at least up to subleading effects of $O(1/mL)$. However, it would be nice to see the exact cancellation in an explicit computation. Moreover, in the limit $m = 0$, corresponding to conformal multiplets, there seems to be a puzzle, in that all sources of R-symmetry breaking disappear from the matter lagrangian! In other words, for $m = 0$ there is, at first sight, no obvious contribution in addition to eq. (2.14). In [4], it was concluded that the contribution in eq. (2.14) does not affect the physical mass (defined in the sense of the representation of AdS), since g^2 runs to zero in the infrared. This explanation is, however, puzzling, as it requires an all-orders resummation of diagrams, while we expect the supersymmetry algebra to be satisfied at each finite order in perturbation theory. Furthermore, this argument cannot be applied to the non-Abelian case. In actual fact, the resolution of the gaugino mass puzzle has to do with the boundary conditions in AdS, which shall be discussed in the next section. What we shall find there is that boundary effects provide a calculable, IR saturated, contribution to the gaugino bilinear in the 1-loop 1PI effective action. This contribution corresponds to a mass m_{IR} which exactly cancels the UV one

$$m_{UV} + m_{IR} = 0\,. \tag{2.15}$$

2.2 Boundary conditions

The most relevant feature of AdS space, for our discussion, is the presence of a (conformal) boundary located at $z = 0$ in the Poincaré patch (2.9). One immediate consequence of the presence of a 2+1-dimensional boundary is that chiral symmetry is always broken in AdS_4 [14]. This is fully analogous to what happens in a field theory on half of flat space: when a fermion travelling towards the boundary is reflected, the momentum flips sign, while J_z is conserved. Thus, helicity is not conserved.

More formally, chiral symmetry is broken by the boundary conditions that are necessary to define the theory. This can be seen by considering a two component spinor propagating on half of flat space, with action

$$S = \frac{1}{2}\int_{z\geq 0} d^4x\left[\left(-i\psi\sigma^m D_m\bar{\psi} - m\psi\psi\right) + \text{h.c.}\right]. \tag{2.16}$$

The variation of the action is

$$\delta S = (EOM) - \frac{i}{2}\left[\delta\psi\sigma^3\bar{\psi} - \text{h.c.}\right]_{z=0}. \tag{2.17}$$

In order to obtain sensible boundary conditions (*i.e.* not over-constraining), a boundary term $-\frac{1}{4}\int_{z=0} e^{-i\varphi}\ \psi\psi + \text{h.c.}$ must be added to the action, where φ is an arbitrary phase. The variational principle then demands that

$$\psi_\alpha\Big|_{z=0} = ie^{i\varphi}\sigma^3_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}\Big|_{z=0}, \tag{2.18}$$

implying that chiral symmetry is broken even for vanishing bulk mass[1].

The generalization to AdS requires some care, because of the divergent scale factor at $z = 0$. The boundary conditions in this case can be derived by considering the behavior of the solutions close to $z = 0$. Without loss of generality, we can choose $mL > 0$. Normalizability of the solution requires that

$$\begin{aligned} mL \geq \frac{1}{2}: &\qquad \psi \propto z^{\frac{3}{2}+mL}\xi \qquad \Longrightarrow \qquad \xi_\alpha = -i\sigma^3_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} \\ 0 \leq mL < \frac{1}{2}: &\qquad \psi \propto z^{\frac{3}{2}\pm mL}\xi \qquad \Longrightarrow \qquad \xi_\alpha = \mp i\sigma^3_{\alpha\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} \end{aligned} \tag{2.19}$$

[1] For $m = 0$, without loss of generality one can choose $\varphi = 0$.

and again chiral symmetry is necessarily broken. Note that for the AdS case, there is no freedom to chose the phase φ. This is basically because the bulk mass operator itself plays the rôle of a boundary mass term. This is easily seen by performing a Weyl rescaling, $\psi = (z/L)^{3/2}\chi$: the lagrangian for χ is just given by eq. (2.16), but with a position dependent mass $m \to ML/z$, which blows up at $z = 0$. The exponent in the asymptotic behavior is precisely the index E of the corresponding representation. Note that for $mL < 1/2$, two inequivalent boundary conditions are possible, corresponding to a double quantization, as happens for scalars in AdS [5]. The existence of one and two solutions respectively for $mL \geq 1/2$ and $0 \leq mL < 1/2$, nicely matches eq. (2.2) and the unitarity bound $E \geq 1$.

