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Black holes as D3–branes on Calabi–Yau threefolds

Matteo Bertolini[a], Pietro Frè[b], Roberto Iengo[a] and Claudio A. Scrucca[a]

[a] *International School for Advanced Studies ISAS–SISSA and INFN Sezione di Trieste, Via Beirut 2–4, 34013 Trieste, Italy*

[b] *Dipartimento di Fisica Teorica, Università di Torino and INFN Sezione di Torino, Via P. Giuria 1, 10125 Torino, Italy*

Abstract

We show how an extremal Reissner–Nordström black hole can be obtained by wrapping a dyonic D3–brane on a Calabi–Yau manifold. In the orbifold limit $T^6/\mathbb{Z}_3$, we explicitly show the correspondence between the solution of the supergravity equations of motion and the D–brane boundary state description of such a black hole.

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In the last couple of years there has been much effort in finding a microscopic description of both extremal and non–extremal black holes arising as compactifications of different p–brane solutions of ten–dimensional supergravity theories. This has been done by considering various solitonic configurations in string theory, such as bound states of D–branes and solitons of different kinds [1] or as intersecting (both orthogonally and at angles) D–branes alone [2]. As far as the microscopic description is concerned, these studies have been mainly devoted to toroidal compactifications and less has been said about Calabi–Yau (CY) ones. On the contrary, from a macroscopic (i.e. supergravity) point of view, these black hole solutions have been known for a long time in both cases and many progresses have been made in the last few years (see [3] and many subsequent works). Different problems arise when trying to find an appropriate D–brane description of these solutions in a non–flat asymptotic space. Moreover, some general results that are valid in the toroidal case no longer hold for CY compactifications. In particular, it is not straightforward to generalize the so called "harmonic function rule" and it is also no longer true that the minimum number of "different" charges (that is, carried by different microscopic objects) must be 4 in order to obtain a regular black hole in four dimensions.

We will be interested in discussing a Reissner–Nordström (R–N) black hole in four dimensions within a CY compactification (whose relevance for obtaining non–singular four–dimensional black hole was already pointed out, see for instance ref. [4]). The R–N solution defined as the usual non-singular black hole solution of Maxwell–Einstein gravity, can also be seen as a particular solution of a wider class of field theories in four dimensions in which the only fields having a non–trivial coordinate dependence are the metric $G_{\mu\nu}$ and a gauge field A_μ, whereas any other field is taken to be constant. In particular, in four–dimensional N=2 supergravity this solution, known as the *double–extreme* black–hole [5], arises in the specific case in which one assumes that the moduli fields belonging to vector multiplets (as well as those belonging to the hyper-multiplets which are anyhow constant in any N=2 black–hole solution) take the same constant values from the horizon to spatial infinity. In order to be consistent with the field equations such constant values are not arbitrary but must coincide with the so called *fixed values*: these are determined in terms of the electric and magnetic charges of all the existing gauge fields by a variational principle that extremizes the central charge and leads to classical formulae expressing the horizon area as a quartic invariant of the U–duality group (see for instance [6, 7, 8] and references therein).

When ten–dimensional supergravity is compactified on a CY threefold $\mathcal{M}_3^{CY}$ we obtain $D = 4, N = 2$ supergravity coupled to matter. As well known the field content of the four–

dimensional theory and its interaction structure is completely determined by the *topological and analytical type* of $\mathcal{M}_3^{CY}$ but depends in no way on its metric structure. Indeed the standard counting of hyper and vector multiplets tells us that $n_V = h^{(1,2)}$ and $n_H = h^{(1,1)}+1$, the numbers $h^{(p,q)}$ being the dimensions of the Dolbeault cohomology groups. Furthermore, the geometrical datum that completely specifies the vector multiplet coupling, namely the choice of the special Kähler manifold and its special Kähler metric is provided by the moduli space geometry of complex structure deformations. To determine this latter no reference has ever to be made to the Kähler metric $g_{ij^\star}$ installed on $\mathcal{M}_3^{CY}$ (for a review of this well established results see for instance [11]). Because of this crucial property careful thought is therefore needed when one tries to *oxidize* the solutions of four–dimensional $N = 2$ supergravity obtained through compactification on $\mathcal{M}_3^{CY}$ to *bona fide* solutions of the original ten–dimensional Type IIB supergravity. To see the four–dimensional configuration as a configuration in ten–dimension one has to choose a metric on the internal manifold in such a way as to satisfy the full set of ten–dimensional equations.

