

MEDIATION OF SUPERSYMMETRY BREAKING IN EXTRA DIMENSIONS

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We review the mechanisms of supersymmetry breaking mediation that occur in sequestered models, where the visible and the hidden sectors are separated by an extra dimension and communicate only via gravitational interactions. By locality, soft breaking terms are forbidden at the classical level and reliably computable within an effective field theory approach at the quantum level. We present a self-contained discussion of these radiative gravitational effects and the resulting pattern of soft masses, and give an overview of realistic model building based on this set-up. We consider both flat and warped extra dimensions, as well as the possibility that there be localized kinetic terms for the gravitational fields.

Keywords: supersymmetry breaking, soft terms, gravity mediation.

1. Introduction

The most important ingredient of supersymmetric extensions of the standard model is certainly the way in which supersymmetry is broken. The standard set-up consists of a visible sector containing the standard matter fields and a hidden sector hosting spontaneous supersymmetry breaking. The effect of supersymmetry breaking in the visible sector is then parametrized by a finite set of soft breaking terms. These soft terms fully characterize the phenomenology of such a model, and are for this reason often treated as free empirical parameters. However, it is an important and challenging question to find a satisfactory and natural microscopic realization of the interesting regions in the space of these parameters.

A particularly interesting and natural possibility is that the visible and the hidden sectors interact only through gravitational interactions and supersymmetry breaking is gravity-mediated [1, 2]. In this situation, the typical scale of the soft masses is given by $m_{\text{soft}} \sim M_{\text{susy}}^2/M_{\text{P}}$, where M_{susy} is the scale at which spontaneous supersymmetry breaking occurs in the hidden sector and M_{P} is the Planck mass characterizing the strength of gravitational interactions. Since gravitational interactions are non-renormalizable, the soft terms depend on the unspecified UV completion of the theory. The best we can do in general is then to parametrize our ignorance about UV effects through higher-dimensional operators suppressed by suitable powers of M_{P} , within a low-energy effective field theory approach. The

dimensionless coefficients controlling these operators are expected to be generically of order one, although their precise values remain out of reach.

The pattern of soft terms predicted by the above scenario is qualitatively compatible with phenomenological requirements. In particular, all the soft masses have the same order of magnitude, which can be tuned to the electroweak scale by choosing M_{susy} to be an intermediate scale. However, at a more quantitative level there are some conceptual problems. Most importantly, the scalar soft masses must be nearly flavour-universal in order not to spoil the Glashow–Iliopoulos–Maiani mechanism suppressing flavour-changing processes in the standard model. There is however no obvious reason for this to be the case in general, even assuming a theoretical description of the flavour structure of the theory relying on some new symmetry that is spontaneously broken at a scale M_{flav} . Indeed, since M_{P} is supposed to be close to the fundamental scale of the theory, it is natural to imagine that M_{flav} is of the same order of magnitude as it, and the soft terms are thus expected to be flavour-generic.

The essence of the above-described supersymmetric flavour problem lies in the fact that even if the soft scalar masses are taken to be flavour-diagonal at the classical level, quantum effects will generically induce unacceptably large flavour-breaking corrections. A natural solution to this problem could then come from some mechanism that is able to screen the soft scalar masses from these dangerous effects. However, it is straightforward to convince one-self that it is impossible to implement this by just imposing some kind of approximate symmetry. Interestingly enough, a radically new possibility turns out to emerge from the notion of locality in the presence of extra dimensions [3]. The crucial idea is that if the visible and the hidden sectors are separated by an extra dimension, any transmission effect for supersymmetry breaking must necessarily correspond to operators that are non-local along the internal dimensions. As a consequence, the soft scalar masses will vanish at the classical level and be dominated by soft quantum effects that are flavour-universal and saturated in the IR, at the compactification scale. These effects are finite and depend in a significant way only on the spectrum of light modes of the higher-dimensional theory. They can thus be reliably computed within an effective supergravity description.

2. Supersymmetry breaking and sequestering

Let us start by describing the way in which supersymmetry is broken in gravity-mediated models. To do so, we consider a supergravity theory with visible and hidden sectors labelled by the index $i = \text{v, h}$. The minimal set-up is obtained by introducing some sample chiral and vector multiplets $\Phi_i = (\phi_i, \chi_i; F_i)$ and $V_i = (A_i^\mu, \lambda_i; D_i)$ in these two matter sectors. Using the superconformal approach to supergravity [4], the gravitational interactions are described by a conformal gravity multiplet $C = (e_\mu^a, \psi_\mu; a_\mu, b_\mu)$ and a conformal compensator chiral multiplet $S = (\phi_S, \psi_S; F_S)$. The full superconformal group is gauged and is reduced to the

ordinary super-Poincaré group by gauge-fixing the extra local symmetries through the conditions $b_\mu = 0$, $\phi_S = 1$ and $\psi_S = 0$.

Using the above framework, the couplings between the matter fields and the scalar auxiliary field of supergravity that are relevant for supersymmetry breaking can be written in a particularly simple and illuminating form. Indeed, the general form of this part of the Lagrangian is obtained by taking ordinary rigid supersymmetry D and F densities of an arbitrary Kähler function Ω and an arbitrary superpotential and kinetic functions W and G_i . At the classical level of the effective theory, the dependence on the conformal compensator is uniquely fixed by Weyl invariance and the matter Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{\text{mat}} = & [\Omega(\Phi_j, \Phi_j^\dagger) S S^\dagger]_D + [W(\Phi_j) S^3]_F + [W(\Phi_j) S^3]_F^\dagger \\ & + [G_i(\Phi_j) \mathcal{W}_i^2]_F + [G_i(\Phi_j) \mathcal{W}_i^2]_F^\dagger . \end{aligned} \quad (1)$$

