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Radiative Origin of the Fermion Mass Hierarchy: A Realistic and Predictive Approach *

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Abstract

The up-down splitting within quark families increases with the family number: $m_u \sim m_d$, $m_c > m_s$, $m_t \gg m_b$,. We show an approach that realizes this feature of the spectrum in a natural way. We suggest that the mass hierarchy is first generated by radiative effects in a sector of heavy isosinglet fermions, and then projected to the ordinary light fermions by means of a seesaw mixing. The hierarchy appears then *inverted* in the light fermion sector. We present a simple left-right symmetric gauge model in which the P- and CP-parities and an isotopical "up-down" symmetry are softly (or spontaneously) broken in the Higgs potential. Experimentally consistent predictions are obtained. The Cabibbo angle is automatically in the needed range: $\Theta_C \sim 0.2$. The top quark is naturally heavy, but not too heavy: $m_t < 150$ GeV.

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Although the idea of radiatively generated fermion mass hierarchy is very attractive, it is difficult to implement it in a realistic way. For instance, it is generally problematic to understand the experimental value of the Cabibbo angle and the large *top-bottom* splitting. In addition dangerous FCNC's have to be kept under control. Recently[1], however, a new approach to the fermion mass puzzle has been suggested. In this approach the mass hierarchy is first radiatively generated in a hidden sector of hypothetical heavy fermions and then transferred to the visible quarks and leptons by means of a *universal seesaw* mechanism[2]. Providing a qualitatively correct picture of quark masses and mixing, this approach solves many problems of the previous models[3,4] of radiative mass generation. In particular, the correct value of the Cabibbo angle can be accommodated, without trouble for the perturbative expansion. Moreover, within the seesaw approach, the effective low energy theory, after integrating out the heavy fermions, is simply the standard model with one Higgs doublet (and with *definite* Yukawa couplings). Thus, flavour changing phenomena, typical of the direct models[4] of radiative mass generation, are naturally suppressed.

The key idea of the model[1] is to suppose the existence of weak isosinglet heavy fermions (Q-fermions) in one-to-one correspondence with the light ones. The model[1] has a field content such that only one family (namely the first) of Q-fermions becomes massive at the tree level. The 2^{nd} Q-family gets a mass at the 1-loop level and the 3^{rd} only at 2 loops. Because of the seesaw mechanism[2], the mass of any usual quark or lepton is inversely proportional to the mass of its heavy partner. Thus the mass hierarchy between the families of light fermions is *inverted* with respect to the hierarchy of Q-fermion families. This feature is very attractive for the following reason. Experimentally we observe a small mass splitting within the lightest quark family (u and d) and an increasing splitting from family to family, with the up-quark masses growing faster: $1 \sim m_u/m_d < m_c/m_s < m_t/m_b$. In our approach it is natural to have $m_u \sim m_d$, since these masses are determined by the tree level masses of the heaviest Q-fermions. On the other hand, the increasing splitting can be related to the difference between the loop-expansion parameters in the up and down quark sectors.

In what follows, we show that the simplest and most economical version of the model[1] provides a predictive ansatz for the quark mass matrices. We assume that the "isotopical" discrete symmetry I_{UD} between up and down quark sectors, as well as the left-right symmetry P_{LR} and CP-invariance, is violated *only* in the loop expansion, due to soft (or spontaneous) breaking in the Higgs potential. The appearance of *both* the mass splitting within the lightest family ($m_d/m_u = 1.5 - 2$) and the large (compared to the other mixing angles) value of the Cabibbo angle ($\sin\Theta_C \simeq 0.22$) is determined by the properties of the seesaw "projection". The troubles for the perturbation expansion are then avoided. The model leads to some successful predictions for the quark mass and mixing pattern. We shall discuss them below.