In this note we will show how an extreme R–N black hole can be obtained by compactification of the self–dual D3–brane on $\mathcal{M}_3^{CY} = T^6/\mathbb{Z}_3$, which is the orbifold limit of a CY manifold with Hodge numbers $h^{(1,1)} = 9$ and $h^{(1,2)} = 0$. Recalling some results obtained in previous works [9, 10], we will explicitly show the correspondence between the supergravity solution and the D–brane boundary state description of such a black hole. In this case, the effective four–dimensional theory is N=2 supergravity coupled to 10 hypermultiplets and 0 vector multiplets, the only vector field in the game being the graviphoton. Since there are no vector multiplet scalars the only regular black hole solution is the double–extreme one. From a supergravity point of view this is somewhat obvious and the same conclusion holds for every Type IIB compactification on CY manifolds with $h^{(1,2)} = 0$. The interest of the $T^6/\mathbb{Z}_3$ case lies in the fact that an explicit and simple D–brane boundary state description can be found. It would be obviously very interesting to find more complicated configurations which correspond to regular $N = 2$ black hole solutions for which an analogous D–brane description can be found.

We will start by showing that the *oxidization* of a *double extreme* black–hole solution of $N = 2$ supergravity to a *bona fide* solution of Type IIB supergravity is possible and quite straightforward. It just suffices to choose for the CY metric the Ricci flat one whose existence in every Kähler class is guaranteed by Yau theorem [12]. Our exact solution of Type IIB supergravity in ten dimensions corresponds to a 3–brane wrapped on a 3–cycle of the generic threefold $\mathcal{M}_3^{CY}$ and dimensionally reduced to 4–dimensions is a double–extreme black hole. Let us then argue how this simple result is obtained.

As well known, prior to the recent work by Bandos, Sorokin and Tonin [13] Type IIB supergravity had no supersymmetric space–time action. Only the field equations could be written as closure conditions of the supersymmetry algebra [14]. The same result could be obtained from the rheonomy superspace formalism as shown in [15]. Indeed, the condition of self–duality for the R–R 5–form $F_{(5)}$ that is necessary for the equality of Bose and Fermi degrees of freedom cannot be easily obtained as a variational equation and has to be stated as a constraint. In the new approach of [13] such problems are circumvented by introducing more fields and more symmetries that remove spurious degrees of freedom. For our purposes these subtleties are not relevant since our goal is that of showing the existence of a classical solution. Hence we just need the field equations which are unambiguous and reduce, with our ansatz, to the following ones:

$$R_{MN} = T_{MN} \tag{1}$$

$$\nabla_M F_{(5)}^{MABCD} = 0 \quad \longleftarrow \quad F^{(5)}_{G_1 \ldots G_5} = \frac{1}{5!}\, \epsilon_{G_1 \ldots G_5 H_1 \ldots H_5}\, F_{(5)}^{H_1 \ldots H_5} \tag{2}$$

$T_{MN} = 1/(2{\cdot}4!)\, F^{(5)}_{M\ldots.}\, F^{(5)}_{N\ldots.}$ being the traceless energy–momentum tensor of the R–R 4–form $A_{(4)}$ to which the 3–brane couples and $F_{(5)}$ the corresponding self–dual field strength.

It is noteworthy that if we just disregarded the self–duality constraint and we considered the ordinary action of the system composed by the graviton and an unrestricted 4–form

$$\mathcal{S} = \frac{1}{2\kappa^2_{(10)}} \int d^{10}x \sqrt{g_{(10)}} \left(R_{(10)} - \frac{1}{2 \cdot 5!} F^2_{(5)} \right) \tag{3}$$

then, by ordinary variation with respect to the metric, we would anyhow obtain, as source of the Einstein equation, a traceless stress–energy tensor:

$$T_{MN} = \frac{1}{2 \cdot 4!} \left(F^2_{(5)MN} - \frac{1}{2 \cdot 5} g_{MN} F^2_{(5)} \right)$$

The tracelessness of T_{MN} is peculiar to the 4–form and signals its conformal invariance. This, together with the absence of couplings to the dilaton (see for instance [16]), allows for zero curvature solutions in ten dimensions.