The functions Ω , W and G_i , which parametrize this Lagrangian, have general expressions consisting of infinite series in inverse powers of M_{P} :

$$\Omega = -3M_{\text{P}}^2 + \Phi_{\text{v}} \Phi_{\text{v}}^\dagger + \Phi_{\text{h}} \Phi_{\text{h}}^\dagger + \frac{h}{M_{\text{P}}^2} \Phi_{\text{v}} \Phi_{\text{v}}^\dagger \Phi_{\text{h}} \Phi_{\text{h}}^\dagger + \dots , \quad (2)$$

$$W = \Lambda^3 + M_{\text{susy}}^2 \Phi_{\text{h}} + \dots , \quad (3)$$

$$G_i = g_i^{-2} + \delta_{i,h} \frac{k}{M_{\text{P}}} \Phi_{\text{h}} + \dots . \quad (4)$$

For simplicity, we have discarded all the dependence on the gauge fields other than their quadratic kinetic terms, and left the gauge quantum numbers of the matter fields unspecified. In the above schematic writing, it should be remembered that the second, third and fourth terms in (2) are always admissible, whereas the second terms in (3) and (4) can occur only for singlets. Moreover, in order to achieve a vanishing cosmological constant, we must in general tune $\Lambda^3 \sim M_{\text{susy}}^2 M_{\text{P}}$. The supersymmetry breaking VEVs of the auxiliary fields are then found to be:

$$F_{\text{h}} \sim M_{\text{susy}}^2 , \quad F_S \sim \frac{\Lambda^3}{M_{\text{P}}^2} \sim \frac{M_{\text{susy}}^2}{M_{\text{P}}} .$$

It follows that the soft masses are given by

$$m_{3/2} \sim |F_S| \sim \frac{M_{\text{susy}}^2}{M_{\text{P}}} , \quad (5)$$

$$m_{1/2} \sim k \frac{|F_{\text{h}}|}{M_{\text{P}}} \sim k \frac{M_{\text{susy}}^2}{M_{\text{P}}} , \quad (6)$$

$$m_0^2 \sim h \frac{|F_{\text{h}}|^2}{M_{\text{P}}^2} \sim h \frac{M_{\text{susy}}^4}{M_{\text{P}}^2} . \quad (7)$$

From the above expressions, we see that the soft gaugino and scalar masses are controlled by the dimensionless coefficients h and k of the leading operators mixing the two sectors. Their values are undetermined within the effective theory, and are

therefore a priori completely generic. In particular, the parameter h can have an arbitrary sign and flavour structure.

At the quantum level, the low-energy effective theory can acquire an anomalous dependence on the compensator multiplet due to the conformal anomaly induced by quantum fluctuations of the light gauge fields. More precisely, Weyl invariance dictates that the dependence on the energy scale μ of all the running couplings must come together with a dependence on the compensator multiplet S , in such a way that only the quantity $\mu/\sqrt{SS^\dagger}$ appears. This leads to anomaly-mediated corrections to the soft masses. In terms of the gauge loop factor $\alpha_{\text{gau}} = g_0^2/(16\pi^2)$, they have the form [3, 5]

$$\delta m_{1/2} \sim a \alpha_{\text{gau}} |F_S| \sim a \alpha_{\text{gau}} \frac{M_{\text{susy}}^2}{M_{\text{P}}}, \quad (8)$$

$$\delta m_0^2 \sim b \alpha_{\text{gau}}^2 |F_S|^2 \sim b \alpha_{\text{gau}}^2 \frac{M_{\text{susy}}^4}{M_{\text{P}}^2}. \quad (9)$$

The coefficients a and b are group-theoretical factors that depend only the quantum numbers of the corresponding fields. In particular, the coefficient b is flavour-universal and has a definite sign, which turns out to be positive for asymptotically free gauge groups and negative otherwise. This implies that the anomaly-mediated contribution is positive for the squarks and negative for the sleptons.

In a generic situation, the coefficients controlling the gravity-mediated contributions to the soft masses are of order one, whereas the coefficients controlling the anomaly-mediated contributions are instead of order α_{gau} . The latter represent thus negligible corrections and the situation is as bad as described in the introduction. This matter of state does however radically change in sequestered models. Indeed, as already explained, in such models the dimensionless coefficients h and k are suppressed and computable. They are forced to vanish at the tree-level in the counting of heavy-mode loops, and their values are therefore determined by the one-loop effect. These are easily estimated by simple power counting to be of the order of $\alpha_{\text{gra}} = M_{\text{KK}}^2/(16\pi^2 M_{\text{P}}^2)$, that is a loop factor for gravity saturated at the compactification scale M_{KK} , which is defined as the mass scale of the Kaluza–Klein modes divided by π and represents the scale at which the constraints put by locality become effectively significant. In sequestered models, we therefore get:

$$k = c \alpha_{\text{gra}}, \quad h = d \alpha_{\text{gra}}. \quad (10)$$

The coefficients c and d are numbers of order one that can be deduced only through an explicit computation. In particular, d is flavour-universal but can a priori have an arbitrary sign.

From the above discussion, we understand that sequestered models can potentially solve the supersymmetric flavour problem of gravity-mediated models and provide a framework where the soft scalar masses are naturally universal and radiatively induced by two distinct effects associated to gauge and gravitational loops. In order to get a interesting model, these two effects must be able to compete, in

order for the slepton masses squared to have a chance to be positive [6]. This can happen if $\alpha_{\text{gra}} \sim \alpha_{\text{gau}}^2$, that is:

$$\frac{M_{\text{KK}}}{M_{\text{P}}} \sim 4\pi\alpha_{\text{gau}}. \quad (11)$$

This is a reasonable possibility, which is compatible with the use of an effective low-energy approach and can therefore be efficiently investigated in concrete models with extra dimensions. It gives a strong motivation for explicitly computing the above discussed gravitational loop contributions to the scalar soft masses in concrete classes of sequestered models and investigate their viability more precisely. This program involves also studying and taking into account the additional effects occurring in these models due to the radion multiplet parametrizing the dynamics of the extra compact dimension. We will see in the following that these are qualitatively similar to those discussed in this section for four-dimensional models, but they will turn out to be absolutely crucial for the possibility of having a positive contribution from the gravitational loop effects.

