

D-type supersymmetry breaking and brane-to-brane gravity mediation

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Abstract

We revisit the issue of gravitational contributions to soft masses in five-dimensional sequestered models. We point out that, unlike for the case of F -type supersymmetry breaking, for D -type breaking these effects generically give positive soft masses squared for the sfermions. This drastically improves model building. We discuss the phenomenological implications of our result.

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1 Introduction

The generic presence of new sources of Flavor and CP violation in the soft supersymmetry breaking parameters is one major conceptual drawback lingering on the supersymmetric solution to the hierarchy problem. This problem was somewhat disregarded in the early models with gravity mediated soft terms with the back thought that gravity is a universal force which therefore naturally delivers flavor preserving soft masses. On the other hand, whatever theory, perhaps string theory, describes gravity at a more fundamental level, that theory should also explain the mass spectrum of quarks and leptons. If quantum gravity is also a theory of flavor then sfermion masses are expected to be flavor breaking as well. Mechanisms to control the size of these effects have been proposed. One idea, more or less satisfactory, is that the same flavor symmetry selection rules that control the hierarchy of fermion masses and mixings control the soft terms as well. A perhaps more ambitious direction, using the universal character of gravity in the infrared, is to sequester in an extra dimensional space the supersymmetry breaking dynamics from the quarks and leptons [1]. In the simplest case of a single extra dimension, soft masses are forbidden by locality at the classical level. In this setup, and in the limit of large radius of compactification, the soft masses are dominated by the superconformal anomaly contribution [1, 2] which is nicely flavor preserving, but unfortunately, the sleptons are tachyons. Furthermore, calculable quantum gravity corrections to the Kähler potential mixing the hidden and the visible sectors give also rise to a universal contribution to sfermion masses. For small enough radius this effect competes with anomaly mediation and has the potential to cure the slepton mass problem [1]. Explicit calculations have concluded that this contribution is unfortunately always negative [3, 4]. However it was also found that yet another contribution, due to the radion field T , becomes positive in the presence of localized kinetic terms with sizable coefficients [3]. It is in principle possible that the radion contribution dominates the sfermion masses, solving the tachyon problem. In practice it turns out to be difficult to construct models where this happens, simply because the radion auxiliary Vacuum Expectation Value (VEV) tends to be suppressed [3, 5]. The more general situation of a warped extra dimension has also been studied in detail, and it was found that the situation remains qualitatively similar [6] (see also [7] for a brief general overview). Finally, models with several extra dimensions also don't seem to provide any clue to the problem, since sequestering is no longer automatic and seems to be lost as soon as important new features are incorporated [8, 9].

The purpose of this paper is to point out that the brane-to-brane mediated soft masses, in the basic case of a single flat extra dimension, can be made positive simply by choosing a hidden sector in which the order parameter of supersymmetry breaking is mostly a gauge auxiliary field D instead of an F auxiliary field ². We

²While we were completing this work, reference [9] appeared, in which a similar mechanism was used to get positive soft masses at tree level in a six-dimensional context.

will do this in the next section. We will then prove that there is no obstacle in the construction of supersymmetry breaking models where $|D| \gg |F|$ (we stress that we are not using Fayet-Iliopoulos terms, which are not acceptable at the supergravity level). Finally we will carefully analyze a simple model of radius stabilization and estimate the size of all possible subleading flavor breaking effects.

2 D-type breaking

Consider a sequestered model on S_1/Z_2 with tree level kinetic function

$$\Omega = -3M_5^3(T + T^\dagger) + \Omega_v(QQ^\dagger) + \Omega_h(Xe^{gV}X^\dagger), \quad (2.1)$$

and superpotential

$$W = W(T) + W_v(Q) + W_h(X). \quad (2.2)$$

T is the radion superfield and Q and X denote collectively the chiral superfields of respectively the visible and hidden sectors³. We also indicate by V the gauge superfield localized at the hidden brane, while we do not explicitly write the visible sector gauge fields because they are irrelevant to the present discussion. In general, X will transform under a reducible representation of the hidden gauge group G . Notice that Ω is related to the Kähler potential by $K \equiv -3M_P^2 \ln(-\Omega/3M_P^{-2})$, so that the absence of direct coupling between hidden and visible sectors is manifest in Ω but not in K . At 1-loop, Ω is corrected by the additive contribution [3, 4, 10] (see also [11]):

$$\Omega_{1\text{-loop}} = \frac{\xi(3)}{6\pi^2} \frac{1}{(T + T^\dagger)^2} \left\{ \frac{3}{2} + \frac{\Omega_v + \Omega_h}{M_5^3(T + T^\dagger)} + \frac{\Omega_v\Omega_h}{M_5^6(T + T^\dagger)^2} + \dots \right\} \quad (2.3)$$

where the dots indicate higher order terms, negligible for our purposes (the full result can be found in ref. [3]). To discuss soft masses it is enough to consider only the lowest-order term in the visible sector kinetic function:

$$\Omega_v = QQ^\dagger. \quad (2.4)$$

Neglecting F_T , the brane-to-brane contribution to the soft masses coming from the third term in eq. (2.3) is the dominant effect. If the supersymmetry breaking dynamics in the hidden sector were accurately described by weakly coupled chiral superfields, then the resulting universal mass would be

$$m_0^2 = -\frac{\xi(3)}{6\pi^2 M_5^6 (T + T^\dagger)^4} (\partial_X \partial_{X^\dagger} \Omega_h) |F_X|^2 < 0, \quad (2.5)$$

where positivity of $\partial_X \partial_{X^\dagger} \Omega_h$ has been used. The situation changes however when the gauge dynamics in the hidden sector cannot be neglected. Basically this means that

