# The Tactical Berth Allocation Problem Integrated optimization in container terminals

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### Outline

- Container terminals
- Berth Allocation & Quay Crane Assignment
- TBAP models & algorithms
- Hierarchical vs integrated approach
- Conclusions





### **Container terminals**





### **Container terminals**







#### Berth Allocation Problem (BAP)

to assign and to schedule ships to berths over a time horizon, according to an *expected handling time*, time windows on the arrival time of ships and availability of berths.

#### **Quay-Crane Assignment Problem (QCAP)**

to assign quay cranes (QC) to ships scheduled by the given berth allocation plan, over a time horizon, taking into account the *QC capacity constraint* in terms of available quay cranes at the terminal.





# **Tactical Berth Allocation Problem (TBAP)**

#### Integration of BAP and QCAP

- *tactical decision level*: we analyze the problem from the terminal point of view, in order to provide decision support in the context of the negotiation between the terminal and shipping lines.
- *quay-crane profiles and handling time*: the handling time becomes a decision variable, dependent on the assigned quay crane profile (i.e. number of cranes per shift, ex. 332). Feasible profiles can vary in length (number of shifts dedicated to the ship) and in size (number of QCs dedicated to the ship in each active shift).

#### **Housekeeping Yard Costs**

• in the context of a *transshipment container terminal*, we take into account the cost generated by the exchange of containers between ships in terms of traveled distance quay-yard-quay.





# The concept of QC assignment profile

TIME	ws=1	ws=2	ws=3	ws=4	ws=5	ws=6	ws=7	ws=8	
	ship 1								
berth I	3	2	2		4	4	5	5	
borth 2		shi	р З				ship 4		
berth 2		4	5			3	3	3	
borth 2					ship 5				
bertii 5			3	3	3	2	2		
QCs	3	6	10	3	7	9	10	8	





# **Transshipment-related housekeeping yard costs**



- Vessels A-B: no housekeeping, straddle carriers
- Vessels C-D: housekeeping, straddle carriers
- Vessels A-D: housekeeping, multi-trailers





Giallombardo, Moccia, Salani and Vacca (Transportation Research Part B, 2010).

#### **Decision variables**

- berth assignment :  $y_i^k \in \{0, 1\}$ ;
- profiles' assignment :  $\lambda_i^p \in \{0, 1\};$
- ship scheduling :  $x_{ij}^k \in \{0, 1\}, T_i^k \ge 0.$

**Objective function :** maximize total value of QC profile assignments & minimize the housekeeping yard cost of transshipment flows:

$$\max \sum_{i \in N} \sum_{p \in P_i} v_i^p \lambda_i^p - \frac{1}{2} \sum_{i \in N} \sum_{k \in M} y_i^k \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_j^w \tag{1}$$





**Berth covering constraints** 

$$\sum_{k \in M} y_i^k = 1 \qquad \forall i \in N,$$
(2)

Flow and linking constraints

$$\sum_{j \in N \cup \{d(k)\}} x_{o(k),j}^k = 1 \qquad \forall k \in M,$$
(3)

$$\sum_{i \in N \cup \{o(k)\}} x_{i,d(k)}^k = 1 \qquad \forall k \in M,$$
(4)

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k - \sum_{j \in N \cup \{o(k)\}} x_{ji}^k = 0 \qquad \forall k \in M, \, \forall i \in N,$$
(5)

$$\sum_{j \in N \cup \{d(k)\}} x_{ij}^k = y_i^k \qquad \forall k \in M, \, \forall i \in N,$$
(6)





#### **Precedence constraints**

$$T_i^k + \sum_{p \in P_i} t_i^p \lambda_i^p - T_j^k \le (1 - x_{ij}^k) M \qquad \forall k \in M, \, \forall i \in N, \, \forall j \in N \cup d(k)$$
(7)  
$$T_{o(k)}^k - T_j^k \le (1 - x_{o(k),j}^k) M \qquad \forall k \in M, \, \forall j \in N,$$
(8)

