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Yukawa Unification: The Good, The Bad and The Ugly[1]

Riccardo Rattazzi[2]

Department of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855

Uri Sarid[3] (presenter)

Physics Department
Stanford University
Stanford, California 94305

Lawrence J. Hall[4]

Theoretical Physics Group, 50A/3115
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720

Abstract

We analyze some consequences of grand unification of the third-generation Yukawa couplings, in the context of the minimal supersymmetric standard model. We address two issues: the prediction of the top quark mass, and the generation of the top-bottom mass hierarchy through a hierarchy of Higgs vacuum expectation values. The top mass is strongly dependent on a certain ratio of superpartner masses. And the VEV hierarchy always entails some tuning of the GUT-scale parameters. We study the RG equations and their semi-analytic solutions, which exhibit several interesting features, such as a focusing effect for a large Yukawa coupling in the limit of certain symmetries and a correlation between the A terms (which contribute to $b \to s\gamma$) and the gaugino masses. This study shows that non-universal soft-SUSY-breaking masses are favored (in particular for splitting the Higgs doublets via D-terms and for allowing more natural scenarios of symmetry breaking), and hints at features desired in Yukawa-unified models. Several phenomenological implications are also revealed.

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[2]E-mail: rattazzi@physics.rutgers.edu
[3]E-mail: sarid@squirrel.stanford.edu
[4] E-mail: hall_lj@theorm.lbl.gov

1 Introduction

There is strong evidence to suggest that the three gauge couplings of the strong, electromagnetic and weak interactions are unified at a high energy scale in a single gauge interaction based on a simple group, such as SU(5) or SO(10), as long as the desert below the unification scale is described by the minimal supersymmetric extension of the standard model (the MSSM). Furthermore, the combination of supersymmetry (SUSY) and grand unification yields models with numerous attractive features: the embedding of the standard-model matter multiplets into a few irreducible representations of the GUT group, the technically natural preservation of a hierarchy between the weak and GUT scales, a longer proton lifetime to allow agreement with current experimental lower bounds, the correct prediction of the ratio of b quark to τ lepton masses, simple ansätze for the remaining fermion masses, and a picturesque scenario for radiative electroweak symmetry breaking. We have chosen, therefore, to look beyond the gauge unification prediction of the weak mixing angle and examine the unification of third-family Yukawa couplings [1], for the most part within its natural context of SO(10) unification. By Yukawa couplings we mean the couplings of the top, bottom and tau to the Higgs doublets which generate their masses when electroweak symmetry is broken. The third family is singled out because of its relatively large Yukawa couplings: it seems reasonable to suppose that they arise at tree-level from the simplest interactions, while the masses and mixings of the other generations require more complex, perhaps also higher-order and certainly very model-dependent structures. Our study also applies more generally to scenarios in which these Yukawa couplings are unified but not in the context of an SO(10) GUT, or even models in which the Yukawas are only comparable near the GUT or Planck scales. We begin by considering the most immediate prediction of this Yukawa unification, namely the top quark mass [2]. However, we are quickly led to consider in some depth the more general question of how the top-bottom mass hierarchy could be generated in the MSSM, and how this hierarchy depends on the initial conditions of the renormalization group (RG) evolution at the GUT scale [3]. We will conclude with a discussion of how natural (or unnatural!) such a hierarchy seems in this context, what its other phenomenological predictions might be, and how one could hope to improve the theoretical picture or obtain experimental corroboration.

One consequence of Yukawa unification is immediate, and independent of other assumptions except for the qualitative nature of the RG evolution equations of the MSSM. Since the Yukawa couplings of the top and bottom quarks (and the τ lepton) are always comparable, the large ratio of the top mass versus the bottom (or tau) mass must be due to a large ratio of the Higgs vacuum expectation values (VEVs) which give rise to their masses. Namely, since the up-type and down-type matter fermion masses arise from couplings to the up-type and down-type Higgs multiplets ($H_{U,D}$), respectively, the large ratio $m_t/m_{b,\tau} = (\lambda_t v_U)/(\lambda_{b,\tau} v_D)$ is not a consequence of a large ratio of Yukawas $\lambda_t/\lambda_{b,\tau}$ but rather of large $v_U/v_D \equiv \tan\beta$. Thus Yukawa unification generically implies $\tan\beta \sim \mathcal{O}(50)$. Further assumptions are necessary to make any precise predictions. We will assume the following three throughout most of this work, although we will point out those conclusions which are more general:

(I) The masses of the third generation, m_t, m_b and m_τ, originate from renormalizable Yukawa couplings of the form $\underline{16}_3\, \mathcal{O}\, \underline{16}_3$ in a supersymmetric GUT with a gauge group containing (the conventional) SO(10); $\underline{16}_3$ denotes the 16-dimensional spinor representation of SO(10) containing the third-generation standard-model fermions (plus the right-handed neutrino which we assume to be superheavy) and their superpartners.