For the metric, we make a block–diagonal ansatz with a Ricci–flat compact part depending only on the internal coordinates y^a (this corresponds to choosing the unique Ricci flat Kähler metric on $\mathcal{M}_3^{CY}$), and a non–compact part which depends only on the corresponding non–compact coordinates x^μ

$$ds^2 = g^{(4)}_{\mu\nu}(x) dx^\mu dx^\nu + g^{(6)}_{ab}(y) dy^a dy^b \tag{4}$$

For $g^{(4)}_{\mu\nu}$ we take the extremal R–N black hole solution, as will be justified below. This ansatz is consistent with the physical situation under consideration. In general, the compact components of the metric depend on the non–compact coordinates x^μ, being some of the scalars of the $N=2$ effective theory. More precisely, using complex notation, the components $g_{ij^\star}$ are related to the $h^{(1,1)}$ moduli parametrizing the deformations of the Kähler class while the g_{ij} ($g_{i^\star j^\star}$) ones are related to the $h^{(1,2)}$ moduli parametrizing the deformations of the complex structure. In Type IIB compactifications, as already stressed, such moduli belong to hypermultiplets and vector multiplets respectively. In our case, however, there are no vector multiplet scalars, that would couple non–minimally to the gauge fields (it is usually said that they "dress" the field strengths) and the hypermultiplet scalars can be set to zero since they do not couple to the unique gauge field of our game, namely the graviphoton (therefore $g_{ab}(x,y)=g_{ab}(y)$).

The 5–form field strength can be generically decomposed in the basis of all the harmonic 3–forms of the CY manifold $\Omega^{(i,j)}$

$$F_{(5)}(x,y)=F^0_{(2)}(x)\wedge\Omega^{(3,0)}(y)+\sum_{k=1}^{h^{(2,1)}}F^k_{(2)}(x)\wedge\Omega^{(2,1)}_k(y)+\text{c.c.}\tag{5}$$

In the case at hand, however, only the graviphoton $F^0_{(2)}$ appear in the general ansatz (5), without any additional vector multiplet field strength $F^k_{(2)}$, and conveniently normalizing

$$F_{(5)}(x,y)=\frac{1}{\sqrt{2}}F^0_{(2)}(x)\wedge\left(\Omega^{(3,0)}+\bar{\Omega}^{(0,3)}\right)\tag{6}$$

Notice that this same ansatz is the consistent one for any double–extreme solution even for a more generic CY (i.e. with $h^{(1,2)}\neq 0$).

With these ansätze, eq. (1) reduces to the usual four–dimensional Einstein equation with a graviphoton source, the compact part being identically satisfied. The latter is a non trivial consistency condition that our ansatz has to fulfil. In fact, in general, eq. (1) taken with compact indices gives rise (after integration on the compact manifold) to various equations for the scalar fields. Indeed, the compact part of the ten–dimensional Ricci tensor R_{ab} is made of the CY Ricci tensor (that with our choice of the metric is zero by definition) plus mixed components (i.e. $R^\mu_{a\mu b}$) containing, in particular, kinetic terms of the scalars. The corresponding stress–energy tensor compact components on the right hand side of the equation would represent coupling terms of the scalars with the gauge fields. In our case, however, these mixed components of R_{ab} are absent. Therefore the complete ten–dimensional Ricci tensor vanishes ($R_{ab}=0$) and self–consistency of the solution requires that also the complete stress–energy tensor T_{ab} should vanish. This follows from our ansatz (6) as it is evident by doing an explicit computation. This conclusion can also be reached by observing that the kinetic term of the 4–form does not depend on g_{ab} when $g_{ij} = 0$, see eq. (7) below.

The four–dimensional Lagrangian is obtained by carrying out explicit integration over the CY. Indeed, choosing the normalization of $\Omega^{(3,0)}$ and $\bar{\Omega}^{(0,3)}$ such that $\left\|\Omega^{(3,0)}\right\|^2 = V_{D3}^2/V_{CY}$ (since the volume of the corresponding 3–cycle is precisely the volume V_{D3} of the wrapped 3–brane) one has ($z^a = 1/\sqrt{2}(y^a + iy^{a+1})$ and $d^6y = id^3zd^3\bar{z}$)

$$\int_{CY} d^6y\sqrt{g_{(6)}} = V_{CY}\ ,\ \ i\int_{CY} \Omega^{(3,0)} \wedge \bar{\Omega}^{(0,3)} = V_{D3}^2 = \int_{CY} d^6y\sqrt{g_{(6)}}\left\|\Omega^{(3,0)}\right\|^2 \tag{7}$$

and then

$$\mathcal{S} = \frac{1}{2\kappa_{(4)}^2}\int d^4x\sqrt{g_{(4)}}\left(R_{(4)} - \frac{1}{2\cdot 2!}\mathrm{Im}\mathcal{N}_{00}F_{\mu\nu}^0 F^{0|\mu\nu}\right) \tag{8}$$