³In what follow, we will denote the scalar component of a chiral multiplet with the same letter as the corresponding superfield.

the vector fields are light and the corresponding auxiliary fields have non-negligible VEVs. Without loss of generality we can limit ourselves to the simple case of a quadratic, renormalizable, kinetic term

$$\Omega_h = X^\dagger e^{gV} X. \quad (2.6)$$

The soft mass m_0^2 is determined by the VEV of the highest component of the superfield $X^\dagger e^{gV} X$. Working in Wess-Zumino gauge this reduces to

$$\begin{aligned} X^\dagger e^{gV} X|_{\theta^2\bar{\theta}^2} &= |F_X|^2 + gD_A X^\dagger T_A X = |\partial_X W_h|^2 - g^2 (X^\dagger T_A X)^2 \\ &\equiv |F_X|^2 - D^2, \end{aligned} \quad (2.7)$$

where in the second equality the equations of motion for F_X and D have been used. A supersymmetry breaking model with $|D| > |F_X|$ would induce a universal positive contribution to the scalar masses, potentially solving the tachyon problem of anomaly mediation. While in the scalar potential $V = |F_X|^2 + D^2/2$ the contributions of the two auxiliary fields add up, in the brane-to-brane mass they enter with opposite signs. This "asymmetric" behavior of F and D arises because $X^\dagger e^{gV} X|_{\theta^2\bar{\theta}^2}$ contains the "kinetic" term for the first and the "source" term for the second. On the other hand the kinetic term for D comes from the hidden gauge superfield strength $W_\alpha W^\alpha|_{\theta^2}$ while the "source" term for F is due to the hidden superpotential. However, based solely on power counting, these latter interactions are found to be unimportant for gravity mediated masses. For example, along with the terms in eq. (2.3), bulk gravity loops could introduce a visible-hidden mixing of the form $[QQ^\dagger W_\alpha W^\alpha/M_5^6(T + T^\dagger)^3]|_{\theta^2\bar{\theta}^2}$. Notice that this is technically a higher derivative interaction than the Kähler potential. As a result, the induced soft mass is suppressed by at least an extra factor $m_{3/2}T$ with respect to the leading contribution in eq. (2.5). Concerning the "source" term for F , one could imagine that similar terms, connecting the visible kinetic term and the hidden sector superpotential, arise. Again, since the superpotential has higher dimension than the Kähler potential, these terms will give soft masses that are suppressed compare to the leading contribution. For example, an operator of the form $[Q^\dagger Q W_h(X)/M_5^6(T + T^\dagger)^3]|_{\theta^2\bar{\theta}^2}$ would also give a contribution to soft masses suppressed by $m_{3/2}T$ (notice that $W_h|_{\theta^2} = |F_X|^2$). At the leading order it is therefore consistent to consider only the effects of the matter kinetic term $X^\dagger e^{gV} X$.

Even though in principle there does not seem to be anything wrong in having a model with $|D| > |F_X|$, we will nonetheless show in the next section that this is indeed the case by presenting a model in which $|D| \gg |F_X|$. Before going to that, it would be useful to give the *on-shell* viewpoint on the above result. The coupling between hidden and visible sectors can also be treated as a correction to the kinetic function of the hidden sector, by defining

$$\tilde{\Omega}_h \equiv (1 + a Q^\dagger Q) X^\dagger e^{gV} X, \quad (2.8)$$

with $a = (\zeta(3)/6\pi^2)M_5^{-6}(T + T^\dagger)^{-4}$. Neglecting the supergravity corrections, which contribute at a higher order in XX^\dagger/M_P^2 , and the effects associated to F_T , the hidden sector potential is found to be

$$\begin{aligned} V_h &= \frac{|\partial_X W_h|^2}{\partial_X \partial_{X^\dagger} \tilde{\Omega}_h} + \frac{g^2}{8} \left(\partial_X \tilde{\Omega}_h T_A X + \text{h.c.} \right)^2 \\ &= \frac{|\partial_X W_h|^2}{1 + a Q^\dagger Q} + \frac{g^2}{2} (1 + a Q^\dagger Q)^2 (X^\dagger T_A X)^2. \end{aligned} \quad (2.9)$$

This equation shows that the fact that the two contributions to the mass of Q from respectively F_X and D terms have opposite signs is simply due to the different powers of $\tilde{\Omega}_h$ appearing in these two terms. The new expression for the scalar soft mass is then:

$$m_0^2 = \frac{\xi(3)}{6\pi^2 M_5^6 (T + T^\dagger)^4} \left(-|F_X|^2 + D^2 \right). \quad (2.10)$$

