

# Anomalies in orbifold field theories

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## Abstract

We study the constraints on models with extra dimensions arising from local anomaly cancellation. We consider a five-dimensional field theory with a  $U(1)$  gauge field and a charged fermion, compactified on the orbifold  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$ . We show that, even if the orbifold projections remove both fermionic zero modes, there are gauge anomalies localized at the fixed points. Anomalies naively cancel after integration over the fifth dimension, but gauge invariance is broken, spoiling the consistency of the theory. We discuss their implications for realistic supersymmetric models with a single Higgs hypermultiplet in the bulk, and possible cancellation mechanisms in non-minimal models.

# 1 Introduction

Theories formulated in  $D > 4$  space-time dimensions may lead to a geometrical understanding of the problems of mass generation and symmetry breaking. Orbifold compactifications [1] of higher-dimensional theories are simple and efficient mechanisms to reduce their symmetries and to generate four-dimensional (4-D) chirality. Phenomenologically interesting orbifold models can be formulated, either as explicit string constructions or as effective higher-dimensional field theories.

The field-theoretical approach to orbifolds is currently fashionable because of its apparent simplicity and flexibility. However, it is well known that the rules for the construction of consistent string-theory orbifolds are quite stringent, and automatically implement a number of consistency conditions in the corresponding effective field theories: in particular, the cancellation of gauge, gravitational and mixed anomalies. Since anomalies are infrared phenomena, if we start from a consistent string model (‘top-down’ approach), anomaly cancellation must find an appropriate description in the effective field theory. Such a description, however, may be non-trivial, as for the Green-Schwarz [2] or the inflow [3] mechanisms. If, instead, we decide to work directly at the field-theory level (‘bottom-up’ approach), great care is needed, since orbifold projections do not necessarily preserve the quantum consistency of a field theory (as discussed, for example, in [4]). In particular, the question of anomaly cancellation must be explicitly addressed.

A first step in this direction was taken in ref. [5], which discussed the chiral anomaly in a five-dimensional (5-D) theory compactified on the orbifold  $S^1/\mathbf{Z}_2$ . It was found that, in such a simple context, naive 4-D anomaly cancellation is sufficient to ensure 5-D anomaly cancellation. For a 5-D fermion of unit charge, and a chiral action of the  $\mathbf{Z}_2$  projection, the 5-D anomaly is localized at the orbifold fixed points, and is proportional to the 4-D anomaly:

$$\partial_M J^M(x, y) = \frac{1}{4} [\delta(y) + \delta(y - \pi R)] \mathcal{Q}(x, y), \quad (1)$$

where<sup>1</sup>  $J^M$  is the 5-D current and

$$\mathcal{Q}(x, y) = \frac{g_5^2}{16\pi^2} F_{\mu\nu}(x, y) \tilde{F}^{\mu\nu}(x, y) \quad (2)$$

is proportional to the 4-D chiral anomaly from a charged Dirac spinor in the external gauge potential  $A_\mu(x, y)$ . In our notation:  $M = [(\mu = 0, 1, 2, 3), 4]$ ;  $x \equiv (x^{0,1,2,3})$  are the first four coordinates,  $y \equiv x^4$  is the fifth coordinate, compactified on a circle of radius  $R$ ;  $y = 0, \pi R$  are the two fixed points with respect to the  $Z_2$  symmetry  $y \rightarrow -y$ ;  $g_5$  is the 5-D gauge coupling constant.

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<sup>1</sup>We work on the orbifold covering space  $S^1$ , and we normalize the  $\delta$ -functions so that, for  $y_0 \in [0, 2\pi R)$  and  $0 < \epsilon < 2\pi R - y_0$ ,  $\int_{-\epsilon}^{2\pi R - \epsilon} dy \delta(y - y_0) f(y) = f(y_0)$ .

In this letter we show that the phenomenon discussed in [5] does not persist in more general cases. To be definite, we consider a 5-D field theory with a  $U(1)$  gauge field  $A_M$  and a massless fermion  $\psi$  of unit charge, compactified on the orbifold  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$ . The action of the two parities are  $y \rightarrow -y$  and  $y' \rightarrow -y'$ , respectively, where  $y' = y - \pi R/2$ . Both the gauge and the fermion fields are taken to be periodic on the circle. We decompose the Dirac spinor  $\psi$  into left and right spinors with parities  $(+, -)$  and  $(-, +)$ , respectively:  $\psi \equiv \psi^{+-} + \psi^{-+}$ . Notice that a standard fermion mass term is forbidden by the  $\mathbf{Z}_2 \times \mathbf{Z}'_2$  symmetry. As for the gauge field, we assign  $(+, +)$  parities to  $A_\mu$ ,  $(-, -)$  to  $A_4$ . Although the theory has no massless 4-D chiral fermion, a non-vanishing anomaly is induced, given by eq. (10) below.