3. Minimal sequestered models

The simplest realization of sequestered models is achieved by introducing a single extra dimension and locating the visible and the hidden sectors at two branes positioned at fixed points in the internal dimension. We shall consider the general case of a warped geometry of the Randall–Sundrum type [7]. The formulation of the locally supersymmetric version of this class of compactifications has been the object of active study in the past years.^a It consists in a gauged 5D supergravity theory compactified on the orbifold S^1/\mathbf{Z}_2 . We denote by y the coordinate of the internal circle and by R its radius. The two fixed points of the orbifold action are located at $y_0 = 0$ and $y_1 = \pi R$, and they represent the boundaries of the physical segment of internal space. Using the notation $\delta_i(y) = \delta(y - y_i)$, the Lagrangian of the theory takes then the form:

$$\mathcal{L} = \mathcal{L}_5 + \delta_0(y)\mathcal{L}_0 + \delta_1(y)\mathcal{L}_1. \quad (12)$$

The kinetic parts of the bulk and boundary Lagrangians are given by

$$\mathcal{L}_5^{\text{kin}} = \sqrt{g_5} \left[-\Lambda_5 - \frac{1}{2}M_5^3 \left(\mathcal{R}_5 + i\bar{\Psi}_M \left(\Gamma^{MRN} D_R - \frac{3i}{2}k\epsilon(y)\Gamma^{MN} \right) \Psi_N + \frac{1}{2}F_{MN}^2 \right) \right], \quad (13)$$

$$\mathcal{L}_i^{\text{kin}} = \sqrt{g_4} \left[-\Lambda_i - \frac{1}{2}M_i^2 \left(\mathcal{R}_4 + i\bar{\Psi}_\mu \gamma^{\mu\rho\nu} D_\rho \Psi_\nu \right) + \left(-|\partial_\mu \phi_i|^2 + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i \right) \right]. \quad (14)$$

In these expressions M_5 is the 5D fundamental energy scale, k is a curvature scale and $M_{0,1}$ are two scales parametrizing possible localized kinetic terms for the bulk fields, whereas Λ_5 and $\Lambda_{0,1}$ are bulk and boundary cosmological constants that are tuned to the values $\Lambda_5 = -6M_5^3 k^2$ and $\Lambda_0 = -\Lambda_1 = 6M_5^3 k$.

^aThere are two different formulations, developed respectively in refs. [8, 9] and [10, 11], which differ by a singular local R -symmetry transformation [12]. We use here the first.

The above theory has a non-trivial supersymmetric warped solution defining a slice of an AdS_5 space that is delimited by the two 4D branes at $y = y_{0,1}$. Defining the function $\sigma(y) = k|y|$, the background is given by [7]:

$$g_{MN} = e^{-2\sigma} \eta_{\mu\nu} \delta_M^\mu \delta_N^\nu + \delta_M^y \delta_N^y, \quad \Psi_M = 0, \quad A_M = 0. \quad (15)$$

At energies much below the compactification scale M_{KK} , the fluctuations around the above background solution are described by an ordinary ungauged 4D supergravity theory with vanishing cosmological constant. The fields of this effective theory are the massless zero modes of the 5D theory and fill out a supergravity multiplet $G = (h_{\mu\nu}, \psi_\mu)$ and a radion chiral multiplet $T = (t, \psi_t)$ of the on-shell surviving supersymmetry. They are defined by parametrizing the fluctuations around the background as follows:

$$\begin{aligned} g_{\mu\nu} &= \exp\left(-2\sigma \frac{\text{Re } t}{\pi R}\right) (\eta_{\mu\nu} + h_{\mu\nu}), \quad g_{\mu y} = 0, \quad g_{yy} = \left(\frac{\text{Re } t}{\pi R}\right)^2; \\ \Psi_\mu &= \psi_\mu, \quad \Psi_y = \frac{\sqrt{2}}{\pi} \psi_t; \quad A_\mu = 0, \quad A_y = \frac{\sqrt{3}}{\sqrt{2}\pi} \text{Im } t. \end{aligned} \quad (16)$$

The relevant low-energy dynamics is then fully controlled by the Kähler function and the superpotential of the effective theory as a function of the radion multiplet T and the matter multiplets Φ_i . These are computed by integrating out the heavy Kaluza–Klein modes of the bulk fields.

At the classical level, the effective Kähler function can be deduced by substituting eqs. (16) into eq. (12) and integrating over the internal dimension. The result has the form [13, 14]:

$$\begin{aligned} \Omega(T+T^\dagger, \Phi_i, \Phi_i^\dagger) &= -3 \frac{M_5^3}{k} \left(1 - e^{-k(T+T^\dagger)}\right) \\ &\quad + \Omega_0(\Phi_0, \Phi_0^\dagger) + \Omega_1(\Phi_1, \Phi_1^\dagger) e^{-k(T+T^\dagger)}. \end{aligned} \quad (17)$$

The first term is the matter-independent contribution from the bulk, whereas the last two terms are the matter-dependent contributions from the branes. Ignoring irrelevant operators involving higher powers of $\Phi_i \Phi_i^\dagger$, the latter have the form

$$\Omega_i^{\text{kin}}(\Phi_i, \Phi_i^\dagger) = -3M_i^2 + \Phi_i \Phi_i^\dagger. \quad (18)$$

Notice that whereas M_0 is arbitrary, M_1 must be smaller than $\sqrt{M_5^3/k}$; otherwise, the radion scalar field parametrized by e^{-kT} would become a ghost [15]. Moreover, when these parameters come close to their maximal values, the effective fundamental scale of the theory is significantly lowered [15]. The effective superpotential, on the other hand, is specified by the superpotentials W_i localized on the branes:

$$W(T, \Phi_i) = W_0(\Phi_0) + W_1(\Phi_1) e^{-kT}. \quad (19)$$

The effective Planck scale can be read off from (17). Assuming that the matter fields have vanishing VEVs and recalling that T has VEV πR , one finds:

$$M_{\text{P}}^2 = \frac{M_5^3}{k} \left(1 - e^{-2\pi k R}\right) + M_0^2 + M_1^2 e^{-2\pi k R}. \quad (20)$$