(II) The evolution of the gauge and Yukawa couplings in the effective theory beneath the SO(10) breaking scale is described by the RG equations of the MSSM.

(III) The two Higgs doublets lie predominantly in a single irreducible multiplet of SO(10).

The first and third assumptions serve to define what we mean by Yukawa unification, while the second allows us to relate this GUT-scale unification to weak-scale observables. From (I) and (III) it follows that the third-generation Yukawas must arise from either a $\underline{16}_3\, \underline{10}_H\, \underline{16}_3$ or a $\underline{16}_3\, \underline{126}_H\, \underline{16}_3$ interaction with an SO(10) Higgs multiplet. The latter leads to the boundary conditions $3\lambda_t^G = 3\lambda_b^G = \lambda_\tau^G \equiv \lambda_G$ at the GUT scale, but the resulting ratio of m_b/m_τ at low energies is far too low to be consistent with experiment (at least within the perturbative regime, and unless very large threshold corrections to the b mass arise at low energies [2]). Thus we are restricted to using the $\underline{10}_H$, and hence the boundary condition

$$\lambda_t^G = \lambda_b^G = \lambda_\tau^G \equiv \lambda_G\,. \tag{1}$$

With this boundary condition, and using the unification of gauge couplings to fix the unification scale and the gauge coupling at that scale, we can now evolve the Yukawa couplings down to the weak scale for any given value of λ_G. The idea is that the three observable masses m_t, m_b and m_τ are functions of the four parameters λ_t, λ_b, λ_τ and $\tan\beta$, and $\lambda_{t,b,\tau}$ are in turn determined by the unification in terms of λ_G and the GUT scale M_G. Since the latter is already known from gauge unification, we are left with three observable masses as functions of only two parameters, λ_G and $\tan\beta$. Thus we use two observables, m_b and m_τ to fix λ_G and $\tan\beta$, and thereby predict the third observable m_t. A detailed analysis of the RG evolution, and the consequent predictions, has already been presented [2]. The results, namely the values of $\lambda_{t,b,\tau}$ and the ratio $R \equiv m_b/m_\tau = \lambda_b/\lambda_\tau$ all at the weak scale, are plotted in Fig. 1 as functions of λ_G. (These curves actually use 2-loop RG evolution, but at this point the difference between 1- and 2-loop equations is not important. For the final predictions of m_t we use the full 2-loop evolution and 1-loop matching conditions.) Evidently, larger λ_G values correspond to a heavy top and to a smaller R ratio. The experimental value $R_{\rm expt}$ is found [2] from the QCD sum rules value and is evolved to the weak scale using 2-loop QCD running. We find, allowing for α_s to vary between roughly 0.11 and 0.12, the shaded range shown in Fig. 1. Thus, in the absence of any large corrections in the matching between the R evolved down from the GUT scale and the $R_{\rm expt}$ in the standard model, we find $\lambda_G > 0.75$, which implies a heavy top.

2 The top and bottom masses

For a precise prediction, 2-loop RG equations must be used along with 1-loop matching functions. These matching functions include logarithmic corrections from infinite counterterms as well as nonlogarithmic contributions from finite graphs. The former are given elsewhere [2]; they are generally quite small, and invariably increase the top mass as the superpartner masses increase. The latter are more interesting, since they can be very large [2, 4]. The dominant corrections arise from the graphs of Fig. 2, which match the value of the b mass as evolved down from the GUT scale to the value in the low-energy theory. Typically the gluino graph dominates, yielding a corrected value $m_b = \lambda_b v_D + \delta m_b$ where $v_D = 174\,\mathrm{GeV}$,

$$\frac{\delta m_b}{m_b} = \frac{8}{3}\, g_3^2\, \frac{\tan\beta}{16\pi^2}\, \frac{m_{\tilde{g}}\mu}{m_{\mathrm{eff}}^2}\,, \tag{2}$$