Let us consider the simple left-right symmetric model based on the gauge group $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R \otimes U(1)_{\bar{B}-\bar{L}}$, suggested in[1]. The left- and right-handed components of usual quarks $q_i = (u_i, d_i)$ and their heavy partners $Q_i = U_i, D_i$ are taken in the following representations:

$$\begin{array}{ll} q_{Li}(I_L = 1/2,\ \bar{B}-\bar{L} = 1/3), & q_{Ri}(I_R = 1/2,\ \bar{B}-\bar{L} = 1/3) \\ U_{Li}(Y_L = 1,\ \ \bar{B}-\bar{L} = 1/3), & U_{Ri}(Y_R = 1,\ \ \bar{B}-\bar{L} = 1/3) \\ D_{Li}(Y_L = -1,\ \bar{B}-\bar{L} = 1/3), & D_{Ri}(Y_R = -1,\ \bar{B}-\bar{L} = 1/3) \end{array} \tag{1}$$

where i=1,2,3 is the family index (the indices of colour $SU(3)_c$ are omitted). Only the nonzero quantum numbers are shown in the brackets: $I_{L,R}$ are the $SU(2)_{L,R}$ weak isospins and $Y_{L,R}$ are the $U(1)_{L,R}$ hypercharges. Let us also introduce *one* additional family of fermions with $\bar{B}-\bar{L} = 1/3$ and with the following hypercharges:

$$\begin{array}{ll} p_L(Y_L = -1/2,\ Y_R = 3/2), & p_R(Y_L = 3/2,\ Y_R = -1/2) \\ n_L(Y_L = 1/2,\ Y_R = -3/2), & n_R(Y_L = -3/2,\ Y_R = 1/2) \end{array} \tag{2}$$

The scalar sector of the theory consists of

$$\begin{array}{ll} H_L(I_L = 1/2,\ \ Y_R = 1), & H_R(I_R = 1/2,\ \ Y_L = 1) \\ T_{uL}(Y_L = -2,\ \ \bar{B}-\bar{L} = -2/3), & T_{uR}(Y_R = -2,\ \ \bar{B}-\bar{L} = -2/3) \\ T_{dL}(Y_L = 2,\ \ \bar{B}-\bar{L} = -2/3), & T_{dR}(Y_R = 2,\ \ \bar{B}-\bar{L} = -2/3) \\ \Phi(Y_L = 2, Y_R = -2), & \varphi(Y_L = 1/2, Y_R = -1/2), \\ \Omega(Y_L, Y_R = 1/2,\ \bar{B}-\bar{L} = -1) & \end{array} \tag{3}$$

where the T-scalars are supposed to be colour triplets. Let us impose also CP, P_{LR} and I_{UD} discrete symmetries. P_{LR}[6], essentially parity, and CP act in the usual way. The isotopical "up-down" symmetry I_{UD} is defined by

$$\begin{array}{l} U_{L,R} \leftrightarrow D_{L,R}, \quad p_{L,R} \leftrightarrow n_{L,R}, \quad H_{L,R} \leftrightarrow \tilde{H}_{L,R} = i\tau_2 H^*_{L,R}, \\ T^u_{L,R} \leftrightarrow T^d_{L,R}, \quad \Phi \leftrightarrow \Phi^*, \quad \varphi \leftrightarrow \varphi^*, \quad A^\mu_{L,R} \leftrightarrow -A^\mu_{L,R} \end{array} \tag{4}$$

where $A^\mu_{L,R}$ are the gauge bosons of $U(1)_{L,R}$. Then the most general Yukawa couplings consistent with gauge invariance, I_{UD}, P_{LR} and CP are

$$\begin{array}{l} \mathcal{L}_1 = \Gamma_{ij}(\bar{q}_{Li}\, U_{Rj}\, \tilde{H}_L + \bar{q}_{Li}\, D_{Rj}\, H_L) + (L \leftrightarrow R) + h.c. \\ \mathcal{L}_2 = \lambda_{ij}(U_{Li}CU_{Lj}T_{uL} + D_{Li}CD_{Lj}T_{dL}) + (L \leftrightarrow R) + h.c. \\ \mathcal{L}_3 = h(\bar{p}_L p_R \Phi^* + \bar{n}_L n_R \Phi) + h_i(\bar{U}_{Li} p_R \varphi^* + \bar{D}_{Li} n_R \varphi) + (L \leftrightarrow R) + h.c. \end{array} \tag{5}$$