The scalar soft masses vanish, independently of which fixed points are chosen to host the visible and the hidden sectors. If $v = 0$ and $h = 1$, this is obvious, since the visible sector multiplet Φ_0 does not couple neither to T nor to Φ_1 . If $v = 1$ and $h = 0$, the visible sector multiplet Φ_1 couples instead to T , although not to Φ_0 , in the Kähler function. But it does in a very particular way, as a conformal compensator, which can be trivialized through a field redefinition, and this guarantees that all the masses cancel. The classical effective theory is thus of the sequestered type.

At the quantum level, there occur two kinds of corrections to the Kähler function. The first class represents a trivial renormalization of the local operators corresponding to the classical expression (17). These UV effects are divergent and incalculable, but anyhow irrelevant, since as explained above they cannot induce soft masses. A second class corresponds instead to new effects that have a field dependence that differs from the one implied by locality and general covariance in (17). These IR effects are finite and calculable, and control relevant contributions to the soft masses. The relevant corrections that we have to compute are therefore non-local and radion-dependent, and can be parametrized in the following general form:

$$\Delta\Omega(T+T^\dagger, \Phi_i, \Phi_i^\dagger) = \sum_{n_0, n_1=0}^{\infty} C_{n_0, n_1}(T+T^\dagger)(\Phi_0\Phi_0^\dagger)^{n_0}(\Phi_1\Phi_1^\dagger)^{n_1}. \quad (21)$$

The functions C_{n_0, n_1} control the leading effects allowing the transmission of supersymmetry breaking from one sector to the other. More precisely, supersymmetry breaking induces in general non-vanishing values for the auxiliary fields of all the non-visible chiral multiplets in the theory, that is the compensator, the radion multiplet and the hidden sector matter multiplets. The operator associated with $C_{0,0}$ yields then a Casimir energy, the one associated to $C_{1,0}$ or $C_{0,1}$, depending on where the visible and the hidden sectors are put, a radion-mediated scalar mass squared, and finally the one associated to $C_{1,1}$ a brane-mediated scalar mass squared. In the flat limit of vanishing warping, $C_{0,0}$ was computed in ref. [16], $C_{1,0}$ and $C_{0,1}$ first in [17] for $M_i = 0$ and then in [18] for $M_i \neq 0$, and finally $C_{1,1}$ in [18, 19] for $M_i = 0$ and in [18] for $M_i \neq 0$. In the general case of finite warping, the first coefficient was computed in refs. [20] (see also [21]) and the other ones in [22].

4. One-loop corrections

There are various techniques that can be used to perform the computation of the above described gravitational quantum corrections. A first approach, which was taken in ref. [18], consists in using the off-shell component description developed in ref. [11] with the same philosophy as ref. [23], and focus on a particular component of each operator. A second approach, which was taken in refs. [19] and [22], consists instead in using the linearized superfield approach of ref. [24] or the generalization to the warped case of [22], and perform a supergraph computation. But happily, after some supergravity gymnastics the corrections we are interested can be mapped to the effective action of a very simple theory involving a single real scalar degree of

freedom propagating in the same geometry and possessing localized kinetic terms at the two branes. We will not give here a rigorous proof of this fact, for which we refer the interested reader to ref. [22], but rather some qualitative hints on the basic properties behind it.

The main ingredients of the computation are the interactions between the brane and the bulk fields. These are fixed by the local symmetries that the theory should possess at the branes, and are most conveniently organized by splitting the bulk fields in background values plus fluctuations, and expanding in powers of the latter. The leading term is given by the minimal coupling between the supercurrent of the matter theory and the supergravity fluctuation multiplet, and leads to cubic vertices involving two brane and one bulk fields. At the next order, we find instead a coupling between the Kähler function of the matter theory and the kinetic term of the supergravity fluctuation, which leads to quartic vertices with two brane and two bulk fields. The one-loop diagrams that contribute to each operator in eq. (21), with given n_0 and n_1 , are obtained by combining these two types of vertices and fall into two classes. The first class consists of all those diagrams that involve some cubic vertices. Fortunately, although individually gauge-dependent, these turn out to sum up to zero in any gauge. The second class consists of the unique diagram that is made of only quartic vertices. This is manifestly gauge-independent and represents the only net contribution. Moreover, it is particularly simple to compute, since the involved interactions have the form of additional kinetic terms for the bulk fields with matter-dependent coefficients and localized at the two branes.

Proceeding along the above lines, and using a linearized superfield formulation of the theory, it is possible to compute the full correction (21) in closed form. The first step is to realize that the kinematical structure of the correction, namely those factors that depend on the tensor and superspace structure of the virtual particles and their interactions, amount to a universal factor of $-4/p^2$, where p is the internal momentum in the loop. The factor 4 takes into account the multiplicity of bosonic and fermionic degrees of freedom and the factor $1/p^2$ the fact that the Kähler function determines the component effective action only after taking its D component. The dynamical part of the correction that is left over can then be described in terms of a single real scalar degree of freedom φ propagating in the same background geometry and having some localized kinetic terms l_i , with Lagrangian

$$\mathcal{L} = -\frac{1}{2}e^{-2\sigma(y)} \left[(\partial_\mu \varphi)^2 + e^{-2\sigma(y)} (\partial_y \varphi)^2 + \left(l_0 \delta_0(y) + l_1 \delta_1(y) \right) (\partial_\mu \varphi)^2 \right]. \quad (22)$$

More precisely, the one-loop correction to the Kähler function is obtained from the one-loop effective action for the above theory by inserting a factor $-4/p^2$ in its virtual momentum representation, and promoting the parameters R and l_i on which it depends to the superfields $(T + T^\dagger)/(2\pi)$ and $-\Omega_i^{\text{kin}}/(3M_5^3)$ respectively.^b

^bThe fact that a displacements of each brane is effectively equivalent to a flow in the local operators on it [25] can be used to relate to some extent the matter dependence to the radion dependence. It allows to derive the former from the latter at leading order and under some restrictions [22].

