Airline disruption recovery and robustness

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ROADEF 2009
February 10-12, Nancy, France
Outline

1. Introduction
2. Aircraft Recovery
3. Passenger Recovery
4. Recoverable robustness
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ROADEF Challenge 2009

Forewords:

We worked on an optimization algorithm for the Aircraft recovery problem with maintenance constraints in collaboration with an IT company (funded by the Swiss government - CTI program).

The problem:

- Recover within a given time horizon an airline schedule in a disrupted state minimizing the recovery costs
- The recent history of the schedule is given to obtain the state of the resources
Data

After data preprocessing, the relevant informations are:

- $F$: a set of scheduled flights, together with an estimation of cancellation cost $c_f$
- $P$: a set of aircrafts
- $R$: a set of passengers (itineraries)
- $I_p, I_r$: a set of initial positions for both aircrafts and passengers
- $S_p, S_r$: a set of required final positions for both aircrafts and passengers
- $T$: a time horizon
- $L$: a set of airport slots
- $q_{l,Dep}, q_{l,Arr}$: slot capacities for take off and landings
Master problem

We model the recovery problem for aircrafts as:

\[
\begin{align*}
\min z_{MP} &= \sum_{r \in \Omega} c_r x_r + \sum_{f \in F} c_f y_f \\
\sum_{r \in \Omega} b^f_r x_r + y_f &= 1 \quad \forall f \in F \\
\sum_{r \in \Omega} b^s_r x_r &= 1 \quad \forall s \in S_p \\
\sum_{r \in \Omega} b^p_r x_r &\leq 1 \quad \forall p \in P \\
\sum_{r \in \Omega} b^{Dep,l}_r x_r &\leq q^{Dep}_l \quad \forall l \in L \\
\sum_{r \in \Omega} b^{Arr,l}_r x_r &\leq q^{Arr}_l \quad \forall l \in L \\
x_r &\in \{0, 1\} \quad \forall r \in \Omega, \ y_f &\in \{0, 1\} \quad \forall f \in F
\end{align*}
\]

Recovery Network

Given $T$, $I_p$ and $S_p$ the R.N. encodes all possible recovery schemes for plane $p$.

Scheduled flights, acyclic, polynomial size
Recovery Network

Given $T$, $I_p$ and $S_p$ the R.N. encodes all possible recovery schemes for plane $p$.

Delay modeling, acyclic network but no more acyclic in terms of flights, exponential size
Recovery Network

Given $T$, $I_p$, and $S_p$ the R.N. encodes all possible recovery schemes for plane $p$.

Time band discretization pseudo-polynomial size but unfeasible recovery schemes are encoded
Generating recovery schemes

Given $\Omega'$, $(x^*_r, y^*_f)$, $(\lambda^*_f, \eta^*_s, \mu^*_p, v^*_l, \rho^*_l)$, new profitable schemes for plane $p$ are computed by solving an ERCSP on the Recovery Network, minimizing:

$$
\tilde{c}^p_r = c^p_r - \sum_{f \in F} b^f_r \lambda^*_f - \sum_{s \in S} b^s_r \eta^*_s - \mu^*_p - \sum_{l \in L} (b^{\text{Dep},l}_r v^*_l + b^{\text{Arr},l}_r \rho^*_l) \quad \forall p \in P
$$

**Remark:** In principle, the R.N. is not necessary (we can use directly the data) but it allows to compute resource bounds and **statically** eliminate most of the unfeasible schemes.

Bi-directional bounded dynamic programming with DSSR. Righini and Salani (2008).
Implementation issues

The algorithm is implemented with BCP framework by COIN-OR.

Speed up, to comply with ROADEF rules:

- Network size is reduced by some parameters: permitted delay, permitted plane swaps
- Pricing problem is solved heuristically with relaxed domination criteria and label elimination
- Heuristic search tree exploration
Passenger routing

An integer solution to $z_{MP}$ gives the aircraft assignment and the flight re-timing or cancellation. From that solution we build a unique connection network which comply with connectivity constraints:

- Arc capacities represent available seats

Passenger itineraries are sorted according to deletion cost and for each itinerary:

- Dummy source and sink connections are the only updated
- Cost of arcs connecting the sink represent the delay cost
- A min-cost flow is solved and decomposed into paths
- Each path is a new itinerary
Conclusions

- The overall recovery procedure is not enough competitive with other methods.