In the QED case, the boundary condition (2.19) for a single charged spinor would break electric charge; in order to conserve electric charge, the boundary conditions must relate ψ_+ to $\bar{\psi}_-$.[2] Repeating the exercise above with the two spinors, normalizability of the solutions requires

$$\begin{aligned} mL \geq \frac{1}{2}: &\qquad \psi_-, \psi_+ \propto z^{\frac{3}{2}+mL} &\Longrightarrow&\qquad \psi_{+\alpha} = -i\sigma^3_{\alpha\dot\alpha}\bar{\psi}_-^{\dot\alpha} \\ 0 \leq mL < \frac{1}{2}: &\qquad \psi_-, \psi_+ \propto z^{\frac{3}{2}\pm mL} &\Longrightarrow&\qquad \psi_{+\alpha} = \mp i\sigma^3_{\alpha\dot\alpha}\bar{\psi}_-^{\dot\alpha} \end{aligned} \tag{2.20}$$

Given the boundary conditions for the fermions, supersymmetry then determines the boundary conditions for the scalars. By acting with the unbroken supersymmetries (2.10) on the fermionic boundary conditions, one finds

$$\begin{aligned} mL \geq \frac{1}{2}: &\qquad z \to 0 &\Longrightarrow&\qquad \phi_+ = \phi_-^* \left[1 + O(z)\right] \\ 0 \leq mL < \frac{1}{2}: &\qquad z \to 0 &\Longrightarrow&\qquad \phi_+ = \pm\phi_-^* \left[1 + O(z)\right] \end{aligned} \tag{2.21}$$

where the sign in the second eq. is correlated with the sign for the fermions. We can see that this is consistent with the equations of motion for the scalars: In the scalar sector, by solving the wave equation for the two mass eigenstates, ϕ_1 and ϕ_2, we find that

$$\begin{aligned} \lim_{z\to 0} \phi_1 &= z^{2+mL} A_2(x) + z^{1-mL} B_2(x) \\ \lim_{z\to 0} \phi_2 &= z^{1+mL} A_1(x) + z^{2-mL} B_1(x)\,. \end{aligned} \tag{2.22}$$

For $mL > 1/2$, normalizability alone implies that $B_1 = B_2 = 0$, corresponding to the first solution in eq. (2.21). For $mL < 1/2$, the mass of the two scalars is in the range where double

[2]Indeed, for $mL > 1/2$ the charge preserving boundary condition is forced on us by normalizability. For $0 \leq mL < 1/2$, compatibly with normalizability, there exist two other, inequivalent, charge-breaking boundary conditions. We will consider these other possibilities elsewhere.

quantization is allowed, and so we can choose $A_1 = A_2 = 0$ (consistently with supersymmetry), corresponding to the second solution in (2.21). Note that, as a combined effect of the boundary conditions for fermions and scalars, $R-$symmetry is broken in the matter sector even for $m = 0$.

Finally, we can also fix the boundary condition for the vector multiplet. By taking Neumann boundary conditions for the gauge field and acting with the supersymmetry transformations, we find that the appropriate sign of the gaugino boundary condition is

$$\partial_z F^{\mu\nu}\big|_{z=0} = 0 \qquad F^{\mu 3}\big|_{z=0} = 0 \qquad \lambda_\alpha = i\sigma^3_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}\big|_{z=0} \tag{2.23}$$

To summarize, the presence of the boundary in AdS_4 always breaks chirality and $R-$symmetry, even when there is no source of explicit breaking in the bulk action. The physics is essentially that of half of flat space. What is special to AdS_4 is that the chiral symmetry is broken, while the maximal number of isometries is preserved. This is, of course, crucial to give a meaning to a mass smaller than the curvature of the space.

3 Gaugino Mass

The boundary conditions derived above provide the necessary 'mass insertions' to give rise to an IR contribution to the gaugino mass. Focussing on the case of massless SQED, let us now compute the gaugino mass at 1-loop order.

3.1 Chiral breaking correction to the self energy

The computation is particularly transparent in the case of massless matter, where the chiral symmetry breaking is entirely due to the boundary effects. (We present the massive case in the appendix.) When $m = 0$, the chiral matter supermultiplet is conformally coupled. As a consequence, the full SQED action in this case is invariant under Weyl transformations at the classical level. This allows us to map the theory in AdS space to one living on half of flat space and perform all the computations using familiar flat space formulae. This is achieved via the superconformal rescaling

$$\phi = \left(\frac{z}{L}\right)\hat{\phi}, \qquad \psi = \left(\frac{z}{L}\right)^{\frac{3}{2}}\hat{\psi}, \qquad \lambda = \left(\frac{z}{L}\right)^{\frac{3}{2}}\hat{\lambda}, \qquad A_M = \hat{A}_M, \tag{3.1}$$

$$s = \left(\frac{z}{L}\right)\hat{s}, \qquad u = \left(\frac{z}{L}\right)^2\hat{u}, \qquad g_{MN} = \left(\frac{z}{L}\right)^2\hat{g}_{MN}. \tag{3.2}$$