A convenient way to compute the effective action for the scalar theory defined by the Lagrangian (22) is to start with $l_i = 0$ and to reconstruct the full result for $l_i \neq 0$ by resumming all the diagrams with l_i insertions. We thus denote by $\Psi_n(y)$ and m_n the wave functions and the masses of the Kaluza–Klein modes of the scalar field φ in the limit $l_i = 0$. The building blocks for the computation of the matter-dependent terms are the boundary-to-boundary propagators connecting the points y_i and y_j , with $y_{i,j} = 0, \pi R$. More precisely, for later convenience we include a suitable power of the induced metric at the two points that the propagator connects, and consider the quantities

$$\Delta_{ij}(p) = \sum_n e^{-\frac{3}{2}\sigma(y_i)} e^{-\frac{3}{2}\sigma(y_j)} \frac{\Psi_n(y_i)\Psi_n(y_j)}{p^2 + m_n^2}. \quad (23)$$

The quantity that is relevant to compute the matter-independent term is instead the following spectral function:

$$Z(p) = \prod_n (p^2 + m_n^2). \quad (24)$$

The above quantities are difficult to evaluate in terms of their definitions as infinite sums and products over the Kaluza–Klein spectrum, because there exist no simple closed form expressions for $\Psi_n(y)$ and m_n . Fortunately, they can be deduced in a much simpler way as solutions to differential equations. The results are most conveniently written in terms of the functions $\hat{I}_{1,2}$ and $\hat{K}_{1,2}$, defined in terms of the standard Bessel functions $I_{1,2}$ and $K_{1,2}$ as

$$\hat{I}_{1,2}(x) = \sqrt{\frac{\pi}{2}} \sqrt{x} I_{1,2}(x), \quad \hat{K}_{1,2}(x) = \sqrt{\frac{2}{\pi}} \sqrt{x} K_{1,2}(x). \quad (25)$$

Consider first the quantities (23). To compute them, we exploit the fact that they are connected by the simple relation $\Delta_{ij}(p) = e^{-\frac{3}{2}\sigma(y_i)} e^{-\frac{3}{2}\sigma(y_j)} \Delta(p, y_i, y_j)$ to the propagator $\Delta(p, y, y')$ of the scalar field φ for $l_i = 0$ in mixed momentum/position space for the non-compact/internal directions. The latter is defined as the solution with Neumann boundary conditions at y equal to 0 and πR of the differential equation $(e^{-2ky} p^2 - \partial_y e^{-4ky} \partial_y) \Delta(p, y, y') = \delta(y - y')$. This is most easily solved by switching to the conformal variable $z = e^{ky}/k$, in which the positions of the two branes are given by $z_0 = 1/k$ and $z_1 = e^{k\pi R}/k$. The result has been derived in ref. [26], and when restricted to the brane positions, it finally yields:

$$\Delta_{00}(p) = \frac{1}{2p} \frac{\hat{I}_1(pz_1)\hat{K}_2(pz_0) + \hat{K}_1(pz_1)\hat{I}_2(pz_0)}{\hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0)}, \quad (26)$$

$$\Delta_{11}(p) = \frac{1}{2p} \frac{\hat{I}_1(pz_0)\hat{K}_2(pz_1) + \hat{K}_1(pz_0)\hat{I}_2(pz_1)}{\hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0)}, \quad (27)$$

$$\Delta_{01,10}(p) = \frac{1}{2p} \frac{1}{\hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0)}. \quad (28)$$

Consider next the formal determinant (24). Although this is not a propagator, it can still be functionally related to it. Indeed, the masses m_n are by definition the

poles of the latter, that is the zeroes of $F(p) = \hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0)$. The infinite product in eq. (24) has the form of an irrelevant constant divergent prefactor times a finite function of the momentum. In order to compute the latter, we consider the quantity $\partial_p \ln Z(p) = \sum_n 2p/(p^2 + m_n^2)$. The infinite sum over the eigenvalues, defined by $F(im_n) = 0$, is now convergent and can be computed with standard techniques. The result is simply $\partial_p \ln F(p)$. This implies that $Z(p) = F(p)$, up to the already mentioned irrelevant infinite overall constant:

$$Z(p) = \hat{I}_1(pz_1)\hat{K}_1(pz_0) - \hat{K}_1(pz_1)\hat{I}_1(pz_0). \quad (29)$$

Having the expressions for $\Delta_{ij}(p)$ and $Z(p)$ for $l_i = 0$, we can compute the full effective action for $l_i \neq 0$ by summing up all the diagrams with an arbitrary number of insertions of these localized kinetic terms. The vacuum diagram without any interaction involves only the spectral function Z . All the other diagrams involve instead the propagators Δ_{ij} , and their sum reconstructs the determinant in the two-dimensional space of branes of the identity plus the interactions times the propagators. Since the interaction localized at z_i involves a factor $(kz_i)^{-2}$, and we have included a factor $(kz_i)^{-\frac{3}{2}}(kz_j)^{-\frac{3}{2}}$ in the definition of the Δ_{ij} 's, each of the latter comes along with a factor $l_i kz_i$. The expression for the correction to the Kähler function is finally found by inserting a factor $-4/p^2$ in the virtual momentum integral, as explained above. The result reads:

$$\Omega_{1\text{-loop}} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{-4}{p^2} \ln \left\{ Z(p) \det \left[\mathbf{1} + kp^2 \begin{pmatrix} l_0 z_0 \Delta_{00}(p) & l_0 z_0 \Delta_{01}(p) \\ l_1 z_1 \Delta_{10}(p) & l_1 z_1 \Delta_{11}(p) \end{pmatrix} \right] \right\}. \quad (30)$$

