The Vehicle Routing Problem with Discrete Split Delivery and Time Windows

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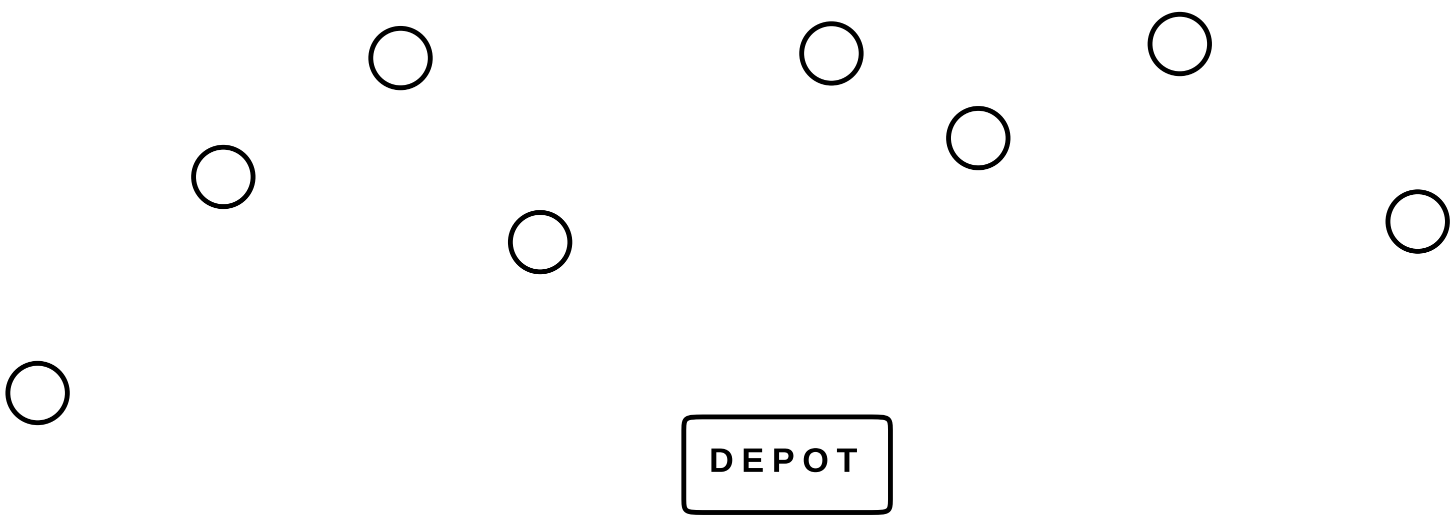
Outline

- Problem description and applications
- MILP formulation and Column Generation approach
- Branch & Price algorithm
- Computational experiments
- Conclusion

The Vehicle Routing Problem (VRP)

Given

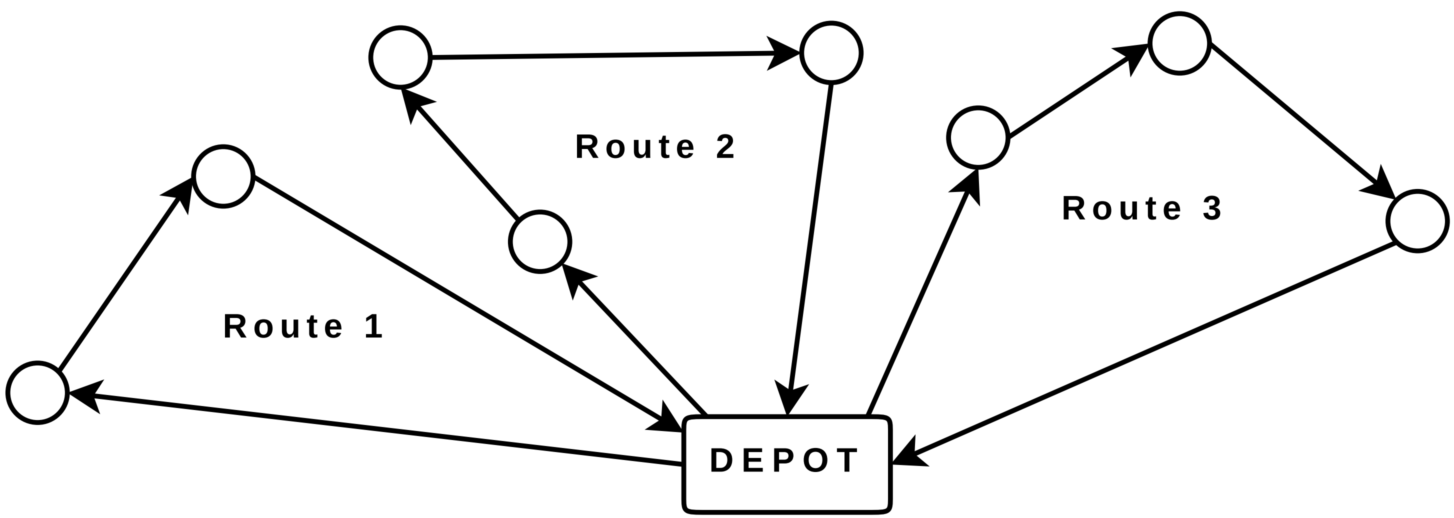
- set of customers with demands to be served (within time windows: VRPTW);
- set of capacitated vehicles available at a single depot;