where C is the charge conjugation matrix. The coupling constants $h, h_i, \lambda_{ij}, \Gamma_{ij} (i,j = 1,2,3)$ are *real* due to CP-invariance ($\lambda_{ij} = -\lambda_{ji}$, since the T-scalars are colour triplets). In what follows we do not make any particular assumption on their structure. We only suppose that they are *typically* $O(1)$, just like the gauge coupling constants. Without loss of generality, by a suitable redefinition of the fermion basis, we can always take $h_2, h_3 = 0$, $\lambda_{13} = 0$, $\Gamma_{12}, \Gamma_{13}, \Gamma_{23} = 0$. In what follows we use this basis.

Let us suppose that the discrete symmetries CP, P_{LR} and I_{UD} are *softly* broken only by the bilinear and trilinear terms in the Higgs potential [1)]. These are given by

$$\mathcal{V}_3 = \Lambda_u T^*_{uL} T_{uR} \Phi + \Lambda_d T^*_{dL} T_{dR} \Phi^* + h.c. \tag{6}$$

where the coupling constants $\Lambda_{u,d}$ are generally *complex*, violating thereby both CP and P_{LR} invariances.

The VEVs $\langle\Phi\rangle = v_\Phi$ and $\langle\varphi\rangle = v_\varphi$, $v_\Phi \gg v_\varphi$, break $U(1)_L \otimes U(1)_R$ down to $U(1)_{L+R}$ (the VEV of Ω then breaks $U(1)_{L+R} \otimes U(1)_{\bar{B}-\bar{L}}$ to the usual $U(1)_{B-L} : B - L = Y_L + Y_R + \bar{B} - \bar{L}$). The fermions p and n become massive, $M_p = M_n = hv_\Phi$, and the Q-fermions of the first family, U_1 and D_1 get masses $M \cong h_1^2 v_\varphi^2 / hv_\Phi$ due to their *seesaw* mixing with the former ones (interactions $\mathcal{L}_3$ in (5)). At the same time the coloured scalars $T_{uL} - T_{uR}$ and $T_{dL} - T_{dR}$ get mixed due to the interaction terms in (6). At this point, radiative mass generation proceeds, following the chain $U_1 \to U_2 \to U_3, \quad D_1 \to D_2 \to D_3$. The Q-fermion mass matrices generated from the loop corrections due to $\mathcal{L}_2$ in (5) can be presented in the following form:

$$M_{U,D} = M(\hat{P}_1 + e^{-i\omega_{u,d}}\, \xi_{u,d}\, \tilde{\lambda}\, \hat{P}_1\, \lambda + C_{u,d} \xi_{u,d}^2\, \tilde{\lambda}^2\, \hat{P}_1\, \lambda^2 + \cdots) \tag{7}$$

where $\hat{P}_1 = diag(1,0,0)$ is a 1-dimensional projector and $\omega_{u,d} = -\arg \Lambda_{u,d}$. The loop expansion factors are

$$\xi_q = \frac{1}{8\pi^2} \sin 2\alpha_q \, \log R_q, \qquad R_q = (M^q_+ / M^q_-)^2 \tag{8}$$

[1)] Actually, this symmetries could be spontaneously broken at the price of introducing $P_{LR}-$ and $I_{UD}-$odd real scalars[1]. The consequences, as far as fermion masses are concerned, would be unchanged.

where M_+^q, M_-^q are the eigenvalues of the mass matrices of the scalars $T_{qL}-T_{qR}$, $q=u,d$, and α_q are the corresponding mixing angles. In a "reasonable" range of parameters ($1<R<10$) the 2-loop factor $C(R)=C(1/R)$ is practically constant[4]: $C_{u,d}\simeq 0.65$. Eq.(8) is valid in the natural regime $M<M_+^q, M_-^q<M_p$.