and $m_{\tilde{g}}$ is the gluino mass while m_{eff} is the mass of the heaviest superpartner in the loop (more exact expressions may be found in our previous work). The important observation here is that the bottom mass gets a large contribution from the up-type Higgs at 1-loop order, whereas its tree-level mass was small due to the small VEV of the down-type Higgs. In the usual scenario with small $\tan\beta$, the bottom was light because its Yukawa coupling was small, or in other words because it was protected by an approximate chiral symmetry. Thus any higher-order corrections would also be suppressed by this approximate symmetry. In the large $\tan\beta$ scenario these corrections are not suppressed: there is an enhancement of $v_U/v_D = \tan\beta$, which overcomes the usual $g_3^2/16\pi^2$ loop factor to give a correction of order 1 to the b mass—at least if $m_{\tilde{g}}\mu \sim m_{\mathrm{eff}}^2$. Phenomenologically, the result is that m_b cannot be predicted with any certainty unless we know something about the superspectrum, and if m_b is uncertain then so is the top mass prediction.

In Fig. 3 we show the top pole mass prediction, now to full 2-loop order, as a function of the $\overline{\mathrm{MS}}$ running parameter $m_b(m_b)$ in two cases. The top curves and the higher horizontal axis correspond to a hierarchical spectrum, in which the squarks are heavy whereas the μ parameter and gaugino masses are light. Then the corrections δm_b are small and the top mass is predicted to be above 180 GeV or so. The bottom curves and the lower horizontal axis correspond to a roughly degenerate spectrum in which the corrections to the b mass are $\mathcal{O}(25\%)$ and negative (i.e. R should be lowered by 25% in Fig. 1 before matching to the experimental value). Now the top can be significantly lighter. In fact, this last argument can be turned around: if the threshold corrections are too large in magnitude and negative then the top mass prediction will be below the experimental lower bound, while if they are large and positive then no value of λ_G will allow agreement with R_{expt}. We thus find the following bounds on the superspectrum: if λ_G is allowed to vary, then

$$-0.37 < \frac{m_{\tilde{g}}\mu}{m_{\mathrm{eff}}^2} < 0.08\,, \tag{3}$$

whereas for fixed λ_G the appropriate limits can be read from Fig. 1.

3 Radiative bottom decay (I)

Do we have any experimental information about these corrections? Of course we have no direct evidence of any of the superpartners, but we can appeal to their indirect appearance in loop diagrams. Consider again the diagrams of Fig. 2, but with one of the b quarks replaced by a strange quark using a flavor-changing vertex, and with a photon attached in all possible ways. We see that the same processes will lead to a contribution to the rare decay $b \to s\gamma$, and with a similar enhancement of $\mathcal{O}(\tan\beta)$ over the usual MSSM scenario [2, 5]. The dependence of these diagrams on the superpartner masses is somewhat similar to that of δm_b, except that (a) now the operator is of higher dimension and so is suppressed by the mass of the heaviest superpartner, and (b) typically the higgsino-mediated diagram dominates. If the parameters appearing in this Higgsino diagram, namely μ, A (the trilinear soft SUSY-breaking parameter) and the squark masses, are all comparable and of order the Z mass, then this diagram gives a contribution to the amplitude for $b \to s\gamma$ many times bigger than the standard model or the usual MSSM amplitudes, and is clearly ruled out by the CLEO limit [6]. To restore agreement with experiment, either the overall superpartner mass scale must be raised far above the electroweak scale, or else μ or A or both must be suppressed relative to the squark mass in the loop. More quantitatively, we find that either the masses must all be raised to at least $\mathcal{O}(\text{TeV})$, or else if both μ and A are near the Z mass then the squarks must be above either ~ 400 GeV or ~ 700 GeV, depending on whether these diagrams interfere destructively or constructively with the 2-Higgs standard model amplitudes. In any case, however, these restrictions do not yet tell us anything about the combination $m_{\tilde{g}}\mu/m_{\text{eff}}^2$ which appears in δm_b; the link between these two will be forged below, when we study the evolution of the entire set of MSSM parameters.

4 Electroweak symmetry breaking

We have assumed in the above that the top-bottom mass hierarchy would arise from a hierarchy of VEVs in the Higgs spectrum, namely $v_U/v_D \equiv \tan\beta \sim \mathcal{O}(50)$. We now consider how such a hierarchy could be generated in the MSSM. Clearly this question is of interest for any model in which the Yukawas themselves do not supply a sufficient hierarchy to explain the large ratio of third-generation quark masses, not just for the equal-Yukawas case to which we have specialized. In studying this question, however, we will pay attention to which spectra are favored by large $\tan\beta$ scenarios, and whether with such spectra and our unification assumptions we can pin down the top mass prediction.