After the rescaling, $\hat{g}_{MN} \equiv \eta_{MN}$ and $\hat{S} = (L/z)(1 + \theta^2/z)$. Since SQED is Weyl invariant (at tree level), the compensator decouples, and we are left with the tree level action for massless, SQED in half of flat space, with a boundary at $z = 0$. The boundary conditions on the fields are most easily implemented by performing an orbifold projection. From the results in the previous section, we have (dropping the circumflexes on the fields),

$$\begin{aligned}
\psi_+(X) &= -i\sigma^3\bar{\psi}_-(\tilde{X}), \\
\phi_+(X) &= \phi^*_-(\tilde{X}), \\
A_\mu(X) &= A_\mu(\tilde{X}), \\
A_z(X) &= -A_z(\tilde{X}), \\
\lambda(X) &= i\sigma^3\bar{\lambda}(\tilde{X}),
\end{aligned} \tag{3.3}$$

where $\tilde{X} = (x, -z)$ is the position of the image point. The flat space propagators can be written down directly using the method of image charges. For the scalars, one has

$$\begin{aligned}
\langle\phi_+(X_1)\phi^*_+(X_2)\rangle &= \langle\phi_-(X_1)\phi^*_-(X_2)\rangle = \frac{1}{4\pi^2}\frac{1}{(X_1 - X_2)^2 + i\epsilon}, \qquad (3.4)\\
\langle\phi_+(X_1)\phi_-(X_2)\rangle &= \langle\phi^*_-(X_1)\phi^*_+(X_2)\rangle = \frac{1}{4\pi^2}\frac{1}{(X_1 - \tilde{X}_2)^2 + i\epsilon}.
\end{aligned}$$

Similarly, for the fermions,

$$\begin{aligned}
\langle\psi_{+\alpha}(X_1)\bar{\psi}_{+\dot{\beta}}(X_2)\rangle &= \langle\psi_{-\alpha}(X_1)\bar{\psi}_{-\dot{\beta}}(X_2)\rangle = \frac{i}{2\pi^2}\frac{(X_1 - X_2)_M\sigma^M_{\alpha\dot{\beta}}}{[(X_1 - X_2)^2 + i\epsilon]^2}, \qquad (3.5)\\
\langle\psi_{+\alpha}(X_1)\psi^\beta_-(X_2)\rangle &= -\frac{1}{2\pi^2}\frac{(X_1 - \tilde{X}_2)_M(\sigma^M\bar{\sigma}^3)_\alpha^\beta}{[(X_1 - \tilde{X}_2)^2 + i\epsilon]^2}.
\end{aligned}$$

One can see that the $i\epsilon$ prescription in Feynman's propagator selects implicitly boundary conditions at $z = \infty$: these are the Hartle-Hawking boundary conditions, appropriate to the Poincaré patch [10].

The off-diagonal propagators determine the chiral-breaking contribution to the gaugino self-energy in Fig. 1

$$\Sigma_\alpha{}^\beta(X_1, X_2) = i\langle J_\alpha(X_1)J^\beta(X_2)\rangle, \tag{3.6}$$

where $J_\alpha = i\sqrt{2}g(\phi^*_+\psi_{+\alpha} - \phi^*_-\psi_{-\alpha})$ and where our convention on the self-energy is defined by $\Gamma_{1PI} \supset \int \frac{1}{2}\lambda^\alpha(X_1)\Sigma_\alpha{}^\beta(X_1, X_2)\lambda_\beta(X_2)$. Performing the Wick contractions, we have

$$\begin{aligned}
\Sigma_\alpha{}^\beta(X_1, X_2) &= 4ig^2\langle\phi^*_+(X_1)\phi^*_-(X_2)\rangle\langle\psi_{+\alpha}(X_1)\psi^\beta_-(X_2)\rangle, \\
&= -\frac{ig^2}{2\pi^4}\frac{(X_1 - \tilde{X}_2)_M(\sigma^M\bar{\sigma}^3)_\alpha^\beta}{[(X_1 - \tilde{X}_2)^2 + i\epsilon]^3}.
\end{aligned} \tag{3.7}$$

Notice that this contribution is non-local, and comes from long-distance physics, as opposed to eq. (2.14). In order to extract from $\Sigma_\alpha{}^\beta(X_1, X_2)$ the correction to the gaugino mass, we must

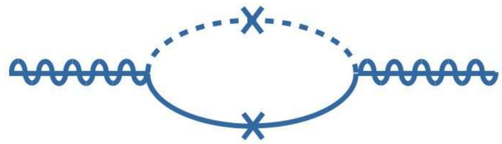

Figure 1: Chiral breaking 1-loop correction to the gaugino self energy. The "mass" insertions correspond to boundary effects.