The theory can be trivially supersymmetrized, by embedding its field content in a  $U(1)$  vector multiplet and a charged hypermultiplet. From the point of view of anomalies, our simple example reproduces the essential features of a recently proposed phenomenological model [6], whose light spectrum contains just the states of the Standard Model (SM), with an anomaly-free fermion content. The underlying 5-D theory is supersymmetric, with vector multiplets containing the SM gauge bosons, and hypermultiplets containing the SM quarks and leptons. In addition, the model of ref. [6] has just one charged hypermultiplet, which contains the SM Higgs boson. Such a model has received some attention because it may give a prediction for the Higgs mass, even though it was recently shown [7] that the Higgs self-energy receives a quadratically divergent one-loop contribution. The latter corresponds to the appearance of a Fayet–Iliopoulos (FI) term, with divergences localized at the orbifold fixed points, which immediately hints at a possible connection with anomalies.

The content of the present letter is organized as follows. We begin by showing that, even if there are no 4-D massless fermions in the spectrum, our simple 5-D theory is actually anomalous. Localized anomalies, with opposite signs, appear at the fixed points of the two orbifold projections. The integrated anomaly vanishes, reflecting the absence of any one-loop anomaly among 4-D massless states, but there are anomalous triangle diagrams when at least one of the external states is a massive Kaluza–Klein (KK) mode. We focus our attention on the  $U(1)^3$  gauge anomaly, which we explicitly compute along the lines of [5]. In realistic extensions, such as [6], similar results would hold for the  $U(1)_Y^3$ ,  $U(1)_Y$ – $SU(2)_L^2$  and  $U(1)_Y$ –gravitational anomalies. We then argue that this anomaly leads to a breakdown of 4-D gauge invariance. Hence, in its minimal form, the model is inconsistent, even as an effective low-energy theory. Next, we consider the supersymmetric extension of our simple theory, and compute the precise expression for the one-loop FI term. Finally, we discuss the possible modifications that could restore the consistency of the theory.

## 2 $U(1)$ anomalies

In this section, we take the theory defined in the Introduction and we compute the  $U(1)^3$  anomaly, following closely the method and the notation of ref. [5] (for an early computation of this type, see also ref. [8]).

The KK wavefunctions  $\xi^{ab}$  for fields  $\varphi^{ab}$  of definite  $\mathbf{Z}_2 \times \mathbf{Z}'_2$  parities ( $a, b = \pm$ ) are defined as:

$$\begin{aligned}\xi_{n \geq 0}^{++}(y) &\equiv \frac{\eta_n}{\sqrt{\pi R}} \cos \frac{2ny}{R}, & \xi_{n > 0}^{+-}(y) &\equiv \frac{1}{\sqrt{\pi R}} \cos \frac{(2n-1)y}{R}, \\ \xi_{n > 0}^{--}(y) &\equiv \frac{1}{\sqrt{\pi R}} \sin \frac{2ny}{R}, & \xi_{n > 0}^{-+}(y) &\equiv \frac{1}{\sqrt{\pi R}} \sin \frac{(2n-1)y}{R},\end{aligned}\quad (3)$$

where  $\eta_n$  is  $1/\sqrt{2}$  for  $n = 0$  and 1 for  $n > 0$ . They form a complete orthonormal basis of periodic functions on  $S^1$ , with given  $\mathbf{Z}_2 \times \mathbf{Z}'_2$  parities. The Fourier modes of a field  $\varphi^{ab}$  are defined as:

$$\varphi_n^{ab}(x) \equiv \int_0^{2\pi R} dy \xi_n^{ab}(y) \varphi^{ab}(x, y), \quad (4)$$

and have a mass given by  $m_{2n+(ab-1)/2}$ , where  $m_n = n/R$ .