The scalar potential of the neutral Higgs bosons which leads to electroweak symmetry breaking is given by

$$V_0 = m_U^2|H_U|^2 + m_D^2|H_D|^2 + \mu B(H_U H_D + \text{h.c.}) + (\text{quartic terms}) \qquad (4)$$

where $m_{U,D}^2 = \mu_{U,D}^2 + \mu^2$ contain the soft-breaking masses and the μ parameter in the superpotential, B is a soft-breaking mass parameter, and the quartic terms arise from

D-terms and so are given by gauge couplings. We will evolve these parameters from the GUT to the weak scale using the 1-loop RG equations of the MSSM, stopping the evolution at some typical scale (of order the squark masses) which minimizes the effects of higher-order corrections. The subscript 0 indicates that we will restrict our attention to this (RG-improved) tree-level potential rather than calculate the full 1-loop effective potential or, better yet, explicitly integrate out massive particles and consider full 1-loop matching conditions. We expect [3] that our qualitative discussion of the radiative symmetry breaking will not be jeopardized by this simplification. That is, a more complete calculation will change the numerical values of the GUT parameters needed for correctly breaking the symmetry, but will not significantly alter the size of the domains in parameter space where such breaking is achieved. The conditions for this breaking are well-known:

$$m_U^2 + m_D^2 \geq 2|\mu B| \tag{5}$$

ensures that the potential is bounded from below, and

$$m_U^2 m_D^2 < \mu^2 B^2 \tag{6}$$

guarantees the existence of a minimum away from the origin and so breaks the symmetry. In practice, since $|\mu B|$ will always be much less than or at most comparable to $|m_U^2|$ and $|m_D^2|$, we can reduce these requirements to $m_A^2 = m_U^2 + m_D^2 > 0$ (using the expression for the pseudoscalar Higgs mass) and $m_U^2 < 0$ (noting that large $\tan\beta$ means that the up-type Higgs gets the large VEV).

In the usual—and very attractive—scenario of radiative breaking, the two mass parameters start out at the GUT scale with a universal positive value: $m_U^2 = m_D^2 = M_H^2 + \mu^2$, where M_H is the soft-breaking mass of the $\underline{10}_H$ of Higgs in SO(10), or of the $\underline{5}_H$ and $\overline{\underline{5}}_H$ in SU(5). Thus the symmetry is not broken at that scale. However, in the RG evolution to the electroweak scale, the large Yukawa coupling of the top quark to H_U, which gives the top its mass, also drives the mass-squared parameter μ_U^2 of H_U negative (with the help of the QCD coupling), while the absence of a large Yukawa in the down sector keeps the mass-squared of H_D positive. In fact, conditions (5) and (6) are easily satisfied for a large range of initial conditions if $\lambda_G \sim \mathcal{O}(1)$, resulting in a very natural picture of radiative symmetry breaking. This picture is essentially lost in the large $\tan\beta$ scenario, for the following two reasons:

1. Since both Yukawas are comparable [and in fact initially equal in the SO(10) case], the two Higgs doublets tend to run in the same way, so either both stay positive at the electroweak scale and the symmetry does not break, or both become negative and the potential becomes unbounded from below (a situation which breaks the symmetry but in a Coleman-Weinberg fashion, yielding an "electroweak scale" orders of magnitude higher than the SUSY-breaking scale). The effects which differentiate the evolution of the two Higgs doublets, namely hypercharge and the absence of a right-handed neutrino, are small and a poor replacement for the usual $\lambda_t \gg \lambda_b$ splitting. Interestingly, an $\mathcal{O}(1)$ splitting between λ_t and $\lambda_{b,\tau}$ is still of little use since it is quickly diminished by the

fixed-point behavior of these couplings. (Some of these observation have been previously made by T. Banks [4].)

2. Even when electroweak symmetry is broken, a large hierarchy of VEVs must be generated between the two similarly-evolving Higgs doublets. Minimizing the potential V_0 when $\tan\beta \gg 1$ yields

$$-2m_U^2 = m_Z^2 \tag{7}$$

and

$$\frac{1}{\tan\beta} = -\frac{\mu B}{m_u^2 + m_D^2} = -\frac{\mu B}{m_A^2}\,. \tag{8}$$

The first equation sets the scale, but from the second equation we see that a large hierarchy in VEVs requires the large hierarchy $\mu B \ll m_U^2 + m_D^2$. This, as we show below, implies a degree of fine-tuning between some parameters in the Lagrangian.