evaluate it on a solution of the massless (tree level) wave equation. This is the analogue of computing the self-energy at zero momentum in flat space. The general solution of the bulk Dirac equation for a massless gaugino is

$$\lambda_0(X) = e^{ip_M X^M}\xi, \qquad \bar{\sigma}^M p_M \xi = 0, \qquad p^M p_M = 0\,. \tag{3.8}$$

Physical states must also satisfy the boundary condition in eq. (2.23). In order to achieve that, two solutions with opposite velocity, p^3/p^0, in the z-direction should be superimposed. However, as we shall explain in a moment, the correct procedure we must follow in the Poincaré patch in order to study the 1-loop corrected wave equation is to work with solutions of the Dirac equation that satisfy boundary conditions at the horizon $z \to \infty$ rather than at the boundary $z = 0$. This is closely related to the AdS/CFT prescription. Alternatively we could overcome this issue by performing an euclidean computation, as in this case Poincaré co-ordinates cover the whole space, but we find it more physical to address directly the Lorentzian point of view.

To obtain the IR contribution to the gaugino mass, we must convolute eq. (3.7) with (3.8). We thus find,

$$\begin{aligned}\int d^4X_2\, \Sigma_\alpha{}^\beta(X_1,X_2)\lambda_{0\beta}(X_2) &= -\frac{ig^2}{8\pi^4}\int d^4X_2 \frac{\partial}{\partial \tilde{X}_2^M}\left(\frac{(\sigma^M\bar{\sigma}^3)_\alpha^\beta}{[(X_1-\tilde{X}_2)^2+i\epsilon]^2}\right)\lambda_{0\beta}(X_2)\\ &= \frac{ig^2}{8\pi^4}\int d^3x_2 \frac{1}{[(x_1-x_2)^2+z_1^2+i\epsilon]^2}e^{ip_\mu x_2^\mu}\xi_\alpha\end{aligned} \tag{3.9}$$

where in the last step we integrated by parts and used $\bar{\sigma}^M\partial_M\lambda_0 = 0$. In the resulting boundary integral, we used the explicit expression for λ_0 in (3.8). Notice that x are coordinates on the boundary. Performing the last integral explicitly we thus find,

$$\frac{1}{2}\int d^4X_2\, \Sigma_\alpha{}^\beta(X_1,X_2)\lambda_{0\beta}(X_2) = \frac{g^2}{16\pi^2}\frac{1}{z_1}e^{i(p_\mu x_1^\mu+|p|z_1)}\xi_\alpha, \tag{3.10}$$

where the $i\epsilon$ in the original integral fixes the sign of $p_3 = \sqrt{-p_\mu p^\mu}$ to be positive. The right hand side of eq. (3.10) is proportional to the original spinor if this satisfies the Hartle-Hawking boundary conditions: positive frequencies purely outgoing and negative frequencies purely incoming. This means that when evaluated on this class of solutions of the bulk Dirac equation, the IR contribution to the self-energy $\Sigma_\alpha{}^\beta$, acts like a mass term m_{IR} which is precisely equal and opposite to the anomaly mediated contribution (see eq. (2.14) after performing the Weyl rescaling in eqs. (3.1),(3.2)). Thus an exact cancellation between UV and IR effects arises, as promised in eq. (2.15). It is the clever relation among these two contributions that ensures the masslessness of the gaugino, as demanded by supersymmetry. This is the main result of our paper.

It remains to be explained why our computation works only for the class of solutions of the form (3.10). These solutions correspond to the creation of incoming particles at the past horizon H^- and to the destruction of outgoing particles at the future horizon H^+ that separate the Poincaré patch from the rest of AdS. Intuitively such processes can be described by causality using solely the fields in the Poincaré patch. Other solutions correspond to processes that are not captured by the Poincaré patch alone and probe other regions of global AdS. In this case there will be extra-contributions from the rest of the space and a computation in global coordinates would be required. That such contributions exist follows from the fact that the Feynman propagator is non-vanishing between a point inside the Poincaré patch and one outside. Had we worked in global coordinates we could have directly checked that the cancellation of the gaugino mass occurs for arbitrary physical states (i.e. solutions of the wave equation that satisfy the boundary conditions).