In the gauge  $A_4 = 0$ , the 4-D Lagrangian for the Fourier modes  $\psi_n \equiv \psi_n^{+-} + \psi_n^{-+}$  can be written as

$$\mathcal{L} = \sum_{m,n} \bar{\psi}_m \left[ (i\partial - m_{2n-1}) \delta_{mn} - g_5 \mathcal{A}_{mn} \right] \psi_n, \quad (5)$$

where  $\mathcal{A}_{mn} \equiv \mathcal{A}_{mn}^{+-} P_+ + \mathcal{A}_{mn}^{-+} P_-$ , with  $P_{\pm} = (1 \pm \gamma_5)/2$ , and <sup>2</sup>

$$A_{\mu mn}^{\pm\mp}(x) \equiv \int_0^{2\pi R} dy \xi_m^{\pm\mp}(y) \xi_n^{\pm\mp}(y) A_{\mu}(x, y) \quad (6)$$

in terms of the  $U(1)$  connection  $A_{\mu}$ .

Interpreting  $\psi_n$  as a single fermion with a flavour index and chiral couplings to the gauge field  $A_{mn}^{\mu}$  through the currents  $J_{\pm mn}^{\mu} = \bar{\psi}_m \gamma^{\mu} P_{\pm} \psi_n$ , it is straightforward to adapt the standard computation of anomalies to obtain:

$$\partial_{\mu} J_{\pm mn}^{\mu} = \pm \left( m_{2m-1} J_{\pm mn}^4 + m_{2n-1} J_{\mp mn}^4 \right) \pm \frac{g_5^2}{32\pi^2} \sum_{k>0} F_{\mu\nu mk}^{\pm\mp} \tilde{F}_{kn}^{\mu\nu \pm\mp}, \quad (7)$$

where  $J_{\pm mn}^4 = \bar{\psi}_m i\gamma_5 P_{\pm} \psi_n$ . Equation (7) can be easily Fourier-transformed back to configuration space by convolution with  $\xi_m^{\pm\mp}(y) \xi_n^{\pm\mp}(y)$ . Using completeness, this yields

$$\partial_M J_{\pm}^M(x, y) = \pm \frac{1}{2} \sum_{k>0} [\xi_k^{\pm\mp}(y)]^2 \mathcal{Q}(x, y), \quad (8)$$

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<sup>2</sup>Notice that the  $A_{\mu mn}^{\pm\mp}$  are not Fourier modes of the type (4), but can be easily related to them. One finds:  $A_{mn}^{\mu \pm\mp} = (\eta_{|m-n|}^{-1} A_{|m-n|}^{\mu} \pm \eta_{|m+n-1|}^{-1} A_{|m+n-1|}^{\mu}) / \sqrt{\pi R}$ .

where the quantity  $\mathcal{Q}$  was defined in eq. (2). The anomaly in the vector current  $J^M(x, y) = J_+^M(x, y) + J_-^M(x, y)$  is then proportional to

$$\sum_{k>0} [(\xi_k^{+-}(y))^2 - (\xi_k^{-+}(y))^2] = \frac{1}{4} e^{-2iy/R} \sum_{l=-\infty}^{\infty} \delta(y - l\pi R/2), \quad (9)$$

hence

$$\partial_M J^M(x, y) = \frac{1}{8} [\delta(y) - \delta(y - \pi R/2) + \delta(y - \pi R) - \delta(y - 3\pi R/2)] \mathcal{Q}(x, y). \quad (10)$$

Therefore, although the integrated anomaly vanishes, there are anomalies, localized at the fixed points, that are equal in magnitude to 1/4 (or 1/2 if we sum the contribution from identified fixed points) of the anomaly from a 4-D Weyl fermion. The full 5-D theory is thus inconsistent (at least in its minimal form).

Let us now rewrite eq. (10) in terms of standard Fourier modes of the current and gauge fields. Recalling that both have  $(+, +)$  parities, the Fourier transform of (10) takes the form:

$$q_M J_n^M(q) = \frac{1}{g_5} \sum_{i,j=0}^{\infty} \int \frac{d^4 p}{(2\pi)^4} q_M T_{nij}^{M\alpha\beta}(p, q) A_{\alpha i}(p) A_{\beta j}(q - p), \quad (11)$$

where

$$q_M T_{nij}^{M\alpha\beta}(p, q) = \frac{g_4^3}{2\sqrt{2}\pi^2} \eta_n \eta_i \eta_j \delta_{n+i+j, \text{odd}} \epsilon^{\alpha\beta\mu\nu} p_\mu q_\nu, \quad (12)$$

with  $g_4 = g_5/\sqrt{2\pi R}$ . This quantity encodes the triangular anomaly between three external KK modes of the photon with indices  $(n, i, j)$ , as illustrated in fig. 1.