The Vehicle Routing Problem (VRP)

Objective

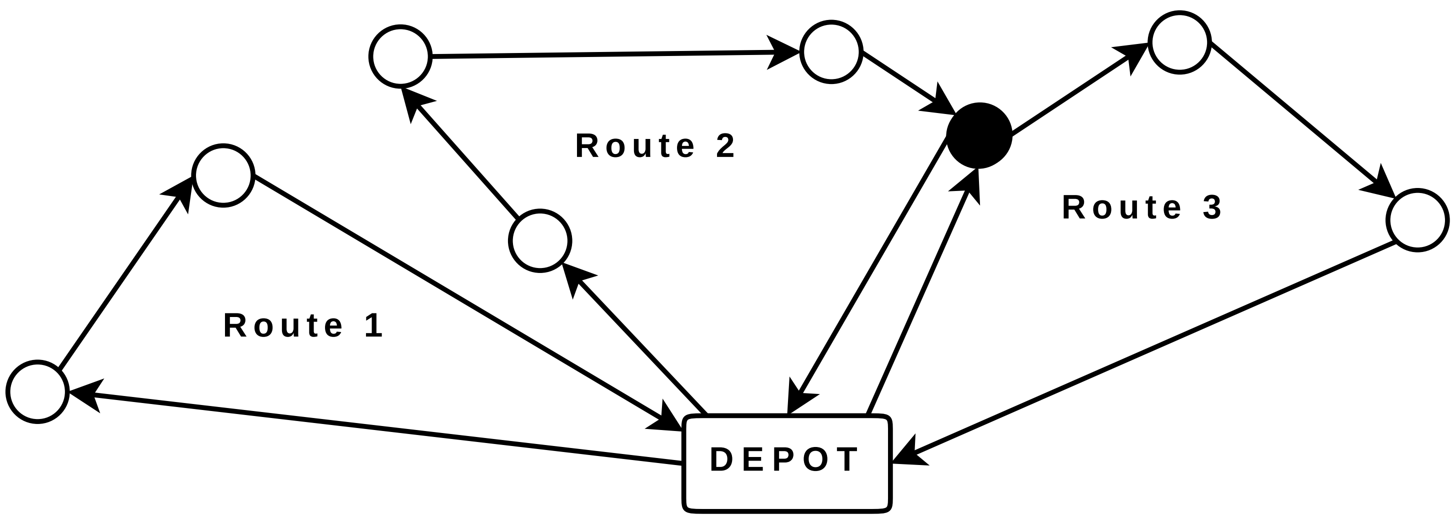
- design the optimal minimum-cost routes for vehicles,
- such that every customer is visited exactly once.



VRP with Split Delivery (SDVRP)

Splittable demand

- demand can be split and thus served by more than one vehicle;
- customers can be visited more than once.



