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A note on supersymmetric D-brane dynamics

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Abstract

We study the spin dependence of D-brane dynamics in the Green-Schwarz formalism of boundary states. In particular we show how to interpret insertion of supercharges on the boundary state as sources of non-universal spin effects in D-brane potentials. In this way we find for a generic (D)p-brane, potentials going like v^{4-n}/r^{7-p+n} corresponding to interactions between the different components of the D-brane supermultiplet. From the eleven dimensional point of view these potentials arise from the exchange of field strengths corresponding to the graviton and the three form, coupled non-minimally to the branes. We show how an annulus computation truncated to its massless contribution is enough to reproduce these next-to-leading effects, meaning in particular that the one-loop (M)atrix theory effective action should encode all the spin dependence of low-energy supergravity interactions.

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1. Introduction

The D-brane description of solitons carrying Ramond-Ramond charges [1] provided us with an explicit tool to study new phenomena in string theory, improving drastically our current understanding of the non-perturbative physics. In particular, the study of soliton interactions or multi-soliton configurations, very non-trivial issues in quantum field theory, are easily performed in the D-brane language. A simple one-loop annulus computation, for instance, is enough to show the BPS "no force" condition between two parallel static D-branes [1] or to study the semiclassical phase shift of one brane moving past an other [2]. The solitons described by these brane configurations presents however a peculiar property, not present in the more familiar solitons appearing in quantum field theory. Their size, indeed, in the limit of small string coupling constant, becomes much smaller than the usual soliton size, fixed basically by the scale of perturbative states, allowing to test distances even shorter than the usual string length [3, 4]. The proposal of [5] for a parton description of M-theory in a given kinematical region as given by an effective Super Yang-Mills $U(N)$ quantum mechanics [6] is an exciting application of these ideas, encouraging to a deeper study of the dynamics of D-branes. Altough there have been several works analyzing D-brane interactions in various configurations, most of them considered the approximation in which a D-brane is a heavy semiclassical spinless state.

Aim of this work is to analyze some non-universal D-brane interactions, due to spin effects, in order to understand the structure of the next-to-leading terms of their potentials. The interaction between two moving D-branes can be written schematically as

$$V(r^2 = b^2 + v^2t^2) \sim J^M(0)\Delta_{MN}(r)J^N(r) \tag{1.1}$$

where $J^M(r)$ denotes generically the eikonal aproximation of the currents $J^\mu(r)$, $T^{\mu\nu}(r)$, etc., according to the possible spin of the fields to which this current couples (vector field, graviton, etc.) and $\Delta_{MN}(r)$ represents the ten dimensional propagator of the corresponding particle exchanged. The sum over M, N run over all the infinite closed string states the two D-branes can exchange.

These currents in momentum space can be decomposed into two pieces :

$$J^M(p,q) = J^M_{univ}(p) + J^M_{spin}(p,q) \tag{1.2}$$

where p is the momentum of the scattered D-brane, in this aproximation much bigger than q, the transfered one. The universal current J^M_{univ} is always determined simply by the ten-momentum p as

$$J^\mu \sim p^\mu, \qquad T^{\mu\nu} \ \sim \ p^\mu p^\nu, \quad \text{etc.}$$

for currents coupled respectively to fields of spin 1, 2, etc.. The large-distance potential is governed by the universal couplings of these currents with the massless string states, that combine in the leading contribution [2]:

$$V(r,v) \sim (\cosh 2v \mp 4\cosh v + 3)/r^{7-p} \tag{1.3}$$

once one substitutes the ten dimensional momentum $p_\mu = M(\cosh v, \sinh v, 0, ...0)$. The minus (plus) sign is for branes with the same (opposite) charge and leads, in the nonrelativistic limit, to a brane-brane potential going like v^4/r^{7-p} [1]. This is indeed the universal, spin-independent potential for two moving D-branes.