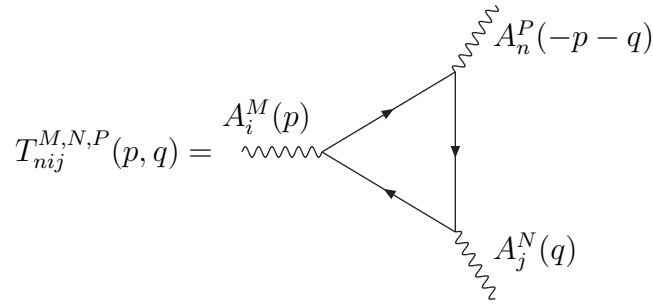


Figure 1: The 1-loop anomalous diagram.

This anomaly vanishes for  $n + i + j = \text{even}$ , and in particular for  $n = i = j = 0$ , reflecting the fact that there is no 4-D anomaly for the massless modes: all non-vanishing anomalous diagrams involve at least one massive mode. These diagrams make the full theory inconsistent. However, it may be asked whether the low-energy effective theory obtained by integrating out all massive modes could be consistent.

This is not the case, because gluing such diagrams through heavy lines produces 4-D gauge symmetry breaking effective interactions among zero-modes. Consider for instance a 3-loop diagram obtained by gluing two anomalous triangles through two massive photons, as depicted in fig. 2. This represents a contribution to the two-point function  $\Pi^{\mu\nu}$  of the zero-mode photon that violates gauge invariance. The non-vanishing longitudinal component of  $\Pi^{\mu\nu}$  is encoded in  $q_\mu q_\nu \Pi^{\mu\nu}(q)$ , which feels only the anomalous part of the triangular subdiagram [9]. Another example is the four-point function involving two longitudinal and two transverse zero-mode photons, which receives a non-vanishing finite two-loop contribution controlled by the anomaly.

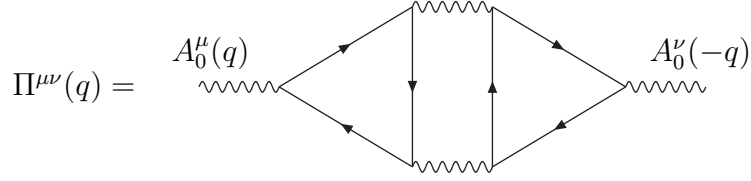


Figure 2: The 3-loop anomalous contribution to the photon two-point function.

These gauge anomalies could be computed in an independent way by using their well-known relation with chiral anomalies and index theorems, which is particularly clear in Fujikawa's approach. In this formalism, the integrated chiral anomaly of a 4-D Dirac fermion is encoded in the quantity  $\text{Tr}_{D=4} [\gamma_5] = \text{index}(\not{D})$ , where  $\not{D}$  is the Dirac operator. In the case of a 5-D theory compactified on  $S^1$ , the anomaly vanishes, since the Hilbert space splits into two identical components of opposite chirality and  $\text{Tr}_{D=5} [\gamma_5] = 0$ . This can be easily extended to an  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$  orbifold compactification. The trace must now be restricted to invariant states only; this can be achieved by inserting into the unconstrained trace a  $\mathbf{Z}_2 \times \mathbf{Z}'_2$  projector  $P$ . Denoting by  $g$  and  $g'$  the generators of  $\mathbf{Z}_2$  and  $\mathbf{Z}'_2$  respectively, the explicit expression of this projector is  $P = \frac{1}{4}(1 + g + g' + gg')$ . Each element in  $P$ , when inserted in the trace, leads to a so-called equivariant index of the Dirac operator. This has a non-vanishing support only at the fixed points of the element. The identity in  $P$  gives a vanishing result as in the  $S^1$  compactification. Similarly, the  $gg'$  element also gives a vanishing contribution, because it generates a translation along the compact direction that does not affect chirality. On the other hand, the elements  $g$  and  $g'$  act chirally on the Dirac fermion  $\psi$  ( $g\psi g^{-1} = \gamma_5\psi$ ,  $g'\psi g'^{-1} = -\gamma_5\psi$ ) and give a non-vanishing contribution. Both have two fixed points, and the integrated anomaly is thus :