3 D versus F auxiliary fields

Consider a globally supersymmetric theory with matter field X belonging to a generally reducible representation of the gauge group G and with scalar potential

$$V = |\partial_X W|^2 + \frac{g^2}{2} (X^\dagger T_A X)^2. \quad (3.1)$$

There is a well know theorem stating that if a solution $F_X = \partial_X W = 0$ exists, then there must also exist a value of X satisfying both conditions $F_X = D_A = 0$.⁴ At first sight, this result suggests that there may be an obstruction in constructing models with $|D_A|$ bigger than $|F_X|$. More precisely, the case in which $|F_X|$ is much smaller than $|D_A|$ approximates well the case $F_X = 0$, suggesting that starting from a point satisfying this condition, D_A can be reduced without significantly affecting F_X . There is however a caveat in this simple reasoning, and, as we will show below, it is possible to find models with $|D_A|/|F_X|$ arbitrarily large. Of course, in order to make the gravity mediated masses positive we do not need that much, but the fact that it is possible suggests to us that the case we care about, $|D_A|^2 > |F_X|^2$, may be pretty generic. In analogy with the standard proof of the theorem stated above, let us start from a point $X = X_0$ in field space, and consider a particular variation of the fields corresponding to a complex gauge rotation:

$$X_0 \rightarrow e^{\alpha_A T_A} X_0, \quad \alpha_A \text{ real}. \quad (3.2)$$

At first order in α_A the potential will change by

$$\begin{aligned} \delta V &= \alpha_A \left\{ [(\partial_X W)^\dagger T_A (\partial_X W) + \text{h.c.}]_{X=X_0} + g^2 (X_0^\dagger \{T_A, T_B\} X_0) (X_0^\dagger T_B X_0) \right\} \\ &= \alpha_A \left\{ 2F_X^\dagger T_A F_X + M_{AB}^2 D_B \right\}, \end{aligned} \quad (3.3)$$

⁴See ref. [12]. This theorem can be considered a corollary of another theorem, which states that the space of D -flat directions is isomorphic to the space of holomorphic gauge invariants [13, 14, 15, 16].

where M_{AB} is the mass matrix of the heavy gauge bosons, and we have used the fact that $\partial_X W$ transforms under complex gauge transformations as the conjugate of X . Consider first the case where $F_X = 0$ at X_0 . We indicate by H the subgroup of G that leaves this point invariant. The D field transforms in the adjoint representation of the gauge group, and its VEV at X_0 is along the subspace G/H . Moreover, the vector mass matrix is a non-singular matrix on the subspace G/H . It is then obvious that for $D \neq 0$ the variation of V is in general non-zero. Therefore points with $D \neq 0$ cannot be stationary. Consider now the case where $F_X \neq 0$. At the minimum, $\delta V = 0$ and thus

$$2F_X^\dagger T_A F_X = -M_{AB}^2 D_B. \quad (3.4)$$

Now, if the Lie algebra charges are $O(1)$ parameters, the above equality roughly implies

$$|F_X|^2 \gtrsim 2|F_X^\dagger T_A F_X| = M_{AB}^2 D_B \gtrsim D^2, \quad (3.5)$$

showing that D^2 can conceivably be of order $|F_X|^2$ but not much bigger. The latter possibility can however be achieved in the presence of large parameters in the charge spectrum. Consider for example a simple $U(1)$ model with matter fields X_{q_i} with charges q_i . Imagine now that the dominant F -auxiliary at the stationary point belongs to a superfield X_{q_k} with charge $q_k \equiv N \gg 1$, whereas all the scalar (non-auxiliary) fields with non-zero VEV belong to superfields with *small* charge, say $O(1)$. In such a situation the left hand side of eq. (3.4) is of order $N|F_X|^2$, while the right hand side can conceivably be of order D^2 . In this way we can parametrically obtain $D^2 \gg |F_X|^2$. Once again, we emphasize that for our application we do not need this parametric separation, but only that $-|F_X|^2 + D^2$ becomes positive and of the same order of magnitude as $|F_X|^2$ in eq. (2.10).

We can make the above more explicit by considering the following O’Raifeartaigh model with a gauged $U(1)$. We introduce 4 chiral superfields $\phi_0, \phi_1, \phi_{-1}, \phi_{-1/N}$ with charges 0, 1, -1, $-1/N$ respectively and superpotential

$$W = \lambda_1 \phi_0 (\phi_1 \phi_{-1/N}^N - 1) + \lambda_2 \phi_1 \phi_{-1}. \quad (3.6)$$