The momentum integral in eq. (30) is of course divergent. This corresponds to the fact that it also contains incalculable corrections to the coefficients of the local operators that are already allowed to appear at the classical level. In order to disentangle the finite corrections associated to the non-local quantities that we are interested in, we thus need to subtract these divergent contributions corresponding to the renormalization of local terms. The appropriate subtraction can be identified by replacing the brane-to-brane propagators $\Delta_{ij}(p)$ and the spectra function $Z(p)$ with their asymptotic behaviours $\tilde{\Delta}_{ij}(p)$ and $\tilde{Z}(p)$ for $p \rightarrow \infty$. Up to exponentially suppressed terms of order $e^{-2p(z_1 - z_0)}$, which are clearly irrelevant, we find:

$$\tilde{\Delta}_{00}(p) = \frac{1}{2p} \frac{\hat{K}_2(pz_0)}{\hat{K}_1(pz_0)}, \quad \tilde{\Delta}_{11}(p) = \frac{1}{2p} \frac{\hat{I}_2(pz_1)}{\hat{I}_1(pz_1)}, \quad \tilde{\Delta}_{01,10}(p) = 0, \quad (31)$$

$$\tilde{Z}(p) = \hat{I}_1(pz_1)\hat{K}_1(pz_0). \quad (32)$$

The divergent part of eq. (30) is then obtained by replacing each untilded quantity with its tilded limit. This yields

$$\Omega_{\text{div}} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{-4}{p^2} \left[\ln \tilde{Z}(p) + \sum_i \ln \left(1 + kp^2 l_i z_i \tilde{\Delta}_{ii}(p) \right) \right]. \quad (33)$$

It can be verified, by using an explicit cut-off and rescaling the integration variable to make the dependence of each term on the z_i 's explicit, that the above expression indeed has the structure of the most general allowed counterterm.

The non-local corrections to the Kähler function can now be computed by subtracting from the total one-loop expression (30) the divergent contribution (33) associated to the local corrections. Our final result is then given by

$$\Delta\Omega = \int \frac{d^4p}{(2\pi)^4} \frac{-2}{p^2} \ln \frac{Z(p)}{\bar{Z}(p)} \frac{\prod_i \left(1 + kz_i l_i p^2 \Delta_{ii}(p)\right) - \prod_i \left(kz_i l_i p^2 \Delta_{ii'}(p)\right)}{\prod_i \left(1 + kz_i l_i p^2 \tilde{\Delta}_{ii}(p)\right)}. \quad (34)$$

As already explained, the parameters l_i and z_i on which this expression depends must be promoted to superfields; defining for convenience the lengths $\alpha_i = M_i^2/M_5^3$, this is done according to the following rules:

$$l_j \rightarrow \alpha_j - \frac{\Phi_j^\dagger \Phi_j}{3M_5^3}, \quad z_j \rightarrow \frac{1}{k} e^{-\frac{i}{2}k(T+T^\dagger)}. \quad (35)$$

The functions C_{n_0, n_1} can now be deduced by expanding eq. (34) in powers of the matter superfields and comparing the result with the general expression (21). They have of course the form of $(n_0 + n_1)$ -point amplitudes involving the dressed brane-to-brane propagators and spectral function, in the presence of localized kinetic terms α_i . Their functional dependence on the radion superfield has the structure of an infinite series in powers of $e^{-k(T+T^\dagger)}$, and can be studied numerically. The most important result is that the functions $C_{0,0}$, $C_{1,0}$ and $C_{0,1}$ are positive for $\alpha_i = 0$ but can become negative for sufficiently large $\alpha_i \neq 0$, the transitions points depending on the warping k , whereas $C_{1,1}$ stays always positive for any value of α_i and k . This implies that the Casimir energy and the radion-mediated squared masses can be either negative or positive, whereas the brane-mediated squared masses are always negative. An important general observation is that the effects of the warping and of the localized kinetic terms are locally similar, as expected from the fact that they affect in similar ways the masses and the wave functions of the Kaluza–Klein modes, but globally differ in crucial aspects. More precisely, a sign flip in the first three coefficients is possible only in the presence of a non-vanishing localized kinetic term. Warping only influences the size of the critical value of the localized kinetic that is needed for the sign flip. This means that to have a chance to obtain positive scalar squared masses in these minimal sequestered modes, one has to rely on a localized kinetic term in the hidden sector, with a size that depends on the warping. To be more quantitative and discuss model building, we shall specialize in the next sections to the two extreme cases of weak and strong warping.

5. Weakly warped models

In the weak warping limit $kR \ll 1$, the low-energy effective theory is most conveniently described by using as radion superfield T . The effective Kähler function can then be organized as a series in powers of the small parameter $k(T+T^\dagger)$. Retaining only the leading term, the tree-level part is given by

$$\Omega = -3M_5^3 (T + T^\dagger + l_0 + l_1). \quad (36)$$

Similarly, the one-loop correction (34) simplifies to

$$\Delta\Omega = \frac{1}{4\pi^2} I_f\left(\frac{l_0}{T+T^\dagger}, \frac{l_1}{T+T^\dagger}, 1\right) (T+T^\dagger)^{-2} \quad (37)$$

and the coefficients defined by eq. (21) are found to be

$$C_{n_0, n_1} = \frac{(-1)^{n_0+n_1}}{4\pi^2 n_0! n_1!} I_f^{(n_0, n_1, 0)}\left(\frac{\alpha_0}{T+T^\dagger}, \frac{\alpha_1}{T+T^\dagger}, 1\right) \frac{(T+T^\dagger)^{-2}}{(3M_5^3(T+T^\dagger))^{n_0+n_1}}. \quad (38)$$