The remaining part of the current J_M^{spin}, at least linear in the transfer momentum, will lead to subleading potentials at large distances whose specific form will depend on the particular state of the D-brane supermultiplet. These sources represent in general non-minimal couplings of super D-branes to the bulk closed string states. In this note we study these next-to-leading effects by using the boundary state technique in the Green-Schwarz formalism [7]. We introduce our formalism, showing how to define a moving boundary

[1]Being interested in nonrelativistic processes, we do not distinguish between velocity and rapidity throughout all the paper.

state in light-cone gauge and the way we use it to analyze spin effects, and then explicity compute the first next-to-leading interactions for generic p-branes. We find in general, besides the universal v^4/r^{7-p} term, a spin-orbit like coupling whose expansion in velocity gives rise to a long range potential v^3/r^{8-p}, and a spin-spin effect $\sim v^2/r^{9-p}$. Analogously, higher effects lead to potentials of order v/r^{10-p} and $1/r^{11-p}$. These leading effects, like in the universal case, are reproduced by a one-loop computation restricted to the massless sector of the open strings stretched between the branes. In particular, for the D0 brane case, this means that a one-loop (M)atrix theory [5] computation in fermionic or more general bosonic backgrounds, corresponding to the studied relative polarizations, should reproduce these effects.

2. Supersymmetric D-branes

In this section we study the spin dependence of D-brane potentials by using the technique of boundary states in the Green-Schwarz formalism, following in particular ref.[7]. D-branes are solitonic BPS saturated configurations of type IIA(B) superstring theory. They are arranged in short-multiplets, that in terms of the little group $SO(9)$ decompose in $\mathbf{128} + \mathbf{84} + \mathbf{44}$, that represent repectively a massive spin 3/2 fermion together with a third-rank antisymmetric and a spin 2 bosonic fields. D-brane interactions, mediated by open strings stretched between them, can be interpreted in the dual channel due to exchange of closed fields of which they are sources. Altough many of our results are valid at all scales, we are mainly interested in this paper to the large distance behaviour of D-brane interactions, so that our analysis will be performed from the closed string point of view. D-brane boundary state techniques are indeed quite useful in this context; a (D)p-brane boundary state $|B\rangle$ is indeed an object that encodes all the (infinite) couplings between a (D)p-brane, considered as a classical source, and the closed string states emitted by it. In the Green-Schwarz formalism, where supersymmetry is manifest, a boundary state can be defined to be the state preserving the linear combination of supercharges

$$\begin{aligned} Q^a_\eta \,|B,\eta\rangle &\equiv (Q^a + i\eta M_{a\dot{b}}\tilde{Q}^{\dot{b}})\,|B,\eta\rangle = 0 \\ Q^{\dot{a}}_\eta \,|B,\eta\rangle &\equiv (Q^{\dot{a}} + i\eta M_{\dot{a}b}\tilde{Q}^{b})\,|B,\eta\rangle = 0 \end{aligned} \tag{2.1}$$

valid in type IIA theory for p even; the case of type IIB (p odd) is easily recovered by switching the dotted and undotted indices in the right-moving charges $\tilde{Q}$. We borrow in (2.1) the notation and definitions of [7], that will be used throughout all the paper. The solution for $|B,\eta\rangle$ is then given by:

$$|B,\eta\rangle = \exp\sum_{n>0}\left(\frac{1}{n}M_{ij}\alpha^i_{-n}\tilde{\alpha}^j_{-n} - i\eta M_{a\dot{b}}S^a_{-n}\tilde{S}^{\dot{b}}_{-n}\right)|B_0,\eta\rangle \tag{2.2}$$

The indices i, a, $\dot{a}$ run over the vector and the two spinor representations of $SO(8)$, $\eta=\pm$ label the brane-antibrane nature and the zero mode part is represented by

$$|B_0,\eta\rangle = \left(M_{ij}|i\rangle|\tilde{j}\rangle - i\eta M_{\dot{a}b}|\dot{a}\rangle|\tilde{b}\rangle\right) \tag{2.3}$$

with the M's given by the 8×8 matrices:

$$M_{ij} = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p}\end{pmatrix}, \quad M_{\dot{a}b} = i\left(\gamma^1\gamma^2\ldots\gamma^{p+1}\right)_{\dot{a}b}, \quad M_{a\dot{b}} = i\left(\gamma^1\gamma^2\ldots\gamma^{p+1}\right)_{a\dot{b}} \tag{2.4}$$