$$\begin{aligned} \text{Tr}_{D=5} [P \gamma_5] &= \frac{1}{4} \sum_{i=1}^4 \text{index}(\not{D})|_{y_i} \\ &= \frac{1}{4} \int d^4x \left[ \mathcal{Q}(x, 0) + \mathcal{Q}(x, \pi R) - \mathcal{Q}(x, \pi R/2) - \mathcal{Q}(x, 3\pi R/2) \right], \end{aligned} \quad (13)$$

where the relative sign between the contributions associated with  $g$  and  $g'$  is due to their opposite action on fermions. This leads to (10).

### 3 Supersymmetric models

The result found for the anomalies in section 2 can be trivially extended to supersymmetric configurations, where the  $U(1)$  gauge field belongs to a 5-D  $N = 1$  vector multiplet and the Dirac fermion  $\psi$  to a charged hypermultiplet. As such, all the above considerations apply also to the model of ref. [6]. In particular, focusing on the Higgs hypermultiplet, doublet under  $SU(2)_L$  with hypercharge  $Y = +1$ , we get a localized  $U(1)_Y^3$  anomaly that is twice the one in (10).

The reader may wonder whether such localized anomaly has any relation with the one-loop FI term recently found in [7]. The method of the previous section can be easily extended to the computation of the full one-loop FI term. The relevant part of the Lagrangian is

$$\begin{aligned} \mathcal{L} = & - \sum_{m,n} (\phi_m^{++})^\dagger \left[ (\square + m_{2n}^2) \delta_{mn} - g_5 D_{mn}^{++} \right] \phi_n^{++} \\ & - \sum_{m,n} (\phi_m^{--})^\dagger \left[ (\square + m_{2n}^2) \delta_{mn} + g_5 D_{mn}^{--} \right] \phi_n^{--}, \end{aligned} \quad (14)$$

where  $\phi_m^{\pm\pm}$  are the modes, defined according to eqs. (3) and (4), of the two scalars in the Higgs hypermultiplet, and

$$D_{mn}^{\pm\pm}(x) \equiv \int_0^{2\pi R} dy \xi_m^{\pm\pm}(y) \xi_n^{\pm\pm}(y) D(x, y), \quad (15)$$

where  $D(x, y)$  is the third component of the triplet of  $N = 2$  auxiliary fields. Considering again the mode indices as flavour indices, we find for the FI term:

$$\mathcal{F}(x) = \sum_{n \geq 0} T_n (D_{nn}^{++} - D_{nn}^{--})(x), \quad (16)$$

where

$$T_n = i g_5 \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - m_{2n}^2} = \frac{g_5}{16\pi^2} \left( \Lambda^2 - m_{2n}^2 \ln \frac{\Lambda^2 + m_{2n}^2}{m_{2n}^2} \right), \quad (17)$$

where  $\Lambda$  is an ultraviolet cutoff. By Fourier-transforming back to configuration space, we can write

$$\mathcal{F}(x) = \int_0^{2\pi R} dy \xi(y) D(x, y), \quad (18)$$

where the exact profile of  $\xi(y)$  can be explicitly evaluated. By first summing over the KK states, the 4-D momentum integral is convergent for generic  $y$ , yielding:

$$\xi(y) = \sum_{n \geq 0} T_n \left[ (\xi_n^{++}(y))^2 - (\xi_n^{--}(y))^2 \right] = \frac{g_5}{8\pi^5 R^3} \left[ \zeta\left(3, \frac{2\tilde{y}}{\pi R}\right) + \zeta\left(3, 1 - \frac{2\tilde{y}}{\pi R}\right) \right], \quad (19)$$

where  $\tilde{y} = y - \pi R/2 \sum_{l>0} \theta(y - l\pi R/2)$  is the restriction of  $y$  to the interval  $[0, \pi R/2[$ . Equation (19) diverges as  $\tilde{y}^{-3}$  when  $\tilde{y}$  tends to 0, which corresponds to  $y$  approaching one of the four fixed points  $y_i = (i-1)\pi R/2$  ( $i = 1, 2, 3, 4$ ). Away from the fixed points, there is only a finite bulk contribution. The divergent part of  $\xi(y)$  is easily evaluated by going back to (17) and using:

$$\begin{aligned} \sum_{k \geq 0} [(\xi_k^{++}(y))^2 - (\xi_k^{--}(y))^2] &= \frac{1}{4} \sum_{i=1}^4 \delta(y - y_i), \\ \sum_{k \geq 0} m_{2k}^2 [(\xi_k^{++}(y))^2 - (\xi_k^{--}(y))^2] &= -\frac{1}{16} \sum_{i=1}^4 \delta''(y - y_i). \end{aligned} \quad (20)$$