Apart from small $O(\xi_{u,d}^2)$ 1-3 entries, the matrices $M_{U,D}$ are *diagonal*. Then the mass hierarchy between the three families of Q-fermions is given by $1:x^{-1}\varepsilon_{u,d}:\varepsilon_{u,d}^2$, where we defined $x=\sqrt{C}\lambda_{23}/\lambda_{12}$ and $\varepsilon_{u,d}=\sqrt{C}\lambda_{12}\lambda_{23}\xi_{u,d}\sim 10^{-2}-10^{-1}$. The parameters ε_u and ε_d are the effective loop-expansion parameters, respectively for the up and down sectors.

The VEVs $\langle H_L\rangle=(0,v_L)$ and $\langle H_R\rangle=(0,v_R)$, $\quad v_R\gg v_L=(2\sqrt{2}\,G_F)^{-1/2}\approx 175$ GeV, break the intermediate $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$ symmetry down to $U(1)_{em}$. Then the ordinary quarks $q=u,d$ acquire masses due to their seesaw mixing with the heavy fermions $Q=U,D$ (interactions $\mathcal{L}_1$ in eq.(5)). The whole mass matrix for up-type quarks, written in block form, is

$$(\bar{u},\bar{U})_L\begin{pmatrix}0 & \Gamma v_L\\ \tilde{\Gamma}v_R & M_U\end{pmatrix}\begin{pmatrix}u\\ U\end{pmatrix}_R \tag{9}$$

and analogously for down-type quarks. When $M_{U,D}\gg v_R,v_L$, the resulting mass matrix for the light states is given by the seesaw formula

$$M_{light}^{u,d}=v_Lv_R\Gamma M_{U,D}^{-1}\tilde{\Gamma} \tag{10}$$

Substituting here eq.(7) we find, in the explicit form,

$$M_{light}=\frac{m}{\varepsilon^2}\begin{pmatrix}\varepsilon^2\gamma_{11}^2 & \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon^2\gamma_{11}\bar{\gamma}_{31}\\ \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon x e^{i\omega}\gamma_{22}^2+\varepsilon^2\gamma_{21}^2 & \varepsilon x e^{i\omega}\gamma_{22}\gamma_{32}+\varepsilon^2\gamma_{21}\bar{\gamma}_{31}\\ \varepsilon^2\gamma_{11}\bar{\gamma}_{31} & \varepsilon x e^{i\omega}\gamma_{22}\gamma_{32}+\varepsilon^2\gamma_{21}\bar{\gamma}_{31} & 1+\varepsilon x e^{i\omega}\gamma_{32}^2+\varepsilon^2\bar{\gamma}_{31}^2\end{pmatrix} \tag{11}$$

where $m=\Gamma_{33}^2 v_L v_R M^{-1}$, $\gamma_{ij}=\Gamma_{ij}/\Gamma_{33}$ and $\bar{\gamma}_{31}=\gamma_{31}+\sqrt{C}x^{-1}$; $\varepsilon=\varepsilon_{u,d}$, $\omega=\omega_{u,d}$ for the up and down quarks, respectively.

It is obvious, from the measured values of quark masses, and from (11), that $\varepsilon_u\ll\varepsilon_d\ll 1$. The up quark mass matrix M_{light}^u is almost diagonal. Neglecting $\sim\varepsilon_u$ corrections we have $m_u=m\gamma_{11}^2$, $m_c=xm\gamma_{22}^2\varepsilon_u^{-1}$ and $m_t=m\varepsilon_u^{-2}$. Thereby, the quark mixing pattern is determined *essentially* by the down quark mass matrix M_{light}^d, where $m_b\approx m\varepsilon_d^{-2}$. The contributions to the parameters of the CKM matrix from M_{light}^u are typically suppressed by the factor $\varepsilon_u/\varepsilon_d$ and we neglect them. After some algebra one can obtain:

$$V_{us} \approx \sqrt{\frac{m_d}{m_s} \mid 1 - \frac{m_u}{m_d} e^{i\delta} \mid} \tag{12}$$

$$V_{ub} \approx \frac{\bar{\gamma}_{31}}{\gamma_{11}} \frac{m_u}{m_b}, \qquad V_{cb} \approx \frac{m_d}{m_u} \left(\sqrt{\frac{m_s}{m_d}} V_{ub} + \frac{\gamma_{32}}{\gamma_{22}} \frac{m_s}{m_b} e^{i\omega_d} \right) \tag{13}$$