In this formalism $X^+ = x^+ + p^+\tau$ and X^- always satisfy Dirichlet boundary conditions, being fixed by the gauge choice. This means in particular that all D-branes are actually euclidean-branes and that our considerations are valid for $-1\le p\le 7$ (actually $-1\le p\le 6$ for moving branes); moreover, supersymmetry is manifest, but the unbroken lorentz group is just $SO(8)$. Since we are also interested to consider the dynamics of moving branes, we should find a way to define the boundary state for moving branes. Following [8], this can be achieved by performing a boost transformation to the static boundary state (2.2). Since it is not trivial to perform a boost in light-cone gauge, where the boost operator is a non-linear and complicated object, we use the following trick to overcome this empasse: we perform an analytic continuation to an euclidean space and identify the “time” with one of the eight transverse directions, say X^{p+1}, and then we realize our boost along a spatial direction, say X^{p+2}, by performing the corresponding $SO(8)$ rotation with parameter v. At the end of the computation we then go back to Minkowski coordinates by identifying the $p+1^{th}$ direction with i times the time direction and sending $v \to iv$. A boundary state boosted by a transverse velocity v in the $p+2^{th}$ direction is then defined by the boosted matrices:

$$\begin{aligned} M_{ij}(v) &\equiv (\sigma_V(v) M \sigma_V(v)^T)_{ij} \\ M_{\dot{a}b}(v) &\equiv (\sigma_s(v) M \sigma_c(v)^T)_{\dot{a}b} \\ M_{a\dot{b}}(v) &\equiv (\sigma_c(v) M \sigma_s(v)^T)_{a\dot{b}} \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} \sigma_V(v) &= \begin{pmatrix} I_p & 0 & 0 & 0 \\ 0 & \cos v & -\sin v & 0 \\ 0 & \sin v & \cos v & 0 \\ 0 & 0 & 0 & I_{6-p} \end{pmatrix} \\ \sigma_s(v) &= \cos(v/2)\, \delta_{\dot{a}\dot{b}} - \sin(v/2)\, \gamma^{[p+1p+2]}_{\dot{a}\dot{b}}, \\ \sigma_c(v) &= \cos(v/2)\, \delta_{ab} - \sin(v/2)\, \gamma^{[p+1p+2]}_{ab} \end{aligned} \tag{2.6}$$

represent the $SO(8)$ rotations on the vector and spinor representations.

The universal part of the potential between D-branes moving with relative velocity v is then easily read from the cylinder computation [2] [7]:

$$\int_0^\infty dt\ \langle B, x| e^{-2p^+(P^- - p^-)t} |B, y, v\rangle, \tag{2.7}$$

where $p^- = i\partial/\partial x^+$ and the boundary states in position space are given in terms of the momentum states as

$$|B, x\rangle = \int \frac{d^{9-p}q}{(2\pi)^{9-p}}\, e^{iq\cdot x}\, |B, q\rangle \tag{2.8}$$

[2]Since we will always consider in the following branes of the same charge, we omit the η index, that is fixed to be plus.

and

$$P^{-} = \frac{1}{2p^{+}}(p^{i})^{2} + \frac{1}{2p^{+}}\sum_{n=1}^{\infty}\left(\alpha^{i}_{-n}\alpha^{i}_{n} + \tilde{\alpha}^{i}_{-n}\tilde{\alpha}^{i}_{n} + n\,S^{a}_{-n}S^{a}_{n} + n\,\tilde{S}^{\dot{a}}_{-n}\tilde{S}^{\dot{a}}_{-n}\right) \qquad (2.9)$$

is the Hamiltonian in light-cone gauge. It is a straightforward exercise to see that eq.(2.7), together with the matrices (2.5), reproduce the Bachas formula [2] after the spin-structures sum. It is worth while, however, to show how arise the dependence (1.3) within this formalism. Considering only the zero mode part of the boundary state, we have