5 Solutions of the RG equations (I)

Before analyzing the implications of these two criticisms, we present the 1-loop solutions [3] of the RG equations for the MSSM mass parameters, integrated between $M_G = 3 \times 10^{16}$ GeV and a typical squark mass of 300 GeV. (None of our results is sensitive to the exact values of these starting and stopping scales.) The solutions depend on the dimensionless initial values of the gauge and Yukawa couplings α_G and λ_G and on the dimensionful GUT-scale parameters M_{sq}, M_H, μ_G, $M_{1/2}$, A_G, B_G and M_X. M_{sq} and M_H are the soft-breaking masses of the $\underline{10}_H$ and the $\underline{16}_3$ respectively, and M_X will be explained below; we note for now that it vanishes for universal soft-breaking masses. Whenever possible, capital letters denote values at the GUT scale. The RG equations themselves are well-known and will not be presented here. These equations dictate that the low-energy values of the various mass parameters depend very simply on the dimensionful initial values, with coefficients that depend only on the dimensionless ones. Since α_G is known from gauge unification, these coefficients depend only on λ_G. For the representative value $\lambda_G = 1$, the solutions are:

$$\begin{aligned}
2\,m_U^2 &= -5.1\,{M_{1/2}^G}^2 + 1.2\,M_H^{G2} - 1.6\,M_{sq}^{G2} + 2\mu^2 - 3.8\,M_X^{G2} && (9)\\
m_A^2 &= -4.9\,{M_{1/2}^G}^2 + 1.1\,M_H^{G2} - 1.7\,M_{sq}^{G2} + 2\mu^2 + .01\,M_X^{G2} && (10)\\
m_Q^2 &= +4.6\,{M_{1/2}^G}^2 - .25\,M_H^{G2} + .51\,M_{sq}^{G2} + 1.0\,M_X^{G2} && (11)\\
m_t^2 &= +4.1\,{M_{1/2}^G}^2 - .27\,M_H^{G2} + .46\,M_{sq}^{G2} + .85\,M_X^{G2} && (12)\\
m_b^2 &= +4.2\,{M_{1/2}^G}^2 - .23\,M_H^{G2} + .55\,M_{sq}^{G2} - 2.9\,M_X^{G2} && (13)\\
m_L^2 &= +.53\,{M_{1/2}^G}^2 - .12\,M_H^{G2} + .77\,M_{sq}^{G2} - 3.1\,M_X^{G2} && (14)\\
m_\tau^2 &= +.15\,{M_{1/2}^G}^2 - .23\,M_H^{G2} + .55\,M_{sq}^{G2} + 1.2\,M_X^{G2} && (15)\\
A_t &= +.09\,A_G + 1.8\,M_{1/2}^G && (16)
\end{aligned}$$

$$A_b = +.07\,A_G + 1.9\,M^G_{1/2} \qquad (17)$$
$$A_\tau = +.20\,A_G - .17\,M^G_{1/2} \qquad (18)$$
$$B = -.86\,A_G - 1.1\,M^G_{1/2} + 1.0\,B_G \qquad (19)$$
$$\mu = .44\,\mu_G \qquad (20)$$

Here Q and L are the squark and slepton doublets, respectively, and t, b and τ are the SU(2)-singlet squarks and sleptons. For clarity of presentation, we have dropped from the first 7 expressions above the terms proportional to A_G^2 and to $A_G M_{1/2}$, since their coefficients are small enough [$\mathcal{O}(0.01 - 0.1)$] and exhibit sufficiently small custodial-SU(2) breaking to be negligible for purposes of symmetry-breaking, at least if A_G is not very much larger than the other mass parameters. We have also used the low-energy value of μ in Eqs. (9-10). These solutions are useful references for the discussions below. These solutions can also be combined [3] with the more analytic approach briefly described in Eqs. (25) to give a complete, semi-analytic solution to the RG equations when custodial SU(2) is an approximate symmetry.