where $\kappa_{(4)}^2 = \kappa_{(10)}^2/V_{CY}$ and $\mathrm{Im}\mathcal{N}_{00} = V_{D3}^2/V_{CY}$. In the general case (eq. (5)) integration over the CY gives rise, of course, to a gauge field kinetic term of the standard form: $\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^\Lambda F^\Sigma + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda *}F^\Sigma$, where $\Lambda, \Sigma = 0, 1, ..., h^{(1,2)}$. As well known (from now on $F_{(2)}^0 \equiv F$), the four–dimensional Maxwell–Einstein equations of motion following from this Lagrangian admit the extremal R–N black hole solution (in coordinates in which the horizon is located at $r = 0$)

$$\begin{aligned} g_{00} &= -\left(1 + \frac{\kappa_{(4)}M}{r}\right)^{-2} &,\quad g_{mm} &= \left(1 + \frac{\kappa_{(4)}M}{r}\right)^{2} \\ F_{m0} &= \kappa_{(4)}\, e_0\, \frac{x^m}{r^3}\left(1 + \frac{\kappa_{(4)}M}{r}\right)^{-2} &,\quad F_{mn} &= \kappa_{(4)}\, g_0\, \epsilon_{mnp}\frac{x^p}{r^3} \end{aligned} \tag{9}$$

where $m, n, p = 1, 2, 3$. The extremality condition is $M^2 = (e^2 + g^2)/4$, where for later convenience we parametrize the solution with

$$M = \frac{\hat{\mu}}{4}\ ,\ \ e = e_0\sqrt{\frac{V_{D3}^2}{V_{CY}}} = \frac{\hat{\mu}}{2}\cos\alpha\ ,\ \ g = g_0\sqrt{\frac{V_{D3}^2}{V_{CY}}} = \frac{\hat{\mu}}{2}\sin\alpha \tag{10}$$

The parameter $\hat{\mu}$ is related to the 3–brane tension μ through $\hat{\mu} = \sqrt{V_{D3}^2/V_{CY}}\,\mu$, and the arbitrary angle α depends on the way the 3–brane is wrapped on the CY. Notice that the charges with respect to the gauge field A^μ are e_0 and g_0, but since the kinetic term, and correspondingly the propagator of A^μ, is not canonically normalized, the effective couplings appearing in a scattering amplitude are rather e and g, which indeed satisfy the usual BPS condition. Further, at the quantum level, e and g are quantized as a consequence of Dirac's condition $eg = 2\pi n$; correspondingly, the angle α can take only discrete values and this turns out to be automatically implemented in the compactification [10].

Now note that in the case of the $T^6/\mathbb{Z}_3$ the square volume of the wrapped $D3$–brane V^2_{D3} defined by the second of eqs. (7) is automatically a constant just because the number of vector multiplets is zero. Notice that for a generic CY compactification we have:

$$i \int_{CY} \Omega^{(3,0)} \wedge \bar{\Omega}^{(0,3)} \;=\; \exp\left[\mathcal{K}\left(\phi, \bar{\phi}\right)\right]$$

where $\mathcal{K}\left(\phi, \bar{\phi}\right)$ is the Kähler potential of the moduli fields $\phi(x)$ associated with complex structure deformations. Hence in the generic case the $D3$–brane volume is dressed by scalar fields and depends on the x–space coordinates. Telling the story in four–dimensional language the graviphoton couples non–minimally to scalar fields. However, on the hand to oxidize the R–N type of black–hole solution we discuss in this paper, it is crucial that we can treat the $D3$–brane square volume V^2_{D3} as x–space independent.

This ends the field theory side of the computation. Let us turn to a microscopic string theory description of the same black–hole.

The problem of describing curved D–branes, such as D–branes wrapped on a cycle of the internal manifold in a generic compactification of string theory, is in general too difficult to be solved. In fact, Polchinsky's description [17] of D–branes as hypersurfaces on which open strings can end relies on the possibility of implementing the corresponding boundary conditions in the CFT describing open string dynamics. Very little has been done for a generic target space compactification (for a recent discussion of this and related issues, see [18]) but there exist special cases, such as orbifold compactifications, which capture all the essential features of more general situations, in which ordinary techniques can be applied.