For the moment we shall not worry about the above charge assignment being anomalous. By defining the invariants

$$I_1 \equiv \phi_1 \phi_{-1/N}^N, \quad I_2 \equiv \phi_{-1} \phi_{-1/N}^{-N}, \quad (3.7)$$

the scalar potential is

$$V = |\lambda_1|^2 |I_1 - 1|^2 + |\lambda_1 \phi_0 + \lambda_2 I_2|^2 |\phi_{-1/N}|^{2N} + |N \lambda_1 \phi_0 I_1|^2 |\phi_{-1/N}|^{-2} \\ + |\lambda_2 I_1|^2 |\phi_{-1/N}|^{-2N} + \frac{g^2}{2} \left(|I_1|^2 |\phi_{-1/N}|^{-2N} - |I_2|^2 |\phi_{-1/N}|^{2N} - \frac{1}{N} |\phi_{-1/N}|^2 \right)^2. \quad (3.8)$$

To further simplify notation, we define $|\phi_{-1/N}|^2 \equiv z$, $|I_2|^2 = w$, $I_1 \equiv ye^{i\theta}$. By extremizing in ϕ_0 and θ , we are left with finding the minimum of

$$V = |\lambda_1|^2(y-1)^2 + |\lambda_2|^2 \frac{N^2 y^2 z^N w}{N^2 y^2 + z^{N+1}} + |\lambda_2|^2 \frac{y^2}{z^N} + \frac{g^2}{2} \left(\frac{y^2}{z^N} - wz^N - \frac{1}{N}z \right)^2. \quad (3.9)$$

In the limit of zero gauge coupling, there is an asymptotic supersymmetric minimum at $w = 0, y = 1$ and $z \rightarrow \infty$. When the gauge coupling is turned on, the vacuum moves in from infinity to settle at some finite value of z , and supersymmetry is broken. To study the vacuum dynamics, it is convenient to define the rescaled couplings $\hat{\lambda}_i \equiv \lambda_i/g$. We can simplify the minimization of the potential by taking $\hat{\lambda}_1 \gg \hat{\lambda}_2 \gg 1$. Then we have that $y \sim 1 + O(1/\hat{\lambda}_1^2)$ and $F_{\phi_0} = \hat{\lambda}_1(y-1) \sim 1/\hat{\lambda}_1$ is negligible. It is also easy to guess that the minimum should lie at $w = 0$. We can then consistently approximate the potential by

$$V/g^2 \simeq \hat{\lambda}_2^2 \frac{1}{z^N} + \frac{1}{2} \frac{z^2}{N^2}. \quad (3.10)$$

This is stationary for $z^{N+2} \sim N^3 \hat{\lambda}_2^2 \gg 1$, and at the stationary point the vacuum energy scales like

$$V_{\min} \sim g^2 (N^3 \hat{\lambda}_2^2)^{\frac{2}{N+2}} \left(\frac{1}{N^3} + \frac{1}{2N^2} \right). \quad (3.11)$$

The F and D contributions correspond respectively to the first and second term in the parentheses. Notice that $D^2/|F|^2 \sim N$ as promised in the qualitative discussion above: the $|F|^2$ term in eq. (3.10) is associated to the auxiliary of a field of charge -1 , while the D^2 term is associated to the VEV of a field of charge $-1/N$.

To get an anomaly free model, it is sufficient to add two additional superfields, ϕ'_0 and $\phi_{1/N}$, with charge 0 and $1/N$ respectively. This makes the model manifestly vector-like. To qualitatively preserve the characteristics of the minimal model, one can then add the following term to the superpotential:

$$\Delta W = \lambda_3 \phi'_0 \phi_{1/N} \phi_{-1/N}. \quad (3.12)$$

It is clear that for large enough $\hat{\lambda}_3$, this will force $\phi'_0 = \phi_{1/N} = 0$, thus leaving the minimization of the potential unaffected.

Note that the above model serves only as an illustration for the possibility of obtaining a large D -type auxiliary field in a perturbative setup. Models of dynamical supersymmetry breaking can also give rise to a non-vanishing D -type auxiliary field, and a priori there is no reason to expect the VEVs of vector multiplet auxiliary fields to be much smaller than the VEVs of chiral multiplet auxiliary fields. For instance, we examined the $4 - 1$ model of ref. [17] (see also [18] for the use of such model to get significant D auxiliary field). This model can be studied numerically, and we found regions of parameter space where $|D| \gtrsim |F|$ (see also [18]).

In conclusion we have proven that, when the supersymmetry breaking dynamics is described by a set of *weakly* coupled superfields (composite or elementary), the VEV $\langle X^\dagger e^{gV} X|_{\theta^2\bar{\theta}^2} \rangle$ is *not* positive definite. At weak coupling, this quantity becomes negative when D -auxiliary fields are big enough. It is reasonable to conclude that it may well be negative also in models that break supersymmetry in the strongly coupled regime (for which there is no weakly coupled description, elementary or composite). An example in such a class is given by an $SU(5)$ gauge theory with matter in a $10 + \bar{5} \equiv X$ representation [19, 20]. Now $X^\dagger e^{gV} X|_{\theta^2\bar{\theta}^2}$ is a composite operator whose VEV does not have an obvious sign. A truly non-perturbative treatment, the lattice, would be needed to infer the sign. But it seems reasonable to conclude that models in this class have an equally good chance to give positive or negative sfermion masses. The only class of models for which $\langle X^\dagger e^{gV} X|_{\theta^2\bar{\theta}^2} \rangle$ is always positive are weakly coupled O’Raifeartaigh models, which are just the simplest to deal with. In all other models we expect the sign not to be definite.