VRP with Discrete Split Delivery (DSDVRP)

Nakao & Nagamochi (2007); Ceselli, Righini & Salani (2009)

- variant of VRP with split delivery;
- demand is discretized;
- the demand of each customer is represented by a set of items;
- items are grouped and delivered in orders (combinations of items);
- demand can be split but items cannot;
- service times depend on the quantity delivered;
- some combinations of items are not allowed.

Field Technician Scheduling Problem

Xu & Chiu (2001)

Problem description

- different types of jobs which require different skills;
- each technician is specialized in a field with certain skills;
- time windows on job starting and completion;
- assignment problem (jobs to technicians) + scheduling problem, where the duration of a job depends on the assignment.

Objective

- maximize the number of jobs completed within a time frame.

TBAP with QC assignment in container terminals

Giallombardo, Moccia, Salani & Vacca (2009)

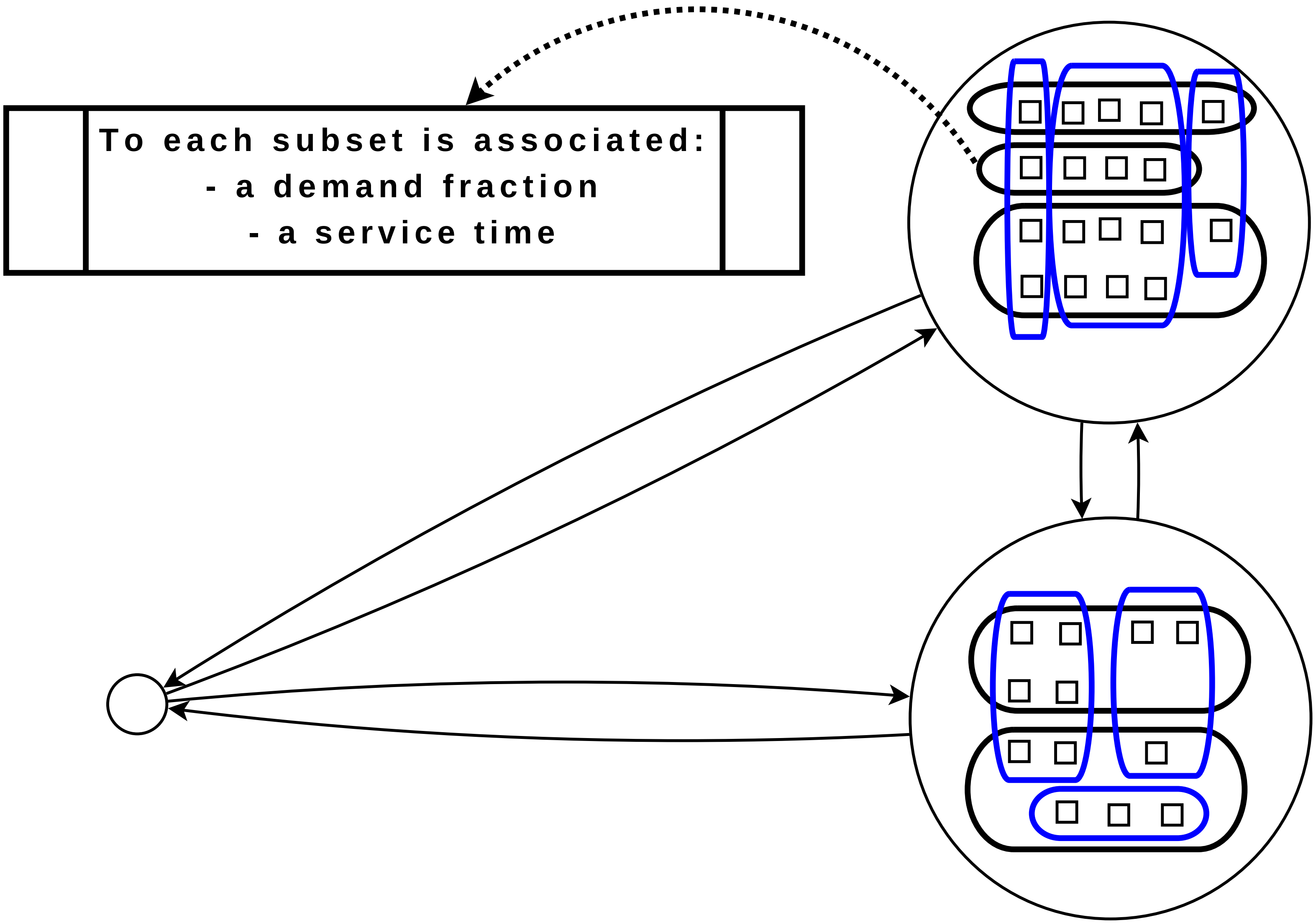
Problem description

- Tactical Berth Allocation Plan (TBAP): assignment and scheduling of ships to berths;
- Quay-Cranes (QC) assignment: a QC profile (number of QCs per shift) is assigned to each ship;
- feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift);
- time windows on ship arrival and on berth availabilities.

Objective

- maximize the value of chosen profiles.

Modeling the Discrete Split Delivery VRPTW



Modeling the Discrete Split Delivery VRPTW

- $G = (V, E)$ complete graph with $V = \{0\} \cup N$;
- (c_{ij}, t_{ij}) : cost and travel time of arc $(i, j) \in E$;
- N : set of customers $\{1, ..., n\}$; node $\{0\}$ represents the depot;
- K : set of vehicles with identical capacity Q;
- R : set of items; $R = \bigcup\limits_{i \in N} R_i, \;\; R_i \cap R_j = \emptyset \; \forall i \neq j, \;\; i, j \in N$;