In previous works [9, 10], a boundary state description of a D3–brane wrapped on 3–cycle of the $T^6/\mathbb{Z}_3$ orbifold has been proposed and applied to various situations. In particular, the semiclassical phase–shift between two of these point–like configurations moving with constant velocities can be obtained simply by computing the tree level (cylinder) closed string propagation between the two boundary states [9]. The result is found to vanish like V^2 for small relative velocities, indicating BPS saturation. The behaviour for large impact parameters, where an effective description in terms of the underlying low energy four–dimensional N=2 supergravity is expected to hold, is

$$\mathcal{A} = \frac{\hat{\mu}^2}{4}\left(\cosh v - \cosh 2v\right)\int dt \Delta_3(r) \tag{11}$$

v being the relative rapidity of the two branes, $\Delta_3(r)$ the three–dimensional Green function,

$r = \sqrt{b^2 + \sinh^2 vt^2}$ and $\vec{b}$ is the impact parameter. In four dimensions, the exchange of scalar, vector and tensor massless particles between the two brane sources give contributions with a peculiar dependence on the rapidity and are proportional to 1, $\cosh v$ and $\cosh 2v$ respectively. This leads to the interpretation of eq. (11) as the exchange of the bosonic part of the N=2 gravitational multiplet, that is the graviton and the graviphoton. The absence of any constant part in (11) signals that there is no scalar exchange between the two branes. Since the two branes are identical and therefore have the same coupling to the scalars of the bulk four–dimensional supergravity, the total scalar exchange is proportional to the sum of the squares of these couplings, and its vanishing implies the vanishing of all the couplings separately. It is interesting to compare (11) to the result for a 0–brane (arising in a corresponding IIA compactification)

$$\mathcal{A} = \frac{\hat{\mu}^2}{4} \left(4 \cosh v - \cosh 2v - 3\right) \int dt \Delta_3(r) \tag{12}$$

for which scalars are exchanged, beside the graviton and the vector. Since the ten–dimensional 0-brane couples only to the dilaton $\phi^{(10)}$ and the world-volume components of the graviton $h^{(10)}_{\mu\nu}$ and the RR vector $A^{(10)}_\mu$, the four–dimensional 0–brane couples only to the corresponding four–dimensional fields $\phi^{(4)}$, $h^{(4)}_{\mu\nu}$ and $A^{(4)}_\mu$ (in particular, in the four–dimensional Einstein frame, it does not couple to the additional scalars and vectors coming from metric).

For the wrapped 3–brane, eq. (11), BPS saturation implies that all the vector repulsion is balanced only by gravitational attraction, whereas for the 0–brane, also the scalars contribute to the attraction, leaving a smaller gravitational potential. Actually both of these four–dimensional configurations come from an effective action of the type

$$\mathcal{S} = \int d^4x \sqrt{g} \left(R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2 \cdot 2!} e^{-a\phi} F^2_{(2)} \right) \tag{13}$$

with $a = 0$ for the R–N black hole and $a \neq 0$ for the 0–brane. The general electric extremal solution of this Lagrangian is [19]

$$ds^2 = -H(r)^{-\alpha} dt^2 + H(r)^\alpha d\vec{x} \cdot d\vec{x} \ , \quad \phi = \beta \ln H(r) \ , \quad A_0 = \gamma \, H(r)^{-1} \tag{14}$$

where

$$\alpha = \frac{2}{1 + a^2} \ , \quad \beta = \frac{2a}{1 + a^2} \ , \quad \gamma = \frac{2}{\sqrt{1 + a^2}} \tag{15}$$

and $H(r)$ satisfies the three-dimensional Laplace equation and can be taken to be of the form $H(r) = 1 + k\Delta_3(r)$. The relevant asymptotic long range fields are thus

$$h_{00} = \alpha \, k \, \Delta_3(r) \ , \quad \phi = \beta \, k \, \Delta_3(r) \ , \quad A_0 = \gamma \, k \, \Delta_3(r)$$

and so the phase-shift between two identical branes moving with relative rapidity v is

$$\mathcal{A} = k^2 \left(\gamma^2 \cosh v - \alpha^2 \cosh 2v - \beta^2\right) \int dt \Delta_3(r) \tag{16}$$

As a consequence of BPS saturation, $\beta^2 - \alpha^2 - \gamma^2 = 0$ and the static force vanishes. Moreover, comparing with eqs. (11) and (12), we learn that the R–N solution corresponds to $a = 0$ and $k = \hat{\mu}/4$, whereas the 0–brane corresponds to $a = \sqrt{3}$ and $k = \hat{\mu}$.