4 Model building and phenomenology

We now discuss the implications of our simple remark. Almost all the results of this section have appeared before (mostly in ref. [5], but also in refs. [3, 6]) and the purpose of this section is to make a synthesis of the important issues in our setup. Let us start by studying the conditions under which anomaly mediation (AM) and brane-to-brane loops (B2B) contribute in a comparable way to the sfermion masses. We first have to relate the compensator VEV F_ϕ to the scale M_S of supersymmetry breaking in the hidden sector. This is done by demanding that the effective four-dimensional cosmological constant vanishes. The hidden sector gives a positive contribution $V_{\text{hid}} = |F|^2 + |D|^2/2 \sim M_S^4$, while the bulk gravity sector, after radius stabilization⁵, gives a negative contribution V_{bulk} roughly equal to $-M_5^3(T+T^\dagger)|F_\phi|^2$. Therefore by demanding $V_{\text{hid}} + V_{\text{bulk}} = 0$ we obtain

$$|F_\phi|^2 \sim \frac{|F|^2 + D^2/2}{M_5^3 T} = \frac{|F|^2 + D^2/2}{M_P^2}. \quad (4.1)$$

In the end, this just corresponds to the usual relation $M_S^4 \sim M_P^2 m_{3/2}^2$. By eq. (2.3), equality of orders of magnitude of AM and B2B then corresponds to

$$\left(\frac{\alpha}{4\pi}\right)^2 \sim \frac{1}{16\pi^2(M_5 T)^3} \equiv \alpha_5. \quad (4.2)$$

Notice that the quantity α_5 corresponds to the loop expansion parameter controlling gravitational quantum corrections at the compactification scale. Numerically, using $M_P^2 = M_5^3 T = 10^{18} \text{GeV}$ the above relation implies a compactification radius $1/T \sim 10^{17} \text{GeV}$, slightly above the GUT scale.

⁵Later in the section, we will illustrate this by considering the model of ref. [5]

With the size of α_5 fixed, we can estimate the size of flavor violation coming from higher order gravitational interactions. The allowed operators are most conveniently classified by using the superfield formulation of linearized supergravity [21, 22, 23] (see also [4, 6]). On the branes, for example, it is possible to have a four-derivative operator of the form

$$\mathcal{L}^{\partial^4} \sim \int d^4\theta \phi^\dagger \phi \frac{Z_{ij}}{\Lambda_5^2} Q_i^\dagger Q_j (K_{mn} V^n)^2. \quad (4.3)$$

The tensor K_{mn} is the two-derivative kinetic operator for supergravity, V_n is the supergravity superfield containing the $4D$ part of the graviton and the gravitino, and ϕ is the conformal compensator chiral multiplet. The scale $\Lambda_5^3 \sim 16\pi^2 M_5^3$ is the NDA cut-off of five-dimensional gravity⁶. We have checked that this operator can be fully covariantized. The quantity Z_{ij} is a matrix in flavor space, which differs in general from the two-derivative kinetic matrix (or equivalently, it differs from the identity δ_{ij} in the basis where the two-derivative term is canonical) and adds a new source of flavor violation, different from the usual Yukawa matrices. From a gravitational viewpoint, the presence of these extra terms corresponds to tiny violations of the equivalence principle at the quantum gravity scale: two different quarks, say the top and the charm, will follow different trajectories in an external field. From the viewpoint of flavor physics, on the other hand, these terms are like any other violation of the GIM mechanism. The latter, when valid, is an accidental property of the truncation of the Lagrangian to the lowest-order operators in the derivative expansion. In the absence of extra flavor selection rules, NDA suggests that $Z_{ij} \sim 1$. This would correspond to maximal flavor violation happening in processes at the quantum gravity scale Λ_5 . Inserting the operators of eq. (4.3) into the graviton loop mediating the B2B effects can produce flavor-violating soft masses that can be estimated by noticing that the loop is dominated at virtuality of order $1/T$. We find the relative size of flavor breaking and flavor preserving B2B masses to be:

$$\frac{\delta m_{ij}^2}{m_0^2} \sim \frac{1}{\Lambda_5^2 T^2} = \alpha_5^{\frac{2}{3}} \sim \left(\frac{\alpha}{4\pi}\right)^{\frac{4}{3}} \sim 10^{-3}. \quad (4.4)$$

In principle there could also be three-derivative flavor-violating operators similar to eq. (4.3), containing two V_n fields and superspace derivatives, with a structure of the form $Q_i^\dagger Q_j V \partial \partial D^2 V$. Such an operator, however, contains only D and no \bar{D} , and is thus charged under R -symmetry. Its contribution to soft masses would thus either be suppressed by an extra power of $m_{3/2}/M_P$, or, if it was combined with a similar operator of opposite R -charge, give an effect comparable to that of the operator in eq. (4.3).