$$\begin{aligned}\langle B_0, v=0|B_0, v\rangle = \mathrm{Tr}_V[M(0)^T M(v)] - \mathrm{Tr}_S\,[M(0)^T M(v)] = \\ = \quad 6 + 2\cos 2v - \cos v\,\mathrm{Tr}\,I + \sin v\,\mathrm{Tr}\,(\gamma^{p+1}\gamma^{p+2}) = 6 + 2\cos 2v - 8\cos v \qquad (2.10)\end{aligned}$$

where the subscript V, S indicate respectively the trace on the vectorial and spinorial indices. After analytic continuation, up to a factor two this is just the velocity dependence of eq.(1.3). The $1/r^{7-p}$ factor comes from integrating in momentum space and reproduces simply the scalar massless propagator in the space transverse to the two (D)p-branes.

Besides this universal force, D-branes feel their spin nature through non-minimal couplings, as seen in the introduction. We construct the currents J^{spin}_{M} by applying the broken supercharges Q^{-} to the boundary state $|B\rangle$. The best way to see that these new boundary states really encode next-to-leading interactions of D-branes with the bulk fields is by computing one-point functions of closed vertex operators on a disk with insertion of supercharges to the boundary. This has been already done for the $p=-1$ D-instanton in ref.[9], (eqs.47-50), in the covariant formalism and for the massless states. They found that, among the usual universal coupling, the insertion of broken supercharges on the boundary of the disk allows new couplings with different closed string states, for which D-branes are in general neutral. In particular, all the terms with even numbers of insertions are formed from powers of the matrix

$$A^{\mu\nu} = \bar{\epsilon}\gamma^{[\mu\nu\rho]}\epsilon\, q_{\rho} \qquad (2.11)$$

where ϵ is the 16-component Majorana-Weyl spinor, parameter of the supersymmetry, and q is the momentum of the emitted closed string state. The generalization of this result for $p > -1$ is straighforward; the presence of Neumann, as well as Dirichlet, boundary conditions will be introduced (in light-cone gauge) by the $M_{IJ}, M_{a\dot{b}}$ matrices that take into account of the fermionic and bosonic correlators on the disk.

The reformulation in terms of light-cone boundary states can also be easily performed. It simply corresponds to apply a bunch of supercharge pairs, one dotted and one undotted, to the boundary state (2.2)[3]. The usual boundary state $|B\rangle$ represents then the universal leading couplings of *all* the D-branes in the supermultiplet with the bulk fields, while the other boundary states $Q^{a_1-}Q^{\dot{a}_1-}|B_0\rangle$, etc. encode the next-to-leading couplings, different for each state of the multiplet, i.e. spin effects. If we want to consider brane-potentials where there is no change in the external states, considered as classical and heavy, we have to restrict our analysis to products of an even number of Q^-'s applied to the boundary state. We will study in the next two sections the first next-to-leading effects encoded in a cylinder or annulus computation with the insertions of up to eight supercharges. For the case of D0-brane potentials, this includes the interactions that in eleven-dimensional supergravity correspond to the non-minimal couplings of the gravitino with the four-form field strength.

3. D-brane dynamics

In this section we apply the general considerations performed before to compute some next-to-leading spin effects. Since we are interested to these interactions at large distances, where the closed string channel description is valid, our analysis will be restricted to the zero-mode part of the boundary state $|B\rangle$. For the same reason, we will consider the supercharges restricted to the massless modes; we then apply to the boundary state $|B_0\rangle$ a

[3] Because of the q^+-integration, a non trivial contribution is obtained only for insertion of dotted-undotted pairs.