Our result can however be readily interpreted from the viewpoint of the AdS/CFT correspondence [9]. Even though Lorentzian AdS/CFT is not nearly as developed as on Euclidean space, we do not see obvious obstructions in the case at hand.[3] From this perspective, the boundary field combination

$$\lambda_-(x) = \lambda(x) - i\sigma^3\bar{\lambda}(x) \tag{3.11}$$

should be viewed as an external source probing the system (the dual CFT). Notice that λ_- is precisely the combination that is set equal to zero for the AdS quantum fields. Performing a path integral over the bulk fields with vacuum boundary conditions at $H^\pm$ one obtains a functional $Z(\lambda_-)$ which generates the correlators of the associated dual operator in the CFT. Given λ_-, a classical source localized at the boundary, the choice of initial and final vacuum states for our path integral fixes the boundary condition for the corresponding bulk field at $z \to \infty$. Working with plane waves, this prescription corresponds precisely to the Hartle-Hawking boundary condition we encountered previously. This gives a prescription for finding

[3] Indeed it is to be expected that, just as there is a procedure to analytically continue a CFT from Euclidean to Lorentzian space, there should also exist a similar procedure to analytically continue the correspondence from Euclidean to Lorentzian AdS. At least in some simple cases this was outlined for instance in Refs. [10,11].

a unique extension of λ_- into the bulk, by requiring that its (effective) action be stationary.

At tree level, we have the boundary effective action

$$\ln Z = S_{bd} = -\frac{1}{4}\int d^3x\left(\lambda\lambda + \bar\lambda\bar\lambda\right) = \int d^3x \lambda_- \frac{\sigma^3\bar\sigma^\mu}{\sqrt{\partial^2 + i\epsilon}}\partial_\mu\lambda_-, \tag{3.12}$$

corresponding to the correlator of a dual fermionic current of scaling dimension $\frac{3}{2}$:

$$\langle O_\alpha(x) O^\beta(0)\rangle = \frac{x^\mu(\sigma^3\bar\sigma_\mu)_\alpha^\beta}{(x^2+i\epsilon)^2}. \tag{3.13}$$

The 1-loop computation we have performed is directly translated into a 1-loop computation of the boundary effective action. The only difference from before is that we need to consider also solutions with Euclidean boundary momenta $p_\mu p^\mu > 0$. In this case the solution in the bulk corresponds to the unique regular solution at $z \to \infty$ as prescribed by Euclidean AdS/CFT. Needless to say the previous computation can be continued to the Euclidean region so that the self energy is diagonal on these solutions. Working at 1-loop accuracy, the corrected boundary effective action is simply obtained by substituting the tree level bulk solution into the 1PI bulk effective action. However our previous result was precisely that the total (UV + IR) 1PI vanishes on the very solution of the massless Dirac equation that satisfied the AdS/CFT boundary conditions at $z \to \infty$ (that is with the same exponent as in eq. (3.10)). Thus we conclude that at the 1-loop level the boundary action is unaffected and thus the dimension of the CFT operator dual to the gaugino field is not renormalized, consistently with supersymmetry.

What we have learned is an amusing lesson on the rôle of the anomaly mediated gaugino mass. The basic reason for its existence is that AdS_4 behaves as 2+1-dimensional field theory as far as chirality is concerned. The mass of fermions is thus additively renormalized by calculable boundary effects. On the other hand, supersymmetry mandates the gaugino to be exactly massless. The simple SQED case, in the end, shows that the only way to achieve this is via the existence of suitable short distance effects, in one-to-one correspondence with the long distance effects. This is yet another illustration of the UV insensitivity of anomaly mediation.

3.2 Chiral preserving correction: wave function renormalization

In the previous section we have shown that the chiral breaking part in the 1-loop self energy does not correct the gaugino mass, nor, similarly, does it correct the boundary effective action. However, strictly speaking there is yet another contribution to the gaugino self-energy that we need to consider. This is the 'chirality-preserving' contribution, $\Sigma_{\alpha\dot\beta}$, the one associated with wave-function renormalization. The issue at hand arises even in the absence of supersymmetry. We will show that this contribution vanishes when acting on a massless spinor. This result may seem obvious at first sight, based on our usual flat space intuition. Indeed, in flat Minkowsky space, Lorentz invariance constrains this term to be proportional to $f(\Box)\,\bar{\not\partial}$, which vanishes on-shell as long as f is not too singular (in fact, f is a logarithmic function). However, the situation is more subtle in AdS, since, at the quantum level, the boundary makes itself felt even inside the bulk, and therefore the z direction is not manifestly equivalent to the others. The purpose of this section is to clarify this issue. An extra complication comes from the need to regularize the divergent part of $\Sigma_{\alpha\dot{\beta}}$. We shall again focus on massless SQED, for which we can work in the conformally rescaled basis (3.2). The general case is briefly considered in the appendix. Working in position space, we find it convenient to use the method of differential regularization [12].