We can compare the above result with the present experimental bounds on flavor violation in the MSSM. The strongest bounds are associated to ϵ_K and to $\text{Br}(\mu \rightarrow$

⁶We are following the remark made in [24] that this is a more accurate estimate of the NDA scale than the usually used one $24\pi^3 M_5^3$. In the end, the extra “floating” π does not make a big difference in our estimates.

$e\gamma$). Assuming $O(1)$ phases, the bound on ϵ_K implies

$$\left(\frac{\delta m^2}{m^2}\right)^2 \lesssim 10^{-7} \left(\frac{m}{300\text{GeV}}\right)^2, \quad (4.5)$$

which compared to our result eq. (4.4), requires either some extra mild suppression of flavor breaking in Z_{ij} or squarks heavier than a TeV. The first possibility is, for instance, comfortably realized in all flavor models based on the Froggat-Nielsen mechanism, where we expect a further suppression of δm_{12}^2 by at least one power of the Cabibbo angle $\theta_C \simeq 0.2$. In practice, however, the second possibility is “unfortunately” almost likely forced on us by the bounds on the SUSY and Higgs particle masses after LEP. A detailed analysis of electroweak breaking would however be needed to make a more precise statement. In one or the other way, it seems rather easy to satisfy the bound on ϵ_K , although it is possible that the supersymmetric contribution amounts to a sizable fraction of the observed value of ϵ_K . The other strong bound comes from $\text{Br}(\mu \rightarrow e\gamma)$. There are various diagrams contributing to this process in the MSSM. Focussing on the chargino induced contribution, which is dominant (or at least not subleading) in a significant variety of situations, and assuming charginos and sleptons of comparable mass, we obtain the rough estimate (see for instance [25, 26]):

$$\text{Br}(\mu \rightarrow e\gamma) \sim 5 \times 10^{-11} \left(\frac{\delta m^2/m^2}{0.001}\right)^2 \left(\frac{150\text{GeV}}{m}\right)^4. \quad (4.6)$$

This is right at the boundary of the experimentally allowed region and significantly above the planned precision $\sim 10^{-13}$ of the ongoing experiment at PSI [27], unless further sizable suppressions from flavor selection rules intervene.

If the flavor physics scale is the cut off Λ_5 then the flavor violating effects we discussed above are the only ones we expect. This is a consistent but restrictive situation, and it is worth considering the perhaps more realistic situations where there is some flavor dynamics at lower scales.

One such situation occurs if the Standard Model gauge group is unified at a scale $\sim 10^{16}$ GeV. Since quarks and leptons become also unified, the flavor violation encoded in the large top Yukawa coupling can generate flavor violation in the lepton sector through renormalization group evolution between the scale at which the soft masses are generated and the GUT scale. This effect was considered in refs. [28, 29], where universality of soft masses was assumed to hold at the Planck scale and the effect on $\text{Br}(\mu \rightarrow e\gamma)$ was found to be close to the bound. In our case, the soft masses are universal at $1/T$, therefore there is less running to the GUT scale, and we expect the effect to be smaller. Moreover, universal A terms at the cut-off scale were also assumed in [28, 29], so that running above M_{GUT} induced flavor violation in the leptonic A -terms. On the contrary, in our setup, A terms exactly follow their AMSB trajectory, and therefore high energy flavor violation decouples from their low energy value [1]. Therefore, in our case, lepton flavor violation only resides in

the dimension 2 slepton masses. It can be estimated by running the third-family soft mass from the compactification scale $1/T$ to the GUT scale M_{GUT} . This effect depends on the value of the top Yukawa coupling at the GUT scale and is typically enhanced by a large group-theory factor. In the case of a minimal $SO(10)$ theory, for example, one finds the following soft mass splitting between the third and the first two generations:

$$\frac{\delta m_3^2}{m^2} \simeq \frac{15}{8\pi^2} \lambda_t^2 \ln(M_{\text{GUT}} T) \simeq 0.3 \left(\frac{\lambda_t}{0.8} \right)^2. \quad (4.7)$$

$SU(5)$ does not lead to bigger effects. After rotating the matter superfields to diagonalize the Yukawa couplings, this induces a soft mass mixing between the first two generations. In general, even for a fully realistic theory of flavor at the GUT scale we expect the rotation matrix V_ℓ in the lepton sector to have entries comparable to those of the CKM matrix V ⁷. We then obtain for the 1-2 entry of the slepton mass matrix the rough estimate

$$\frac{\delta m^2}{m^2} \sim V_{13} V_{23} \frac{\delta m_3^2}{m^2} \simeq 10^{-4} \left(\frac{\lambda_t}{0.8} \right)^2. \quad (4.8)$$

Neglecting the running down to the weak scale, which does not qualitatively change the result, and using eq. (4.6) we find

$$\text{Br}(\mu \rightarrow e\gamma) \sim 5 \times 10^{-13} \left(\frac{\lambda_t}{0.8} \right)^4 \left(\frac{150\text{GeV}}{m} \right)^4, \quad (4.9)$$

which is below the present bound, but again within reach of future experiments⁸.