The function $I_f(a_0, a_1, b)$ appearing in these equations satisfies the scaling law $I_f(\gamma a_0, \gamma a_1, \gamma b) = \gamma^{-2} I_f(a_0, a_1, b)$, and is defined as

$$I_f(a_0, a_1, b) = - \int_0^\infty dx x \ln \left[1 - \frac{1-a_0 x/2}{1+a_0 x/2} \frac{1-a_1 x/2}{1+a_1 x/2} e^{-bx} \right]. \quad (39)$$

The effect of the localized kinetic terms is parametrized by the dimensionless variables $\epsilon_i = \alpha_i/(T+T^*)$. For $\epsilon_i = 0$, all the coefficients are positive and proportional to $c_f = I_f(0, 0, 1) \approx 1.202$. More precisely, for the first four operators, the functional factor $(-1)^{n_0+n_1} I_f^{(n_0, n_1, 0)}$ is found to be $c_f, 2c_f, 2c_f$ and $6c_f$. For $\epsilon_i \neq 0$, the situation changes in an interesting way [18]. In particular, for small ϵ_0 and large ϵ_1 , these functional factors tend to $-(3/4)c_f, -(3/2)c_f, (4 \ln 2/\zeta(3)) \epsilon_h^{-2} c_f$ and $(4 \ln 2/\zeta(3)) \epsilon_h^{-2} c_f$. The reflected situation, with large ϵ_0 and small ϵ_1 , is perfectly similar.

In this case, the two branes are completely equivalent. For concreteness, let us put the visible sector at y_0 and the hidden sector at y_1 , that is $v = 0$ and $h = 1$ in our notation, and take $\epsilon_0 = 0$ and $\epsilon_1 = \epsilon_h$. We have then $M_{KK} = 2(T+T^*)^{-1}$ and $M_P^2 = M_5^3(T+T^*)(1+\epsilon_h)$. The radion-mediated and brane-mediated contributions to the soft scalar squared masses are given respectively by $-\partial_T \partial_{T^\dagger} C_{1,0} |F_T|^2$ and $-C_{1,1} |F_h|^2$. After rewriting the derivatives with respect to $T+T^\dagger$ as derivatives with respect to the last argument of the function (39), we find:

$$m_0^2 = b \alpha_{\text{gau}}^2 |F_S|^2 + \left[\frac{1}{3} (1 + \epsilon_h) I_f^{(1,0,2)}(0, \epsilon_h, 1) \right] \alpha_{\text{gra}} \frac{|F_T|^2}{(T+T^*)^2} + \left[-\frac{1}{9} (1 + \epsilon_h)^2 I_f^{(1,1,0)}(0, \epsilon_h, 1) \right] \alpha_{\text{gra}} \frac{|F_h|^2}{M_P^2}. \quad (40)$$

For ϵ_h much larger than 1, the numerical coefficient in the first bracket becomes positive and grows like ϵ_h , whereas the one in the second stays negative and of order 1. The possibility of achieving a satisfactory result depends however on the dynamics of the model, through the values of $F_S, F_T/(T+T^*)$ and F_h/M_P .

A simple class of viable models is obtained by considering the following effective superpotential, which can arise for example through gaugino condensation [27]:

$$W = \Lambda_a^3 e^{-n\Lambda_a T} + \Lambda_b^3 + M_{\text{susy}}^2 \Phi_h. \quad (41)$$

In the limit $\Lambda_a \gg \Lambda_b$, there is a stable solution with large $T \sim [n\Lambda_a \ln(\Lambda_a/\Lambda_b)]^{-1}$, with a cosmological constant that can be made to vanish by tuning one of the scales as a function of the others. The solution is then a function of two parameters, which can be taken to be the supersymmetry breaking scale M_{susy} and the large

dimensionless radius parameter $t = n\Lambda_a T$. In terms of these quantities, the F -terms are given by

$$F_S \sim \frac{M_{\text{susy}}^2}{M_{\text{P}}}, \quad \frac{F_T}{T+T^*} \sim \frac{1}{t} \frac{M_{\text{susy}}^2}{M_{\text{P}}}, \quad \frac{F_{\text{h}}}{M_{\text{P}}} \sim \frac{M_{\text{susy}}^2}{M_{\text{P}}}. \quad (42)$$

In this situation, the sum of the two radion-mediated and brane-mediated gravitational contributions becomes positive for $\epsilon_{\text{h}} \sim t^2$, and competes with the anomaly-mediated contribution if $\epsilon_{\text{h}} t^{-2} \alpha_{\text{gra}} \sim \alpha_{\text{gau}}^2$, that is $M_{\text{KK}}/M_{\text{P}} \sim 4\pi\alpha_{\text{gau}}$ as very generically estimated in eq. (11). In this situation, we get flavour-universal and non-tachyonic soft masses given by $m_0 \sim m_{1/2} \sim \alpha_{\text{gau}} m_{3/2}$. The radion mass is found to be of comparable magnitude: $m_{\text{radion}} \sim m_{3/2}$.

6. Strongly warped models

In the strong warping limit $kR \gg 1$, it is convenient to parametrize the radion superfield by $\omega = e^{-kT}$. The effective Kähler function can then be organized as a series in powers of the small parameter $\omega\omega^\dagger$. The tree-level part is given by

$$\Omega = -3M_5^3(k^{-1} + l_0) + 3M_5^3(k^{-1} - l_1)\omega\omega^\dagger. \quad (43)$$

Similarly, retaining only the leading term, the one-loop correction (34) simplifies to

$$\Delta\Omega = \frac{1}{4\pi^2} I_{\text{w}}(l_0 k, l_1 k) (k\omega\omega^\dagger)^2 \quad (44)$$

and the coefficients defined by eq. (21) to

$$C_{n_0, n_1} = \frac{(-1)^{n_0+n_1}}{4\pi^2 n_0! n_1!} I_{\text{w}}^{(n_0, n_1)}(\alpha_0 k, \alpha_1 k) \frac{(k\omega\omega^\dagger)^2}{(3M_5^3 k^{-1})^{n_0+n_1}}. \quad (45)$$