where $\delta = -\omega_d + \arg(x e^{i\omega_d} \gamma_{22}^2 + \varepsilon_d \gamma_{21}^2) \approx -\omega_d + \arg(1 + e^{i\omega_d})$ is a phase measuring the amount of CP-violation in the CKM matrix. Within uncertain (but supposed to be ~ 1) numerical factors the formulae (13) fit the experimental values of V_{ub} and V_{cb} (notice that for $\Gamma_{32} = 0$ one has $V_{ub}/V_{cb} = m_u/\sqrt{m_d m_s} = 0.11 - 0.15$). The small values of V_{ub} and V_{cb} imply that the corresponding entries in M_{light}^d *cannot* significantly affect the eigenvalues. As for the 1-2 mixing, the situation is different. The relation $m_u \neq m_d$ requires a correction to m_d from the 1-2 entry in M_{light}^d. This correction is of the right order of magnitude, provided $\Gamma_{21}/\Gamma_{11} = O(\sqrt{m_s/m_u}) \approx 6$. We consider such a spread, in the value of the Yukawa coupling, perfectly acceptable. As a result, the formula (12) appears which implies the Cabibbo angle to be in the needed range: $V_{us} = 0.22 \pm 0.07$ within all uncertainties. The comparison of (12) with the experimental value $V_{us} \approx 0.22$ implies a large CP-phase, $\delta \sim 1$, in agreement with the recent data.

From the mass matrices (11) one can also derive the relations

$$\frac{\varepsilon_d}{\varepsilon_u} = \frac{m_u m_c}{m_d m_s} = \sqrt{\frac{m_t}{m_b}} \tag{14}$$

which allows to calculate the top quark mass using the known masses[7] of the other quarks. The large value of the former implies that the "seesaw" corrections[8] to equation (10) have to be taken into account. Doing so, we obtain the physical top quark mass

$$m_t^* = m_t^0 \left[1 + \left(\frac{m_t^0}{\Gamma_{33} v_L} \right)^2 \right]^{-1/2} \tag{15}$$

where m_t^0 is the "would be" mass, calculated from eq.(14). Obviously, the analogous corrections are negligible for other quark masses since we demand all Γ's to be ~ 1. On the other hand, from (11) one can easily derive that $\Gamma_{21}/\Gamma_{33} \approx \varepsilon_d^{-1} \sqrt{m_d m_s / m_u m_b} \geq 0.17 \varepsilon_d^{-1}$. In order to be consistent with perturbation theory we assume that all the Yukawa coupling constants, including Γ_{21} and λ's, are less than 2. This implies $\Gamma_{33} \leq 1$. Consequently, from

(14) and (15) we obtain $m_t^* = 50 - 150$ GeV. The large spread here is related mainly with the uncertainties in the light quark masses. It is also interesting to turn the logic around and say that the experimental lower bound[9] $m_t^* > 91$ GeV favours the lower values of m_d/m_u and m_s among those allowed in[7].

The inclusion of leptons in this model is straightforward and will be presented elsewhere. In fact $U(1)_{\bar{B}-\bar{L}}$ can be unified with $SU(3)_c$ within Pati-Salam[10] type SU(4). The $U(1)_L \otimes U(1)_R \otimes I_{UD}$ part can also be enlarged to $SU(2)'_L \otimes SU(2)'_R$, in which case the isotopical symmetry is obviously continuous.

Last but not least we wish to remark that in our approach the strong CP-problem can be automatically solved *without* axion. Owing to P and/or CP-invariances the initial $\Theta_{QCD} = 0$ and $\Theta_{QFD} = \arg Det\hat{M}$, where $\hat{M}$ is the *whole* mass matrix $\hat{M}$ of all fermions q, Q and p, n is also vanishing at tree level, because of the seesaw pattern[11]. The loop corrections can provide, however, $\bar{\Theta} = 10^{-9} - 10^{-10}$, which makes this scenario in principle accessible to the search of the $DEMON$ - dipole electric moment of neutron.

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