Another likely source of lepton flavor violation, in principle independent of unification, is associated to the neutrino Yukawa couplings. If neutrinos are Dirac particles, then these Yukawa couplings are so small that their effect is totally negligible. However, if, as it seems more plausible, neutrinos turn out to be light because of the see-saw mechanism, then they could have sizable and even $O(1)$ Yukawas. This biggest value would correspond to a right handed neutrino mass around $\sim 10^{14}$ GeV. The induced effect is analogous to the one just discussed in Grand Unified models: Yukawa flavor violation feeds into the slepton masses via RG evolution from the scale $1/T$ or M_{GUT} down to the relevant right-handed neutrino mass M_N , at which

⁷For instance in the naive minimal $SU(5)$ model with renormalizable Yukawa interactions, this V_ℓ is exactly the CKM matrix, but at the same time one also gets the wrong predictions $\lambda_e = \lambda_d$, $\lambda_\mu = \lambda_s$ at the GUT scale.

⁸In the case of $SO(10)$, as noticed in ref. [29], the presence of mixing in both right-handed and left handed sleptons makes another diagram, involving A -terms and enhanced by a factor m_τ/m_μ , important. We have checked, that because of other suppressions (this diagram involves the smaller hypercharge coupling and needs a double insertion of the mass splitting in eq. (4.7)) our estimate eq. (4.9) is not qualitatively modified. A more detailed analysis, accounting for the specific features of our spectrum, is clearly needed in order to make more precise statements.

the theory flows to the MSSM [30]. The relevance of this effect however strongly relies on the size of the neutrino Yukawa, which unlike λ_t in the previous discussion, is not fixed directly by experiments, and may well be, say, $O(10^{-2})$ in which case the effects would not be dramatic. The only case in which we necessarily expect some sizable neutrino Yukawa is for GUTs, like $SO(10)$ or bigger groups, where neutrinos and quarks are unified. In this case however the mixings due to neutrino Yukawa matrices are favored to be CKM-like, with the large lepton flavor mixing angles measured at low energy coming from the interplay between the Yukawa and the right-handed neutrino Majorana mass matrix. Then in these models we expect extra contributions that are similar, but not bigger than the estimate in eq. (4.8). Therefore we conclude that, while it is possible that neutrino induced lepton flavor violation becomes quite large, there is no necessary reason for it to be the case [31].

To summarize, we have identified at least 3 sources of flavor violation in our scenario that could contribute in an important way to the rate for $\mu \rightarrow e\gamma$. While all these effects are more or less model dependent, they seem both small enough to be safely compatible with the present bound and big enough to raise hopes for a signal in the ongoing experimental search for $\mu \rightarrow e\gamma$.

To conclude this section, we focus on a simple explicit model of radius stabilization [5] to illustrate that there needn't exist flavor breaking corrections bigger than the ones we have discussed and coming from the radius stabilization mechanism. The model involves two Super Yang-Mills sectors, one on the hidden brane and one in the bulk. Gaugino condensation in the low energy effective theory leads to a superpotential of the form

$$W = \frac{1}{16\pi^2} (\Lambda_1^3 + \Lambda_2^3 e^{-a\Lambda_2 T}), \quad (4.10)$$

where $\Lambda_{1,2}$ are the strong coupling scales of respectively the boundary and bulk gauge interactions, while a is an $O(1)$ coefficient. As emphasized in ref. [5], at the leading order in an expansion in $1/T \propto 1/M_P^2$, it is consistent to study the radius potential by neglecting the supersymmetry breaking sector localized at the hidden brane. The first remark is that, apart from the Goldstino, all the fields in that sector can conceivably have a mass of order $M_S \gg m_{3/2}$, and they can be integrated out before studying radius stabilization. Secondly, as it can be directly checked, the supersymmetry breaking corrections to the radion potential induced by integrating out the hidden sector are controlled by the gravitino mass and are parametrically smaller than the potential induced by W . At the minimum of the radion potential, we obtain

$$\Lambda_2 T \simeq 3 \ln(\Lambda_2/\Lambda_1) + \ln(\Lambda_2 T), \quad (4.11)$$

$$F_\phi \simeq \frac{\Lambda_1^3}{16\pi^2 M_S^3 T} \simeq m_{3/2}, \quad (4.12)$$

$$\frac{F_T}{T} \simeq \frac{m_{3/2}}{\Lambda_2 T}, \quad (4.13)$$

$$m_T \simeq m_{3/2}(\Lambda_2 T). \quad (4.14)$$

Notice that in the perturbative limit, where $\Lambda_2 T \gg 1$, we obtain $m_T \gg m_{3/2}$ and $F_T/T \ll m_{3/2}$, which is consistent with neglecting gravity mediated effects associated to the radion, as just discussed. A reasonable assumption, that allows us to proceed further, is that the quantum scales of the SYM theory and gravity in the bulk coincide: $\Lambda_5 \sim \Lambda_2$. In this case, by using eqs. (4.11) and (4.12) we can simply express the gravitational expansion parameter as

$$\alpha_5 \equiv \frac{1}{(\Lambda_5 T)^3} \sim \left(\ln \frac{M_P}{m_{3/2}} \right)^{-3}. \quad (4.15)$$

As remarked in ref. [5], by inputting $m_{3/2} \sim 10^5$ GeV, and for $M_P \sim 10^{18}$ GeV, we automatically obtain $\alpha_5 \sim (1/30)^3 \sim 10^{-4}$, in such a way that AM and B2B are effortlessly of the same order.