#### Ship and Berth time windows

 $a^k$ 

$$a_i y_i^k \le T_i^k \qquad \forall k \in M, \, \forall i \in N,$$
 (9)

$$T_i^k \le b_i y_i^k \qquad \forall k \in M, \, \forall i \in N,$$
(10)

$$\leq T_{o(k)}^k \qquad \forall k \in M, \tag{11}$$

$$T_{d(k)}^k \le b^k \qquad \forall k \in M,$$
(12)





**Profile covering & linking constraints** 

$$\sum_{p \in P_i} \lambda_i^p = 1 \qquad \forall i \in N,$$
(13)

$$\sum_{h \in H^s} \gamma_i^h = \sum_{p \in P_i^s} \lambda_i^p \qquad \forall i \in N, \forall s \in S,$$
(14)

$$\sum_{k \in M} T_i^k - b^h \le (1 - \gamma_i^h) M \qquad \forall h \in H, \, \forall i \in N,$$
(15)

$$a^{h} - \sum_{k \in M} T_{i}^{k} \le (1 - \gamma_{i}^{h})M \qquad \forall h \in H, \, \forall i \in N,$$
(16)

$$\rho_i^{ph} \ge \lambda_i^p + \gamma_i^h - 1 \qquad \forall h \in H, \, \forall i \in N, \, \forall p \in P_i,$$
(17)

Quay crane and profile feasibility

$$\sum_{i \in N} \sum_{p \in P_i} \sum_{u=max\{h-t_i^p+1;1\}}^{h} \rho_i^{pu} q_i^{p(h-u+1)} \le Q^h \qquad \forall h \in H^{\bar{s}}$$
(18)





# **Solving the TBAP**

- Model implemented and validated using general-purpose solvers (CPLEX, GLPK).
- Test instances based on real data provided by MCT, Port of Gioia Tauro, Italy.
- Up to 30 ships over a time horizon of 1 week; up to 60 ships over a time horizon of 2 weeks. Up to 30 quay crane profiles per ship.
- Only small-size instances (10 ships) solved at optimality. Often, no feasible solution provided.
- Efficient heuristic for TBAP (based on tabu search and mathematical programming).





# **Nested tabu search for TBAP**

Our heuristic algorithm for TBAP consists of 2 steps:

- 1. identify a QC profile assignment for the ships;
- 2. solve the resulting tactical berth allocation problem.

#### Algorithm 1: Nested tabu search

**Initialization :** Assign a QC profile to every ship.

repeat

- 1. solve BAP via tabu search;
- 2. update profiles using reduced cost arguments.

until stop criterion;

The BAP tabu search was adapted from Cordeau, Laporte, Legato and Moccia (2005).





# **TBAP computational results**

10x3				20x5			
Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
H1_10	99.17	98.52	7	H1_10	-	97.26	81
H1_20	97.96	98.36	15	H1_20	94.33	97.19	172
H1_30	98.76	98.33	27	H1_30	93.74	97.37	259
H2_10	99.26	98.92	7	H2_10	-	97.27	82
H2_20	96.97	98.48	16	H2_20	96.66	97.38	173
H2_30	96.79	98.17	28	H2_30	-	97.26	274
L1_10	100.00	99.12	6	L1_10	-	97.30	74
L1_20	100.00	99.01	15	L1_20	-	97.25	158
L1_30	99.99	98.29	26	L1_30	-	97.06	254





# **TBAP computational results**

30x5				40x5			
Instance	CPLEX	HEUR	Time (sec)	Instance	CPLEX	HEUR	Time (sec)
H1_10	-	95.67	340	H1_10	-	97.38	1104
H1_20	-	95.31	677	H1_20	-	97.38	2234
H1_30	-	95.54	1009	H1_30	-	97.25	3387
H2_10	-	95.88	316	H2_10	-	97.40	1095
H2_20	-	95.81	684	H2_20	-	97.33	2198
H2_30	-	95.30	969	H2_30	-	97.27	3296
L1_10	-	96.55	324	L1_10	94.92	97.41	1421
L1_20	-	96.43	652	L1_20	94.47	97.14	2996
L1_30	-	96.18	966	L1_30	-	96.20	4862





# **Column generation for TBAP**

Vacca, Salani and Bierlaire (Proceedings of TRISTAN VII, June 2010).