Hence, the structure of the FI term is:

$$\mathcal{F}(x) = \frac{g_5}{64\pi^2} \sum_{i=1}^4 \left[ \Lambda^2 D(x, y_i) + \frac{1}{2} \ln(\Lambda R) D''(x, y_i) \right] + \int_0^{2\pi R} dy K(y) D(x, y), \quad (21)$$

with  $K(y)$  being a finite function. Therefore, the divergent part of the induced FI term is localized at the orbifold fixed points, as the anomaly. This is a remnant of the relation between FI terms and mixed  $U(1)$ –gravitational anomalies in supersymmetric theories.

## 4 Outlook

We have seen that orbifold field theories can be anomalous even in the absence of an anomalous spectrum of zero modes. It is then important to understand whether there exist anomaly cancellation mechanisms, and whether they can be consistently implemented: for definiteness, we discuss this issue by making reference once more to the case of  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$ .

One possibility would be to add localized fermions at the fixed points, analogous to the twisted sectors of string compactifications. However, as we have seen in section 2, a bulk fermion produces only half of the anomaly of a Weyl fermion at each fixed point. Therefore, this possibility may be generically cumbersome to realize without the guidance of an underlying string theory.

Another possibility would be to implement an anomaly cancellation mechanism of the Green–Schwarz [2] or inflow [3] type. The former would lead to a spontaneous breaking of the  $U(1)$  gauge symmetry [10]. For the latter, we must cope with the fact that a 5-D Chern–Simons term  $\epsilon^{MNO PQ} A_M F_{NO} F_{PQ}$  (see [11]) cannot be added to the bulk Lagrangian, because it is not invariant under the two orbifold projections. However, we can imagine more general possibilities. For example <sup>3</sup>, we could introduce

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<sup>3</sup>We thank R. Rattazzi for having suggested this possibility to us.



a bosonic field  $\chi$  with  $(-, -)$  periodicities and try to introduce the Chern–Simons term in combination with  $\chi$ . The field  $\chi$  should then dynamically get a vacuum expectation value with a non-trivial  $y$ -profile

$$\langle \chi(y) \rangle \propto \begin{cases} +1, & 0 < y < \frac{\pi R}{2} \\ -1, & \frac{\pi R}{2} < y < \pi R \end{cases} \quad (\text{mod. } \pi R) \quad (22)$$

(breaking spontaneously the  $\mathbf{Z}_2 \times \mathbf{Z}'_2$  discrete symmetry), thereby generating a sort of magnetic charge for the fixed points and leaving only very massive fluctuations. The resulting Chern–Simons term could then cancel, for an appropriate value of the coefficient, the one-loop anomaly. Notice that this mechanism can work only when the integrated anomaly vanishes.

It is interesting to observe that the presence and the structure of the anomalies that we have found in the  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$  orbifold could have been anticipated <sup>4</sup> by analysing the intermediate models on  $S^1/\mathbf{Z}_2$  or  $S^1/\mathbf{Z}'_2$ . Indeed, the  $S^1/\mathbf{Z}_2$  and  $S^1/\mathbf{Z}'_2$  models have anomalies given by eq. (1) and localized at the two  $\mathbf{Z}_2$  and  $\mathbf{Z}'_2$  fixed points respectively, but with opposite sign, reflecting the difference between the  $\mathbf{Z}_2$  and  $\mathbf{Z}'_2$  actions on the fermions.

For supersymmetric models, all the above considerations apply, but supersymmetry poses further constraints. It seems then quite difficult to get a consistent SUSY field theory on the  $S^1/(\mathbf{Z}_2 \times \mathbf{Z}'_2)$  orbifold with a single bulk Higgs hypermultiplet. On the contrary, the addition of a second Higgs hypermultiplet in the bulk, as in [12], would cancel at the same time the anomaly and the one-loop-induced FI term. It therefore seems that the necessity of having two Higgs doublets in 4-D supersymmetric extensions of the SM persists also in these higher-dimensional constructions.

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