Let us now consider the potentially dangerous effects mediated by the bulk gauge fields in the above model. Since the observable and hidden sector fields are not charged under the bulk gauge group, the relevant couplings between fields localized on the branes and bulk gauge fields are non-renormalizable interactions of the form

$$\Omega_v \supset \left((A_v)_{ij} \frac{W^\alpha W_\alpha}{\Lambda_2^3} + (B_v)_{ij} \frac{D^\alpha W^\beta D_\alpha W_\beta}{\Lambda_2^4} + \dots + \text{h.c.} \right) Q^{i\dagger} Q^j, \quad (4.16)$$

$$\Omega_h \supset \left(A_h \frac{W^\alpha W_\alpha}{\Lambda_2^3} + B_h \frac{D^\alpha W^\beta D_\alpha W_\beta}{\Lambda_2^4} + \dots + \text{h.c.} \right) X^\dagger e^{gV} X. \quad (4.17)$$

The quantities $(A_v)_{ij}$ and $(B_v)_{ij}$ are a priori generic flavour non-universal matrices, whereas A_h and B_h are numbers, and all of them are expected by NDA to be $O(1)$. Notice however that the leading first term in each parenthesis violates R -symmetry, and its coefficient could therefore be further suppressed compared to the one of the second term in each parenthesis, which preserves R -symmetry, depending on the extent to which R symmetry is broken. The interactions in Ω_v and Ω_h , through the exchange of the bulk gauge fields, give rise to potentially dangerous flavour violating contributions to sfermion masses. (Notice that since $\Lambda_2 T \sim 30$, any non-perturbative effect associated to gaugino condensation is highly suppressed, and can therefore be neglected). The leading contribution comes from a one-loop diagram involving a vertex proportional to A_v/Λ_2^3 and one proportional to A_h/Λ_2^3 . As usual, the loop integral is saturated at momenta of the order of the compactification scale $1/T$, and by dimensional analysis it yields a factor of order $g_5^4/(4\pi^2 T^6)$. Since $\Lambda_2 = 8\pi^2/g_5^2$, this can be rewritten as $16\pi^2/(\Lambda_2^2 T^6)$. This results finally in the following effective operator:

$$\Delta\Omega_{1\text{-loop}}^{\text{gau}} \sim (A_v)_{ij} A_h \frac{16\pi^2}{\Lambda_2^8 T^6} Q^{i\dagger} Q^j X^\dagger e^{gV} X + \text{h.c.} \quad (4.18)$$

This has to be compared with the gravitational effect encoded in the third term of eq. (2.3), which can be rewritten as

$$\Delta\Omega_{1\text{-loop}}^{\text{gra}} \sim \delta_{ij} \frac{16\pi^2}{\Lambda_2^6 T^4} Q^{i\dagger} Q^j X^\dagger e^{gV} X. \quad (4.19)$$

We see that the effect (4.18) is down by at least a factor $1/(\Lambda_2 T)^2$ compared to (4.19), even in the limit where R symmetry is maximally broken. Since $\Lambda_2 \sim \Lambda_5$, this effect has therefore at worst the same size as the one due to subleading gravitational effects. It is straightforward to verify that the effects induced by the subleading R -preserving couplings in eqs. (4.16) and (4.17) are further suppressed by extra powers of $1/(\Lambda_2 T)^2$ and are therefore less important than the subleading gravitational effect. The typical size of flavour violating effects remains therefore the one of eq. (4.4).

5 Conclusions

We have shown that when D -type auxiliaries in the hidden sector have a big enough VEV, then the universal soft scalar masses squared induced by gravitational loops in five-dimensional sequestered models is positive. We have argued, by showing two explicit examples, that this situation is not hard to achieve and that it may be pretty generic. This opens up the possibility to build viable models where satisfactory gaugino and sfermion masses are naturally generated by a combination of anomaly mediation and brane-to-brane gravity loops. We have also estimated the flavor non-universal subleading corrections that are generically expected to occur and found that they do not contradict to present experimental bounds. Moreover we found that the rate for $\mu \rightarrow e\gamma$ could plausibly be within the reach of the planned sensitivity of the ongoing experiment. An issue that remains to be studied concerns the generation of μ and $B\mu$, which is known to be a generic weakness of the anomaly mediated scenario, and the breaking of the electroweak symmetry. One interesting and perhaps natural direction to explore would be to extend the model by adding a light singlet (NMSSM). We hope to do that in future work.

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