Altogether, these arguments lead to evidence that the boundary state constructed in refs [9, 10] actually represents a R–N black hole. An equivalent way of analyzing this configuration, to see again that it indeed correctly fits the general solution R–N $\times$ CY discussed before, is to compute one–point functions $\langle \Psi \rangle = \langle \Psi | B \rangle$ of the massless fields of supergravity and compare them with the linearized long range fields of the supergravity R–N black hole solution (9). This second method presents the advantage of yielding direct informations on the coulpings with the massless fields of the low energy theory.

The original ten–dimensional coordinates are organized as follows: the four non–compact directions X^0, X^1, X^2, X^3 span $\mathcal{M}_4$, whereas the six compact directions X^a, X^{a+1}, $a = 4, 6, 8$, span $T^6/\mathbb{Z}_3$. The three T^2's composing T^6 are parametrized by the 3 pairs X^a, X^{a+1}, and the $\mathbb{Z}_3$ action is generated by $2\pi/3$ rotations in these planes. The boundary state $|B\rangle$ of the D3–brane wrapped on a generic $\mathbb{Z}_3$–invariant 3–cycle can be obtained from the boundary state $|B_3(\theta_0)\rangle$ of D3–brane in ten dimensions with Neumann directions X^0 and $X'^a(\theta_0)$, where the $X'^a(\theta_0)$ directions form an arbitrary common angle θ_0 with the X^a directions in each of the 3 planes X^a, X^{a+1} (actually, we could have chosen 3 different angles in the 3 planes, but only their sum will be relevant, as it could be inferred from eq. (20) below). First, one projects onto the $\mathbb{Z}_3$–invariant part and then compactifies the directions X^a, X^{a+1}. The $\mathbb{Z}_3$ projection is implemented by applying the projector $P = 1/3(1+g+g^2)$ on $|B_3(\theta_0)\rangle$, where $g = \exp i2\pi/3(J^{45} + J^{67} + J^{89})$ is the generator of the $\mathbb{Z}_3$ action and J^{aa+1} is the X^a, X^{a+1} component of the angular momentum operator. This yields

$$|B\rangle = \frac{1}{3} \sum_{\{\Delta\theta\}} |B_3(\theta = \Delta\theta + \theta_0)\rangle \tag{17}$$

where the sum is over $\Delta\theta = 0, 2\pi/3, 4\pi/3$. It is obvious form this formula that $|B\rangle$ is a periodic function of the parameter θ_0 with period $2\pi/3$. Therefore, the physically distinct values of θ_0 are in $[0, 2\pi/3]$ and define a one parameter family of $\mathbb{Z}_3$–invariant boundary states, corresponding to all the possible harmonic 3–forms on $T^6/\mathbb{Z}_3$, as we will see. Notice that requiring a fixed finite volume V_{D3} for the 3–cycle on which the D3–brane is wrapped implies discrete values for θ_0 [10]. The compactification process restricts the momenta entering the Fourier decomposition of $|B\rangle$ to belong the momentum lattice of $T^6/\mathbb{Z}_3$. Since the massless supergraviton states $|\Psi\rangle$ carry only space time momentum, the compact part of the boundary state will contribute a volume factor which turns the ten–dimensional D3–brane tension $\mu = \sqrt{2\pi}$ into the four–dimensional black hole charge $\hat{\mu} = \sqrt{V_{D3}^2/V_{CY}}\mu$ [10], and some trigonometric functions of θ_0 to be discussed below.

Using the technique of ref. [20], the relevant one–point functions on $|B_3(\theta)\rangle$ for the graviton and 4–form states $|h\rangle$ and $|A\rangle$ with polarization h^{MN} and A^{MNPQ}, are

$$\langle B_3(\theta)|h\rangle = -\frac{\hat{\mu}}{2}T\,h_{MN}\,M^{MN}(\theta)\ ,\ \ \langle B_3(\theta)|A\rangle = -\frac{\hat{\mu}}{8}T\,A_{MNPQ}\,M_{ab}(\theta)\,\Gamma^{MNPQ}_{ba} \tag{18}$$