bunch of Q^{a-} and $Q^{\dot{a}-}$ that are given by:

$$
\begin{aligned}
Q^{a-} &= (2q^+)^{1/2}(S_0^a - iM_{a\dot{b}}\tilde{S}_0^{\dot{b}}) \\
Q^{\dot{a}-} &= (2q^+)^{-1/2} q_i \gamma^i_{\dot{a}a}(S_0^a - iM_{a\dot{b}}\tilde{S}_0^{\dot{b}})
\end{aligned}
\tag{3.1}
$$

where, according to the general coniderations of last section, we take for a (D)p-brane the direction $p+1$ as our "time". After having applied the supercharges, we can boost the new boundary state along the direction $p+2$ to obtain the generic moving current. The S_0 operators are realized as usual by

$$
S_0^a|i\rangle = \gamma^i_{a\dot{a}}|\dot{a}\rangle/\sqrt{2}, \qquad S_0^a|\dot{a}\rangle = \gamma^i_{a\dot{a}}|i\rangle/\sqrt{2} \tag{3.2}
$$

and the analogous for the right-moving states. The boundary state obtained by the insertion of two of these broken supercharges is then the following:

$$
|B\rangle_{a_1\dot{a}_1} \equiv Q^{-a_1}Q^{-\dot{a}_1}|B\rangle = M^{a_1\dot{a}_1}_{ij}|i\rangle|\tilde{j}\rangle + iM^{a_1\dot{a}_1}_{\dot{a}b}|\dot{a}\rangle|\tilde{b}\rangle. \tag{3.3}
$$

where now the a-dependent M matrices are given by

$$
\begin{aligned}
M^{a_1\dot{a}_1}_{ij} &\equiv M_{kj}\, q_l\, \gamma^{[lki]}_{a_1\dot{a}_1} \\
M^{a_1\dot{a}_1}_{\dot{a}b} &\equiv q_j\, [(\gamma^j\gamma^i)_{\dot{a}_1\dot{a}}(M\gamma^i)_{ba_1} - (\gamma^j\gamma^i M)_{\dot{a}_1 b}\gamma^i_{\dot{a}a_1}]
\end{aligned}
\tag{3.4}
$$

The boost of these matrices is defined as before through eqs.(2.5). Sustituing the M's (2.4) in (3.4), a simple algebra leads to the first spin correction to D-brane potentials

$$
\langle B, v|B, v=0\rangle_{a_1\dot{a}_1} = 2(\gamma^{[p+1,p+2]}\gamma^i)_{a_1\dot{a}_1}\, q_i\, \sin v(\cos v - 1) \tag{3.5}
$$

where q is the momentum transfer between the two D-branes.

We immediately see from eqs.(2.7) and (2.8) that eq.(3.5) produces at large distances a spin-orbit like coupling going like v^3/r^{8-p}.

The next-to-leading effect (next power in q) comes from the insertion of four supercharges; in this case the amplitude is given by

$$
{}_{a_1\dot{a}_1}\langle B, v|B, v=0\rangle_{a_2\dot{a}_2} = \mathrm{Tr}_V\left(M^{a_1\dot{a}_1}\sigma_v(v)M^{a_2\dot{a}_2}\sigma_v(v)^T\right) - \mathrm{Tr}_S\left(M^{a_1\dot{a}_1}\sigma_s(v)M^{a_2\dot{a}_2}\sigma_c(v)^T\right) \tag{3.6}
$$

where the trace and the matrix multiplication in both terms are over the vectorial and spinorial indices respectively. It is not difficult to see that for any choice of polarizations $a_{1,2}, \dot{a}_{1,2}$, the static force is zero and the first non-vanishing contribution goes as v^2/r^{9-p}. In particular the static force cancels, due to a compensation between the vectorial and spinorial contributions in eq.(3.6), corresponding in the NS-R formalism, to exchange of NSNS and RR states, respectively. Note that the non-minimal coupling arising from the eleven dimensional gravitino-four form field strenght interaction is just encoded in this amplitude. There is only an other interaction that can produce effects of order $1/r^{9-p}$ and it is the one obtained by inserting four supercharges on the same boundary state. Again, this leads to a potential of order v^2/r^{9-p}.

The six and eight supercharge insertions can be analyzed similarly although the gamma matrix algebra become more laborious. The ending result is however simple; as already anticipated in the introduction, they give rise in general to interactions linear in velocity $\sim v/r^{10-p}$ and to a static force $\sim 1/r^{11-p}$ respectively. As far as the leading contribution of these higher order effects is concerned, it is easier to show their general dependence in the open string channel, as we will see in next section.