- A route represents the sequence of ships visiting a berth.
- A quay crane profile is assigned to each ship belonging to the route.
- Profitable routes are generated in the pricing subproblem.
- The underlying network has one node for every ship, for every quay crane profile and for every time step.





# **Integrated vs hierarchical approach**

The hierarchical approach consists of the following steps:

- 1. determine the expected handling time for every ship;
- 2. BAP: solve the classical berth allocation problem (no qc profile assignment, no capacity constraint);
- 3. QCAP: assign a qc profile to every ship, taking into account the capacity constraint and the provided bap schedule.





# **Integrated vs hierarchical approach**

We consider 2 scenarios for the handling time:

- scenario A : longest feasible profile for every ship;
- scenario B : max-value profile for mother vessels, longest feasible profile for feeders.

Scenario A allows for comparison with TBAP, since all quay crane profiles are feasible for the QCAP.

Scenario B is more realistic, although it may lead to infeasibility of QCAP.





# **Integrated vs hierarchical approach**

Scenario A, 10 ships, 3 berths, 1 week.

Improvement of the integrated approach w.r.t. hierarchical approach:

instance (tbap gap)	objective	housekeeping cost	profiles' value
H1_10 (0.8%)	0.6%	2.8%	0.2%
H1_20 (1.7%)	-0.3%	0.0%	-0.3%
H1_30 (1.3%)	0.1%	0.4%	0.0%
H2_10 (0.0%)	0.7%	5.1%	0.0%
H2_20 (0.0%)	0.9%	6.4%	0.0%
H3_30 (0.0%)	0.9%	6.4%	0.0%
L1_10 (0.4%)	0.6%	4.0%	0.0%
L1_20 (0.6%)	0.7%	5.0%	0.0%
L1_30 (0.0%)	1.3%	8.9%	0.0%





# **BAP** objective functions

We compare the following BAP models:

**Min-cost BAP** 

$$\min \sum_{i \in N} \sum_{k \in M} \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_i^k y_j^w$$

**Min-delay BAP** 

 $\min\max_{i\in N}(T_i-a_i)$ 

Min-cost-bounded-delay BAP

$$\min \sum_{i \in N} \sum_{k \in M} \sum_{j \in N} \sum_{w \in M} f_{ij} d_{kw} y_i^k y_j^w$$

s.t. 
$$T_i - a_i \leq (1 + \epsilon)T^* \quad \forall i \in N.$$





## Scenario A, 10 ships, 3 berths, 1 week

	MIN	-COST BA	١P	MIN-DELAY BAP			
instance	obj	gap t(sec)		obj	t (sec)	cost	
H1_10	232'794	8.93%	3600	OPT	103.8	277'062 (+19%)	
H2_10	145'770	5.13%	3600	OPT	3.6	174'120 (+19%)	
L1_10	162'061	6.10%	3600	OPT	4.5	196'417 (+21%)	





## Scenario A, 10 ships, 3 berths, 1 week

	MC-BD BAP ( $\epsilon$	= 0.2)	MC-BD BAP ( $\epsilon$	MIN-COST	
instance	obj*	t (sec)	obj*	t (sec)	obj
H1_10	252'324 (+8%)	1471	239'808 (+3%)	1401	232'794
H2_10	165'300 (+13%)	75	159'672 (+9%)	167	145'770
L1_10	178'105 (+10%)	90	178'105 (+10%)	92	162'061





#### Conclusions

- Integration of BAP and QCAP
- Model and algorithms for TBAP
- Hierarchical vs integrated approach
- Next step: exact method (improve solution and/or bounds)





# Thanks for your attention!





#### References

- Cordeau, J. F., Laporte, G., Legato, P. and Moccia, L. (2005). Models and tabu search heuristics for the berth-allocation problem, *Transportation Science* **39**: 526–538.
- Giallombardo, G., Moccia, L., Salani, M. and Vacca, I. (2010). Modeling and solving the tactical berth allocation problem, *Transportation Research Part B: Methodological* 44(2): 232–245.
- Vacca, I., Salani, M. and Bierlaire, M. (2010). Column generation for the tactical berth allocation problem, *Proceedings* of TRISTAN VII, Tromso, Norway.