T is the total time and μ is correctly changed to $\hat{\mu}$ by the volume factor that the compact part of the boundary state contributes [10]. The numerical coefficients appearing in (18) have been choosen at our convenience by relying on the scattering amplitude [10], where the relative normalization is easily fixed, as already discussed. The matrices $M(\theta) = \Sigma(\theta)M\Sigma^T(\theta)$ are obtained from the usual ones corresponding to Neumann boundary conditions along X^0, X^4, X^6, X^8

$$M_{MN} = \mathrm{diag}(-1,-1,-1,-1,1,-1,1,-1,1,-1)\ ,\ \ M_{ab} = \Gamma^{0468}_{ab}$$

through a rotation of angle θ in the 3 planes X^a, X^{a+1}, generated in the vector and spinor representations of each $S0(2)$ subgroup of the rotation group $S0(8)$ by

$$\Sigma_V(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\ ,\ \ \Sigma_S(\theta) = \cos\frac{\theta}{2} - \sin\frac{\theta}{2}\,\Gamma^{aa+1}$$

After some simple algebra, one finds

$$\begin{aligned}
\langle B_3(\theta)|h\rangle &= \frac{\hat{\mu}}{2}T\left\{h^{00}+h^{11}+h^{22}+h^{33}-\sum_a\left[\cos 2\theta\left(h^{aa}-h^{a+1a+1}\right)-2\sin 2\theta\,h^{aa+1}\right]\right\} \\
\langle B_3(\theta)|A\rangle &= 2\hat{\mu}T\left[\cos^3\theta\left(A^{0468}-A^{0479}-A^{0569}-A^{0578}\right)\right. \\
&\quad +\sin^3\theta\left(A^{0579}-A^{0568}-A^{0478}-A^{0469}\right) \\
&\quad \left.+\cos\theta\left(A^{0479}+A^{0569}+A^{0578}\right)+\sin\theta\left(A^{0568}+A^{0478}+A^{0469}\right)\right]
\end{aligned} \tag{19}$$

The one–point functions for the $D3$–brane wrapped on $T^6/\mathbb{Z}_3$ are then obtained by averaging over the allowed $\Delta\theta$'s: $\langle\Psi\rangle = 1/3\sum_{\{\Delta\theta\}}\langle B_3(\theta)|\Psi\rangle$. One easily finds the only non–vanishing averages of the trigonometric functions appearing in eq.s (19) to be

$$\frac{1}{3}\sum_{\{\Delta\theta\}}\cos^3\theta = \frac{1}{4}\cos 3\theta_0\ ,\ \ \frac{1}{3}\sum_{\{\Delta\theta\}}\sin^3\theta = -\frac{1}{4}\sin 3\theta_0 \tag{20}$$

so that finally, meaning now with h and A all the four–dimensional fields arising from the graviton and the 4–form respectively upon compactification,

$$\langle h \rangle = \frac{\hat{\mu}}{2} T \left(h^{00} + h^{11} + h^{22} + h^{33} \right) \; , \;\; \langle A \rangle = \frac{\hat{\mu}}{2} T \left(\cos 3\theta_0 A^0 - \sin 3\theta_0 B^0 \right) \tag{21}$$

where we have defined the graviphoton fields

$$A^\mu \equiv A^{\mu 468} - A^{\mu 479} - A^{\mu 569} - A^{\mu 578} \; , \;\; B^\mu \equiv A^{\mu 579} - A^{\mu 568} - A^{\mu 478} - A^{\mu 469} \tag{22}$$

Using self–duality of the 5–form field strength in ten dimension, one easily see that $F_B^{\mu\nu} = {}^*F_A^{\mu\nu}$ so that A^μ and B^μ are not independent fields, but rather magnetically dual. Using the A^μ field, we get electric and magnetic charges

$$e = \frac{\hat{\mu}}{2} \cos 3\theta_0 \; , \;\; g = \frac{\hat{\mu}}{2} \sin 3\theta_0 \tag{23}$$

or viceversa using the B^μ field. Comparing with eq. (10) one finds that $\alpha = 3\theta_0$ and therefore the ratio between e and g depends on the choice of the 3–cycle, as anticipated. Also, as explained, only discrete values of θ_0 naturally emerge requiring a finite volume.