The unregulated $\Sigma_{\alpha\dot{\beta}}$ is given by,

$$\begin{aligned}\Sigma_{\alpha\dot{\beta}}(X_1,X_2) &= i\langle J_\alpha(X_1)J_{\dot{\beta}}(X_2)\rangle \\ &= -4ig^2\langle\phi_+(X_1)\phi_+^*(X_2)\rangle\langle\psi_{+\alpha}(X_1)\bar{\psi}_{+\dot{\beta}}(X_2)\rangle\end{aligned} \tag{3.14}$$

This corresponds to the following correction to the effective action

$$\Gamma = -\frac{g^2}{2\pi^4}\int d^4X_1 d^4X_2 \bar{\lambda}(X_1)\frac{X_{12M}\bar{\sigma}^M}{(X_{12}^2+i\epsilon)^3}\lambda(X_2), \tag{3.15}$$

where $X_{12} = (X_1 - X_2)_M$. This expression has, however, a non-integrable singularity at $X_{12}=0$, which must be regulated. Naïvely, using differential regularization amounts to replacing

$$\frac{X_{12M}}{(X_{12}^2+i\epsilon)^3} \to \frac{1}{16}\frac{1}{\partial X_1^M}\left(\Box_1\frac{\ln(X_{12}^2M^2)}{X_{12}^2+i\epsilon}\right), \tag{3.16}$$

where M plays the rôle of the renormalization mass scale. This cannot, however, be the full story, since the explicit mass scale M breaks dilatation invariance $X\to kX$. In the rescaled basis, $SO(3,2)$ arises as the subgroup of $SO(4,2)$ which is left unbroken by the compensator background $\tilde{s} = L/z$ [13]. Consequently the regulated self-energy in eq. (3.16) does not respect the AdS isometries. As the lack of invariance follows from the regularization, the counterterm needed to restore the symmetry must be local, and must of course involve the compensator. By simple reasoning one can quickly derive the unique form of this counterterm. In order to do so, let us imagine that we had regulated the loop in a manifestly covariant fashion, by introducing Pauli Villars fields with mass M. The crucial aspect of Pauli-Villars fields is that, being massive, their quadratic lagrangian depends directly on the compensator, $\tilde{s}$, via the substitution

$$M \to M\times\tilde{s}(z) = M\times\frac{L}{z}, \tag{3.17}$$

which formally restores conformal invariance. However it does not make any sense to simply perform this replacement in eq. (3.16). To find out how eq. (3.16) is modified we must be a tad more careful. We just need to focus on the M-dependent part of the regulated self-energy. Using the identity

$$\Box \frac{1}{x^2 + i\epsilon} = 4\pi^2 i \delta^4(x), \tag{3.18}$$

the M-dependent part of the effective action is given by

$$\Delta\Gamma_{UV} = -\frac{ig^2}{8\pi^2} \ln M^2 \int d^4X \bar{\lambda}(X) \bar{\sigma}^M \partial_M \lambda(X), \tag{3.19}$$

whose unique local covariantization is[4]

$$\Delta\Gamma_{UV} = -\frac{ig^2}{8\pi^2} \ln(M\tilde{s}(z)) \left[\bar{\lambda}\bar{\sigma}^M \partial_M \lambda - \partial_M \bar{\lambda}\bar{\sigma}^M \lambda\right]. \tag{3.21}$$

The local $\ln \tilde{s}$ term gives the following correction to $\delta\Gamma/\delta\bar{\lambda}$

$$-\frac{ig^2}{8\pi^2} \partial_M \ln \tilde{s} \bar{\sigma}^M \lambda = \frac{ig^2}{8\pi^2} \frac{1}{z} \bar{\sigma}^3 \lambda. \tag{3.22}$$

On the other hand, from eq. (3.16) the 'IR' contribution to the equation of motion is

$$\frac{g^2}{2\pi^4} \frac{1}{16} \int d^4X_2 \; \frac{\partial}{\partial X_2^M} \left(\Box_1 \frac{\ln(X_{12}^2 M^2)}{X_{12}^2} \right) \bar{\sigma}^M \lambda(X_2). \tag{3.23}$$

To investigate how this non-local contribution affects the gaugino mass we must compute it on the solution λ_0 of the massless wave equation specified by (3.8). Integrating by parts and using $\bar{\sigma}^M \partial_M \lambda_0 = 0$, eq. (3.23) becomes

$$\begin{aligned} -\frac{g^2}{32\pi^4} \int d^3x_2 \left(\Box_1 \frac{\ln M^2 X_{12}^2}{X_{12}^2} \right) \bar{\sigma}^3 \lambda_0(X_2)\big|_{z_2=0} &= \frac{g^2}{8\pi^4} \int d^3x_2 \frac{1}{(X_{12}^2 + i\epsilon)^2} \bar{\sigma}^3 \lambda_0(X_2) \\ &= -\frac{ig^2}{8\pi^2} \frac{1}{z_1} \bar{\sigma}^3 \lambda_0(X_1), \end{aligned} \tag{3.24}$$

where the final integral is identical to the one computed in the previous section, eq. (3.9). Again the last identity is only valid for solutions satisfying the Hartle-Hawking boundary conditions. We thus find that the contributions in eqs. (3.22) and (3.24) again cancel so that the propagation of the gaugino is not affected. In particular the gaugino remains massless. Note that, while the cancellation in the previous section relies on supersymmetry, this effect is independent of supersymmetry. This cancellation between UV and IR contributions, dictated by