- C : set of combinations of items; $C = \bigcup\limits_{i \in N} C_i, \;\; C_i \cap C_j = \emptyset \; \forall i \neq j, \;\; i, j \in N$;
- e_c^r : 1 if item $r \in R$ belongs to combination $c \in C$;
- t_c : service time of combination $c \in C$ such that $\max_{r \in R} e_c^r t^r \leq t_c \leq \sum_{r \in R} e_c^r t_r$;
- q_c : size of combination $c \in C$; $q_c = \sum_{r \in R} e_c^r t^r$;
- $[a_i, b_i]$: time window for customer $i \in N$.

Modeling the Discrete Split Delivery VRPTW

Decision variables

- x_{ij}^k binary: 1 if arc $(i,j) \in E$ is used by vehicle $k \in K$, 0 otherwise;
- y_c^k binary: 1 if vehicle $k \in K$ delivers combination $c \in C$, 0 otherwise;
- $T_i^k \geq 0$: time when vehicle $k \in K$ arrives at customer $i \in N$.

Objective function

- minimize the total traveling costs: $$z^* = \min \sum_{k \in K} \sum_{(i,j) \in E} c_{ij} x_{ij}^k$$

Constraints

- flow and precedence constraints;
- demand-satisfaction constraints;
- time-windows constraints;
- capacity constraints.

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Modeling the Discrete Split Delivery VRPTW

Flow and linking constraints

$$\sum_{j \in V} x_{0j}^k = 1 \qquad \forall k \in K, \tag{1}$$

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{ji}^k = 0 \qquad \forall k \in K, \forall i \in V, \tag{2}$$

$$\sum_{j \in V} x_{ij}^k = \sum_{c \in C_i} y_c^k \qquad \forall k \in K, \forall i \in N, \tag{3}$$

Covering constraints

$$\sum_{k \in K} \sum_{c \in C} e_c^r y_c^k = 1 \qquad \forall r \in R, \tag{4}$$

$$\sum_{c \in C_i} y_c^k \leq 1 \qquad \forall k \in K, \forall i \in N, \tag{5}$$

Modeling the Discrete Split Delivery VRPTW

Precedence constraints

$$T_i^k + \sum_{c \in C_i} t_c y_c^k + t_{ij} - T_j^k \leq (1 - x_{ij}^k) M \qquad \forall k \in K, \forall i \in N, \forall j \in V, \tag{6}$$

$$T_i^k - t_{0i} \geq (1 - x_{0i}^k) M \qquad \forall k \in K, \forall i \in N, \tag{7}$$