Further evidence for the identifications (23) comes from the computation of the electromagnetic phase–shift between two of these configurations with different θ_0's, call them $\theta_{1,2}$. Since the four–dimensional electric and magnetic charges of the two black holes are then different, there should be both an even and an odd contribution to the phase–shift coming from the corresponding R–R spin structures. Indeed, one correctly finds [10]

$$\mathcal{A}_{even} \sim \frac{\hat{\mu}^2}{4} \cos 3\left(\theta_1 - \theta_2\right) = e_1 e_2 + g_1 g_2 \; , \;\; \mathcal{A}_{odd} \sim \frac{\hat{\mu}^2}{4} \sin 3\left(\theta_1 - \theta_2\right) = e_1 g_2 - g_1 e_2 \tag{24}$$

Notice that all the compact components h^{ab} of the graviton have cancelled in (21), reflecting the fact the black hole has no scalar hairs. Moreover, the one–point function of the R–R 4–form is precisely of the form of our ansatz (6), with the unique holomorphic and antiholomorphic 3–forms $\Omega^{(3,0)}$ and $\bar{\Omega}^{(0,3)}$ showing up in (21). Indeed

$$\Omega^{(3,0)} = \Omega \, dz^4 \wedge dz^6 \wedge dz^8 \; , \;\; \bar{\Omega}^{(0,3)} = \Omega^* \, d\bar{z}^4 \wedge d\bar{z}^6 \wedge d\bar{z}^8 \tag{25}$$

so that the real 3–form appearing in (6) is given by

$$\Omega^{(3,0)} + \bar{\Omega}^{(0,3)} = \mathrm{Re}\Omega \left(\omega^{468} - \omega^{479} - \omega^{569} - \omega^{578} \right) + \mathrm{Im}\Omega \left(\omega^{579} - \omega^{568} - \omega^{478} - \omega^{469} \right) \tag{26}$$

where $\omega^{abc} = 1/\sqrt{2} \, dy^a \wedge dy^b \wedge dy^c$. The precise correspondence between the boundary state result (21) and the purely geometric identity (26) is then evident. The combination of components of the 4–form appearing in (21) is proportional to the integral over the D3–brane world–volume V_{1+3}

$$\langle A\rangle = \frac{\mu}{2}\,\mathrm{Re}\int_{V_{1+3}} (A+iB)\wedge\Omega^{(3,0)} = \int_{V_1} (eA+gB) \tag{27}$$

This formula yields an interesting relation between the parameters $\mu, \hat{\mu}, \theta_0$ and the complex component Ω in (25) defining the 3–cycle; one gets $\Omega = (\hat{\mu}/\mu)e^{-i3\theta_0}$. Notice that one correctly recovers $|\Omega| = \sqrt{V_{D3}^2/V_{CY}}$, the arbitrary phase being the sum of the arbitrary overall angles θ_0 appearing in the boundary state construction. Finally, dropping the overall time T, inserting a propagator $\Delta = 1/\vec{q}^2$ and Fourier transforming eqs. (21) with the identification (27), one recovers the asymptotic gravitational and electromagnetic fields of the R–N black hole, eqs. (9).

This confirms that our boundary state describes a $D3$–brane wrapped on $T^6/\mathbb{Z}_3$, falling in the class of regular four–dimensional R–N double–extreme black holes obtained by wrapping the self–dual D3–brane on a generic CY threefold. This boundary state encodes the leading order couplings to the massless fields of the theory, and allows the direct determination of their long range components, falling off like $1/r$ in four dimensions. The subleading post–Newtonian corrections to these fields arise instead as open string higher loop corrections, corresponding to string world–sheets with more boundaries; from a classical field theory point of view, this is the standard replica of the source in the tree-level perturbative evaluation of a non–linear classical theory. In a series expansion for $r\to\infty$, a generic term going like $1/r^l$ comes from a diagram with l open string loops, that is l branches of a tree-level closed string graph (each branch brings an integration over the transverse 3–momentum, two propagators and a supergravity vertex involving two powers of momentum, yielding an overall contribution of dimension $1/r$).

Let us end with few final comments. As pointed out by the authors of [4], heuristically speaking the reason why single D–brane black holes are non–singular in CY compactifications, as opposed to the toroidal case, is that the brane is wrapped on a topologically non–trivial manifold and therefore can intersect with itself. This intersection mimics the actual intersecting picture of different branes holding in toroidal compactifications that is the essential feature in order to get a non–singular solution in that case. In our case, such analogy is particularly manifest since the boundary state $\mathbb{Z}_3$–invariant projection (17) can be seen as a three D3–branes superposition at angles $(2\pi/3)$ in a T^6 compactification. As illustrated in [21] such intersection preserves precisely 1/8 supersymmetry, as a single D3–brane does on $T^6/\mathbb{Z}_3$. For toroidal compactification this is not enough, of course, because at least 4 intersecting D3–branes are needed in order to get a regular solution [2].

Finally, since this extremal R–N configuration is constructed by a single D3–brane, it naturally arises the question of understanding the microscopic origin of its entropy.

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