4. Open string channel

It is instructive to see how the leading orders of D-brane potentials found before are reproduced by the corresponding annulus computation in the open string framework. The spin potentials are represented, as before, by the insertion of supersymmetric charges in the partition function of the open strings stretched between the two moving branes. The boundary conditions for these open strings[4] are given by [2]

$$X^{p+2} + v\,X^{p+1} \;=\; \partial_\sigma(v\,X^{p+2} - X^{p+1}) = 0 \quad \text{at} \;\; \sigma = 0$$

[4]We recall that the term ‘p-brane’, as in the preceeding sections, denotes really a (D)p-instanton, related to standard (D)p-branes by a Wick rotation.

$$X^{p+2} \;=\; \partial_\sigma X^{p+1} = 0 \quad \text{at} \;\; \sigma = \pi \tag{4.1}$$

in the $p+1^{th}$ (time) and $p+2^{th}$ (brane velocity) directions, while they satisfy the standard Neumann and Dirichlet conditions for the remaining $1,..,p$ and $p+3,...,8$ light-cone directions respectively.

Spin potentials can then be read from $2n$-point functions of fermionic vertex operators at zero momentum, i.e. supercharges, at one-loop:

$$\mathcal{A}^{a_i,\dot{a}_i}_{(2n)} \equiv \int \mathcal{D}X\mathcal{D}S\, e^{-(S_0+S_v)} \prod_{i=1}^{n} Q^{a_i-} Q^{\dot{a}_i-} \tag{4.2}$$

where S_0 is the free worldsheet string action and

$$S_v \equiv v \oint d\tau \left[(X^{p+1}\partial_\sigma X^{p+2} - \frac{i}{4}(\bar{S}\rho^1\gamma^{[p+1,p+2]}S) \right] \tag{4.3}$$

represents the term that twists the usual Neumann-Dirichlet boundary conditions in the $(p+1, p+2)$ plane, according to eqs.(4.1). ρ^1 is the 2×2 matrix as defined in [10] and the functional integration $\mathcal{D}X\mathcal{D}S$ in eq.(4.2) includes also grassmannian integrations over the eight fermionic zero modes of the untwisted S_0 action. Expanding in powers of v, this leads to a vanishing result unless the eight fermionic zero modes are soaked up. If no supercharges are inserted, then, the $v-$twisted partition function is zero up to v^4, that is the minimum power in velocity that soak up all the eight zero modes. The t-modulus integration leads to the standard $1/b^6$ impact parameter dependence for the universal phase-shift. Each pair of supercharges provides two fermionic and one bosonic zero modes producing an additional bt^2 insertion in the partition function, the impact parameter b being the zero mode of $\partial_\sigma X$, appearing in the dotted supercharge. In this way we have generically

$$(vt)^m\, (S_0^-\gamma^{[p+1p+2]}S_0^-)^m t^{2n} \prod_{i=1}^{n} b_k \gamma^{[ijk]}_{a_i\dot{a}_i} (S_0^-\gamma^{[ij]}S_0^-) \tag{4.4}$$

where use has been made of the 'Fierz' identity

$$S_0^{a-}S_0^{b-} = \frac{1}{16}(S_0^-\gamma^{[ij]}S_0^-)\gamma^{[ij]}_{ab} \tag{4.5}$$

with $2(n+m)=8$, which provide the eight fermionic zero modes needed in order to get a non-vanishing result and n the number of dotted-undotted pairs of supercharge insertions. We are left then with an additional $(bt)^n$ which after the t-modulus integration leads to spin effects going like $v^{(4-n)}/r^{7-p+n}$. We should recall, however, that the matching between the two channels is just in these leading orders, and the complete expression in terms of the twisted theta functions will differ of course for higher orders effects, in exactly the same way happens for the v^4/r^{7-p} universal term [4]. As already mentioned in the introduction, this suggests that for the case of D0 branes, a one loop M(atrix) [5] theory computation will be able to capture these leading large distance supergravity spin effects, being all fixed by the massless spectrum.

Note added: Once this work was completed, a revised version of the paper [11] appeared, whose results partially overlap with those presented here.

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