[4] Indeed, compatibly with locality and power counting, another term is naïvely possible:

$$[\partial_z \ln \tilde{s}(z)] \bar{\lambda} \bar{\sigma}^3 \lambda. \tag{3.20}$$

This term must however be discarded as it explicitly breaks CP (the regulated theory is formally CP-invariant, even though parity is, of course, 'spontaneously' broken by the expectation value of $\tilde{s}$).

the AdS isometry (a subgroup of the conformal group), can be viewed as an $N = 0$ counterpart of the one found previously. This is perhaps not surprising, as anomaly mediation is itself the supersymmetric counterpart of the trace anomaly. Indeed, in a superfield formalism, these two separate cancellations would be manifestly related.

4 Summary

We studied the rôle played by anomaly mediated (AM) mass terms in $N = 1$ theories on AdS_4 with unbroken supersymmetry. For simplicity we focussed on the gaugino mass term in SQED with massless matter. We showed that the AM gaugino mass term is required by the super-AdS algebra in order to exactly cancel another 1-loop contribution, of infrared origin and associated with the AdS boundary. The latter effect originates because chirality ($R-$symmetry in this case) is necessarily broken by reflection at a 2+1-dimensional boundary.

Indeed, by computing first this finite IR effect (which does not require the introduction of a regulator) and by using the fact that the algebra dictates a massless gaugino, we could have argued the need for a local, UV generated, AM contribution. Since the latter is independent of whether the theory lives in flat or curved space, that would have provided yet another derivation of AM gaugino masses. The possibility of relating the AM mass to purely IR quantities illustrates the "UV insensitivity" of this effect, a property which makes it potentially relevant in phenomenological applications. The fact that AM effects represent local parts of non-local structures in the 1PI action is well known. Our result provides a new twist on that perspective: the AM gaugino mass is just a reflection of the breakdown of chirality at the 2+1-d boundary of AdS_4.

There are several directions in which one might extend and improve our result. One obvious possibility is to perform the same computation in the non-abelian case, where, unlike in the abelian case, proper gauge-fixing will be needed. Another problem concerns the rôle of all other AM terms, such as sfermion masses and "A-terms": it should be possible to derive them from consistency conditions as well, but probably in a more subtle way than for the gaugino mass.

In this paper we worked on the Poincaré patch. This procedure is clean for the euclidean case and from the AdS/CFT standpoint: our computation corresponds to checking that, as expected by supersymmetry, the scaling dimension of the operator dual to the gaugino field is not renormalized. The Lorentzian computation is more delicate, as we have to deal with boundary conditions at the horizons which separate the chosen patch from the rest of AdS. It would then be interesting to try to perform the same computation in global coordinates, and check that, in that case, the 1-loop self energy does vanish when convoluted with the normalizable solutions. Finally, it would be interesting to understand the rôle of anomaly mediation purely from the CFT viewpoint. The AdS bulk picture is that the gaugino must be massless even though chirality is broken, corresponding to non-vanishing ${\Sigma_\alpha}^\beta$ off-shell. In the CFT picture, the non-vanishing of ${\Sigma_\alpha}^\beta$, shows up in the 4-point function of operators dual to the AdS matter fields. However it is not immediately obvious how to translate the bulk picture to the boundary, since there is no notion of chirality in 2+1-d field theory.

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Appendix: Massive Charged Matter

The cancellation of the UV and IR contributions to the gaugino mass, being a consequence of the algebra, is a general effect which must hold for any mass of the matter fields. In this appendix we check explicitly the cancellation for arbitrary values of m in the superpotential. This computation can also be interpreted as the derivation of the anomaly mediated UV contributions (2.14), (3.21) using Pauli-Villars fields.

For massless matter, the only source of chiral symmetry breaking is due to the presence of the boundary, while when $m \neq 0$, chiral symmetry is broken also in the bulk. In this case, the matter is not conformally-coupled and, therefore, the propagators cannot be obtained by simply rescaling the flat space results. A full AdS computation is required.

We will need the propagators for a chiral multiplet with arbitrary mass. The scalar propagator associated to the representation $D(E,0)$ ($(mL)^2 = E(E-3)$) is given by[5]

$$\Delta(E,0) = \frac{1}{(4\pi)^2 L^2} \frac{\Gamma[E]\Gamma[E-1]}{\Gamma[2E-2]} \left(\frac{2}{u}\right)^E {}_2F_1\left(E, E-1; 2E-2, -\frac{2}{u}\right),$$

where we have introduced the AdS invariant length,

$$u = \frac{(X_1 - X_2)^2 + i\epsilon}{2z_1 z_2}, \tag{A.1}$$

[5]This formulae hold for $E > 3/2$ where both scalars in the chiral multiplet have standard boundary conditions. This is the range where a single quantization is possible.