- We easily adapted the code for aircraft disruption recovery with maintenance constraints to comply with ROADEF rules.

- Identified issues: neglected some cost structures, pricing “too heuristic”, sequential approach.

- Outlook: solution quality can be improved by a post-processing phase.
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Approaches toward robustness

(Airline) schedule disruptions occur because of unpredicted events (noise in the nominal data) which are of stochastic nature.

**Reactive and proactive approaches**

- Online optimization (Albers (2003))
- Stochastic optimization (with recourse) (Kall and Wallace (1994))
- Worst-case (robust) optimization (Bertsimas and Sim (2004))
- Risk-management/Light robustness (Kall and Mayer (2005), Fischetti and Monaci (2008))
Uncertainty set

Often uncertainty sets (characterization of data fluctuation) are difficult to estimate. Wrong estimation of uncertainty set may lead to bad or too conservative solutions.

We aim to design an optimization framework which:

- simple, has the same complexity as the deterministic problem
- provides solutions with guaranteed deviation from optimum
- does not need for probabilistic uncertainty sets
- accounts for reactive strategies

We search a robust recoverable solution.
Robustness features

Given a deterministic optimization problem:

\[
\begin{align*}
\min & \quad f(x) \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x \in X
\end{align*}
\]

Identify \textbf{structural properties} \( \mu(x) \) of a solution which are exploited by the reactive strategy. Solve a multi-objective optimization problem:

\[
\begin{align*}
\min & \quad f(x), \max \quad \mu(x) \\
\text{s.t.} & \quad Ax \leq b \\
& \quad x \in X
\end{align*}
\]

Relax original objective in a (budget) constraint:

\[
\begin{align*}
\max & \quad \mu(x) \\
\text{s.t.} & \quad Ax \leq b \\
& \quad f(x) \leq (1 + \rho)f(x^*) \\
& \quad x \in X
\end{align*}
\]
Robust recoverable aircraft scheduling

**Tactical** planning: Re-timing of flights is permitted in the definition of $r \in \Omega$ within a range of 60 minutes.

\[
\max z_{RF} = \mu(x)
\]

\[
(14) - (15)
\]

\[
(17) - (21)
\]

\[
\sum_{r \in \Omega} d_r x_r \leq C
\]

\[
x_r \in \{0, 1\} \quad \forall r \in \Omega
\]

\[
y_f \in \{0, 1\} \quad \forall f \in F
\]
Robust recoverable aircraft scheduling

The recovery algorithm performs better in presence of slack time between flights and effective possibilities of swapping planes.

Increase the minimal idle time of schedule $r$

$$\mu_{IT}(x) = \sum_{r \in \Omega} \delta_{r}^{\min} x_r$$

Quadratic formulation

$$\mu_{CROSS}(x) = \sum_{r \in \Omega} \sum_{p \in \Omega} b_{rp} x_r x_p$$
Robust recoverable aircraft scheduling

The recovery algorithm performs better in presence of slack time between flights and effective possibilities of swapping planes.

Increase the minimal idle time of schedule $r$

$$\mu_{IT}(x) = \sum_{r \in \Omega} \delta_{r}^{\text{min}} x_{r}$$

We define meeting points $m$

$$\sum_{r \in \Omega} b_{r}^{m} x_{r} - y_{m} \geq 0 \quad \forall m \in M$$

$$\mu_{CROSS}(x) = \sum_{m \in M} (y_{m} - 1)$$
Robust results

Results on ROADEF09 set A instances (average)

<table>
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<tr>
<th></th>
<th>Original</th>
<th>CROSS</th>
<th>CROSS</th>
<th>IT</th>
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<tbody>
<tr>
<td>BUDGET [min]</td>
<td>0</td>
<td>5000</td>
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<tr>
<td>RECOVERY COST</td>
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<td>555400.3</td>
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<td># Canceled Flts</td>
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<td>6.9</td>
<td>5.3</td>
<td>5.8</td>
<td>5.9</td>
</tr>
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<td>Total Delay [min]</td>
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<td>2421.8</td>
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<tr>
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<td>385.3</td>
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<tr>
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<td>454.1</td>
<td>501.1</td>
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<tr>
<td>Avg Psg Delay [min]</td>
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<td>38.7</td>
<td>24.6</td>
<td>29.5</td>
<td>29.8</td>
</tr>
</tbody>
</table>

Eggenberg And S. (2008b).
Thanks

Thanks for your attention

Any question?