Time windows

$$T_i^k \geq a_i \sum_{j \in V} x_{ij}^k \qquad \forall k \in K, \forall i \in N, \tag{8}$$

$$T_i^k + \sum_{c \in C_i} t_c y_c^k \leq b_i \sum_{j \in V} x_{ij}^k \qquad \forall k \in K, \forall i \in N, \tag{9}$$

Capacity constraints

$$\sum_{c \in C} q_c y_c^k \leq Q \qquad \forall k \in K. \tag{10}$$

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Dantzig-Wolfe reformulation

- P : set of feasible routes;
- c_p : cost of route $p \in P$;
- e_p^r : binary parameter equal to 1 if item $r \in R$ is delivered in route $p \in P$;
- λ_p : binary decision variable equal to 1 if route $p \in P$ is chosen;

Master problem

$$\min \sum_{p \in P} c_p \lambda_p \tag{11}$$

$$\sum_{p \in P} e_p^r \lambda_p = 1 \qquad \forall r \in R, \tag{12}$$

$$\sum_{p \in P} \lambda_p \leq |K|, \tag{13}$$

$$\lambda_p \geq 0 \qquad \forall p \in P. \tag{14}$$

Column generation scheme

Pricing subproblem

$$p^* = \arg\min_{p \in P} \{\tilde{c}_p\} = \arg\min_{p \in P} \{c_p - \sum_{r \in R} \pi_r e_p^r - \pi_0\} \tag{15}$$

where π_r are the dual variables associated to constraints (12) and π_0 is the dual variable associated to constraint (13).

Column generation

- if $\tilde{c}_{p^*} < 0$, then column p^* is added to the (restricted) master problem;
- otherwise, the current master problem solution is proven to be optimal.

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Column generation scheme

Remarks

- the pricing subproblem is an Elementary Resource Constrained Shortest Path Problem (ERCSPP);
- the underlying network has one node for each order and transit time ($t_{ij} + t_c$); Ceselli et al. 2009: one node for each item.

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Branch & Price for the DSDVRPTW

- column generation approach;
- exact algorithm based on Branch&Price;
- pricing solved using bi-directional dynamic programming (Righini & Salani, 2006);
- branching rules:
  1. total number of vehicles;
  2. number of vehicles visiting a customer;
  3. flow on arcs.
- no additional cuts at master level.

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Instances

- derived from Solomon's data sets R1, C1 and RC1 for the VRPTW;
- $N = 25, 50$ customers;
- $Q = 30, 50, 100;$
- the demand of each customer is discretized in 12 items;
- we generated 3 scenarios:

scenario	orders	description
A	3	full, 50-50%;
B	5	full, 50-50%, 75-25%;
C	7	full, 50-50%, 75-25%, 90-10%;
O	1	full order (unsplit case).

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Computational results: 25 customers

			A		B		C	
class	nb_inst	Q	nb_solved	t	nb_solved	t	nb_solved	t
R1	12	30	12	87	10	694	6	1554
		50	11	342	6	463	5	522
		100	9	16	10	129	9	551
C1	9	50	9	273	0	x	0	x
		100	3	947	0	x	0	x
RC1	8	30	8	317	0	x	0	x
		100	8	222	2	1542	0	x

Computational results: 50 customers

			A		B		C	
class	nb_inst	Q	nb_solved	t	nb_solved	t	nb_solved	t
R1	12	30	1	3011	0	x	0	x
		50	1	1527	0	x	0	x
		100	2	120	2	509	1	93
RC1	8	50	7	723	0	x	0	x
		100	1	1953	0	x	0	x

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Summing up

- time limit: 1 hour;
- 69% (A), 32% (B) and 23% (C) instances solved for 25 customers;
- 14% (A), 2% (B) and 1% (C) instances solved for 50 customers;
- difficulty increases with the number of customers and with the number of orders;
- split deliveries are more frequent with small values of Q;
- in some cases, split deliveries not only decrease the total traveling costs but also allow to save one vehicle.

Conclusions

- finding optimal solutions is difficult, already with a small number of orders;
- only a limited number of instances with 50 customers could be solved;
- the bottleneck is the pricing problem:
  - the underlying network is huge (one node per each order!)
  - how to efficiently handle this feature of the problem?

Thanks for your attention!