The fermion propagator associated to the representation $D(E+1/2,1/2)$ can be found in Ref. [14],

$$\begin{aligned}
\langle\psi_{+\alpha}(X_1)\psi_-^{\beta}(X_2)\rangle &= \frac{-\Gamma[E]\Gamma[E+1]}{(32\pi^2L^3)\Gamma[2E-1]}\left(\frac{2}{u+2}\right)^{E+1} {}_2F_1\left(E+1,E-1;2E-1,\frac{2}{u+2}\right)\times\Gamma_\alpha^{\ \beta},\\
\langle\psi_{\pm\alpha}(X_1)\bar{\psi}_{\pm\dot{\beta}}(X_2)\rangle &= \frac{i\Gamma[E]\Gamma[E+1]}{(32\pi^2L^3)\Gamma[2E-1]}\left(\frac{2}{u+2}\right)^{E+1} {}_2F_1\left(E+1,E;2E-1,\frac{2}{u+2}\right)\times\Gamma_{\alpha\dot{\beta}}
\end{aligned} \tag{A.2}$$

where,

$$\begin{aligned}
\Gamma_\alpha^{\ \beta} &= \frac{(X_1-\tilde{X}_2)_M(\sigma^M\bar{\sigma}^3)_\alpha^{\ \beta}}{\sqrt{z_1z_2}}\\
\Gamma_{\alpha\dot{\beta}} &= \frac{(X_1-X_2)_M\sigma^M_{\alpha\dot{\beta}}}{\sqrt{z_1z_2}}
\end{aligned} \tag{A.3}$$

As in the massless case, the contribution of the matter loop to the gaugino mass arises from the the self-energy (3.6),

$$\Sigma_\alpha^{\ \beta}(X_1,X_2) = 4ig^2\langle\phi_+^*(X_1)\phi_-^*(X_2)\rangle\langle\psi_{+\alpha}(X_1)\psi_-^{\beta}(X_2)\rangle \tag{A.4}$$

where now

$$\langle\phi_+(X_1)\phi_-(X_2)\rangle = \frac{\Delta(E+1,0)-\Delta(E,0)}{2}, \tag{A.5}$$

and the fermion belongs to the representation $D(E+1/2,1/2)$.

In order to compute the contribution to the gaugino mass, we evaluate the self-energy on the solution of the massless gaugino equation as in section 3.1. This highly non-trivial integral of hypergeometric functions can be evaluated numerically by choosing the simplest solution of the massless equation of motion, $\lambda_0(X_1) = z^{3/2}\xi_0$,

$$\int dX_2\sqrt{-g}\,\Sigma_\alpha^{\ \beta}(X_1,X_2)\lambda_{0\beta}(X_2) = -\frac{g^2}{8\pi^2L}\lambda_{0\alpha}(X_1). \tag{A.6}$$

Following the discussion in section 3.1 we expect the same to hold for any solution satisfying the appropriate boundary conditions. This contribution as expected does not depend on the mass and cancels the anomaly-mediated UV contribution, proving for general m that this term is necessary for the consistency of the supersymmetric theory. As a check of this result, one can consider the limit $m \gg 1/L$, as done in [4]. In this limit, the curvature is a small effect and the loop can be computed using flat-space propagators, but with the AdS mass splitting.

For completeness we also checked the wave functions contribution. The chiral preserving contribution to self-energy in general reads,

$$\Sigma_{\alpha\dot{\beta}}(X_1, X_2) = -2ig^2 \left[\langle\phi_1(X_1)\phi_1^*(X_2)\rangle + \langle\phi_2(X_1)\phi_2^*(X_2)\rangle\right] \langle\psi_{+\alpha}(X_1)\psi_{+\dot{\beta}}(X_2)\rangle \tag{A.7}$$

Repeating the same steps as in section 3.2, we find numerically,

$$\int dX_2\sqrt{-g}\Sigma_{\alpha\dot{\beta}}(X_1, X_2)\lambda^{\dot{\beta}}(X_2) = -\frac{g^2}{8\pi^2 L}\lambda_\alpha(X_1). \tag{A.8}$$

independently of the mass. This calculation also proves that by regulating the theory with Pauli-Villars fields there is an $N = 0$ anomaly mediated contribution of the form considered before. In this case the contribution of the heavy fields with $m \gg 1/L$ cannot be obtained with the flat space propagators since this effect is entirely due to the fact that the theory lives in curved space.

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