The function $I_{\text{w}}(a_0, a_1)$ appearing in these expressions is defined by

$$I_{\text{w}}(a_0, a_1) = \frac{\pi}{4} \int_0^\infty dx x^3 \frac{\hat{K}_1(x)}{\hat{I}_1(x)} \frac{1}{1+a_0} \frac{1 - a_1 x/2}{1 + a_1 x/2} \frac{\hat{K}_2(x)/\hat{K}_1(x)}{\hat{I}_2(x)/\hat{I}_1(x)}. \quad (46)$$

The effect of the localized kinetic terms is parametrized in this case by $\epsilon_i = \alpha_i k$. For $\epsilon_i = 0$, all the C_{n_0, n_1} 's are positive and proportional to $c_{\text{w}} = I_{\text{w}}(0, 0) \approx 1.165$. More precisely, for the first four operators, the factor $(-1)^{n_0+n_1} I_{\text{w}}^{(n_0, n_1, 0)}$ is found to be c_{w} , c_{w} , $2c_{\text{w}}$ and $2c_{\text{w}}$. For $\epsilon_i \neq 0$, the situation again becomes more interesting [22]. For small ϵ_0 and sizeable ϵ_1 , $C_{0,0}$ and $C_{1,0}$ become negative, whereas $C_{1,0}$ and $C_{1,1}$ stay positive. For large ϵ_0 and small ϵ_1 , on the other hand, all the coefficients stay positive and get just suppressed.

In this case, the two branes are not equivalent and there are two possible kinds of models. We will however consider only the case where the visible sector is at y_0 and the hidden sector at y_1 , that is $v = 0$ and $h = 1$ in our notation, in order to have a chance to get positive scalar squared masses, and take $\epsilon_0 = 0$ and $\epsilon_1 = \epsilon_{\text{h}}$.

We have then $M_{\text{KK}} = k|\omega|$ and $M_{\text{P}}^2 = M_5^3 k^{-1}$. The radion-mediated and brane-mediated contributions to the soft scalar squared masses are given in this case by $-\partial_\omega \partial_{\omega^\dagger} C_{1,0} |F_\omega|^2$ and $-C_{1,1} |F_h|^2$, and we find:

$$m_0^2 = b \alpha_{\text{gau}}^2 |F_S|^2 + \left[\frac{16}{3} I_{\text{w}}^{(1,0)}(0, \epsilon_h) \right] \alpha_{\text{gra}} |F_\omega|^2 + \left[-\frac{4}{9} I_{\text{w}}^{(1,1)}(0, \epsilon_h) \right] \alpha_{\text{gra}} \frac{|\omega|^2 |F_h|^2}{M_{\text{P}}^2} . \quad (47)$$

For ϵ_h close to 1, the numerical coefficient in the first bracket can become positive whereas the one in the second remains negative, and both are of order 1. Again, the possibility of achieving a satisfactory result depends however on the dynamics of the model, through the values of the parameters F_S , F_ω and $\omega F_h/M_{\text{P}}$.

A class of viable models can be obtained with the following effective superpotential, which can be induced for example through gaugino condensation [14, 28]:

$$W = \Lambda_a^3 \omega^n + \Lambda_{b_0}^3 + \Lambda_{b_1}^3 \omega^3 + M_{\text{susy}}^2 \Phi_h \omega^3 . \quad (48)$$

We assume that Λ_{b_0} is small compared to Λ_{b_1} but not too small compared to Λ_a , in such a way that we can neglect the ω -dependent term in the negative gravitational contribution to the potential. In the limit $\Lambda_a \gg \Lambda_{b_1}$ with $n > 3$ or $\Lambda_a \ll \Lambda_{b_1}$ with $n < 3$, there is a stable solution with small $\omega \sim (\Lambda_{b_1}/\Lambda_a)^{\frac{3}{n-3}}$, with a cosmological constant that can be adjusted to zero by tuning again one of the scales. The solution is then a function of three parameters, which can be chosen to be the supersymmetry breaking scale M_{susy} , the small dimensionless warp factor ω , and one more independent dimensionless combination of scales. Omitting the dependence on the latter parameter, that we shall assume to be generic, the F -terms are given by

$$F_S \sim \omega^2 \frac{M_{\text{susy}}^2}{M_{\text{P}}}, \quad F_\omega \sim \frac{\omega^2}{1 - \epsilon_h} \frac{M_{\text{susy}}^2}{M_{\text{P}}}, \quad \frac{\omega F_h}{M_{\text{P}}} \sim \omega^2 \frac{M_{\text{susy}}^2}{M_{\text{P}}} . \quad (49)$$

In this situation, the sum of the two radion-mediated and brane-mediated gravitational contributions becomes positive for $\epsilon_h \sim 1$, and competes with the anomaly-mediated contribution if $\alpha_{\text{gra}} \sim \alpha_{\text{gau}}^2$, which implies again $M_{\text{KK}}/M_{\text{P}} \sim 4\pi\alpha_{\text{gau}}$ as advocated in eq. (11). In this situation, we get flavour-universal and non-tachyonic soft masses given by $m_0 \sim m_{1/2} \sim \alpha_{\text{gau}} m_{3/2}$. The radion mass is in this case found to be of even larger magnitude: $m_{\text{radion}} \sim \omega^{-1} m_{3/2}$.

7. Conclusions

The results of the previous sections show that minimal sequestered models can become viable, with gaugino masses dominated by anomaly mediation and scalar squared masses dominated by radion mediation. This requires however the presence of a large localized kinetic term for the bulk gravitational fields in the hidden sector, with a value that is close to its warping-dependent maximal value. It would be interesting to further investigate this extreme situation, and in particular to understand whether it can occur in any natural way and to what extent subleading flavour-violating effects are suppressed.

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