6 Obtaining a hierarchy of VEVs

To understand the second criticism above, let us examine in more detail how Eq. (8) may be satisfied. We concentrate for the moment on six relevant electroweak-scale parameters: the up- and down-type Higgs masses m_U^2 and m_D^2, a typical squark mass m_0^2, the B parameter in the scalar potential, a gaugino mass (specifically the wino mass) $m_{1/2}$, and the μ parameter. This last one may be set to zero by imposing a Peccei-Quinn symmetry on the Lagrangian, so the size of μ measures the breaking of this $\mathcal{PQ}$ symmetry; consequently, μ is multiplicatively renormalized. The previous two, B and $m_{1/2}$, along with the A parameter, transform in the same way under a continuous $\mathcal{R}$ symmetry (so they enter into each other's RG equations) and may be made arbitrarily small by imposing this $\mathcal{R}$ symmetry. With these two symmetries in mind [2], we consider three possible spectra having splittings which lead to a large $\tan\beta$ according to Eq. (8):

mass:	scenario A:	scenario B:	scenario C:
$7\,m_Z$			$m_D\,m_0$
m_Z	$m_U\,m_D\,m_0$	$m_U\,m_D\,m_0$ $\quad m_{1/2}\,\mu$	m_U $\quad B\,m_{1/2}\,\mu$
$\frac{1}{7}\,m_Z$	$B\,m_{1/2}\,\mu$		
$\frac{1}{50}\,m_Z$		B	

Of course there are many other ways to split the parameters and obtain the correct hierarchy, but these will suffice to demonstrate the fine-tuning involved in the splittings. The value of $\tan\beta$ is determined directly only by m_U, m_D, μ and B; we include m_0 and $m_{1/2}$ to illustrate the symmetries. Scenario A involves no tuning at all (at this stage of the analysis): the only hierarchies present are those enforced by the two symmetries. However, as also pointed out by Nelson and Randall [7], such a scenario is ruled out for large $\tan\beta$ since it would imply a light chargino, in disagreement with bounds from LEP [8]. In fact, both μ and $m_{1/2}$ must be comparable to or above the Z mass to satisfy this bound. Therefore we *must* widely split some parameters without a symmetry justification, and this will entail fine-tuning. For example, in scenario B all parameters are kept at the Z mass but the B has been tuned to be light (namely, its initial value at the GUT scale is chosen to almost completely cancel the contributions induced by A and $m_{1/2}$ through the RG evolution). Alternatively, in scenario C, m_U is adjusted to end up much below the other scalar masses (yielding $m_A^2 \simeq 50 m_Z^2$) while the other parameters are kept at the Z mass using the approximate symmetries. In these two scenarios, and in fact *generically* whenever $\mu > m_Z$ and $m_{1/2} > m_Z$, the initial conditions at the GUT scale must be adjusted to at least a relative accuracy of $1/\tan\beta$ to obtain the necessary hierarchy of VEVs. We should point out, however, that such a tuning is no worse than the one which would be needed in the small $\tan\beta$ case if the squarks were experimentally determined to be above 700 GeV or so.

7 Splitting the Higgs doublets

We return now to the first criticism above, and address the splitting between the two Higgs doublets. Recall that after running we need $2m_U^2 < 0$ while $m_U^2 + m_D^2 > 0$. However, the two masses evolve almost in parallel, since custodial symmetry breaking effects, namely hypercharge and the absence of ν_R, are small. Thus, if at the GUT scale the mass parameters are custodial-SU(2) symmetric, the splitting of the two Higgs masses at the weak scale is small relative to a typical SUSY mass M_S at the GUT scale: $m_D^2 - m_U^2 \equiv \epsilon_c M_S^2$ ("c" for custodial). Putting these together, we learn that only within a window of size $\sim \epsilon_c$ in the GUT-scale parameter space can we simultaneously satisfy $m_U^2 < 0$ and $m_A^2 > 0$; if they are satisfied, then $m_Z^2 = -2m_U^2 < \epsilon_c M_S^2$ and $m_A^2 = m_U^2 + m_D^2 < \epsilon_c M_S^2$. In practice, this is usually accomplished [1] using the gaugino mass as the largest mass parameter, so $M_S = M_{1/2} \geq M_{sq,H}$: this is because, according to Eqs. (9,10), custodial breaking effects proportional to $M_{1/2}^2$ lower m_U^2 with respect to m_D^2, while those from the scalar masses $M_{sq,H}^2$ act in the opposite way. Furthermore μ must also typically be $\mathcal{O}(M_{1/2})$ in order to keep m_A^2 positive. Then, in addition to the $\mathcal{O}(\epsilon_c)$ fine-tuning of the Z mass, the large values of $m_{1/2}$ and μ mean that the B parameter must be adjusted beyond the $\mathcal{O}(1/\tan\beta)$ accuracy of the previous paragraph. To see this, we rewrite Eq. (8) in the form

$$\frac{B}{m_{1/2}} = \frac{1}{\tan\beta} \frac{m_U^2 + m_D^2}{\mu\, m_{1/2}} \tag{21}$$

which quantifies the needed suppression of the electroweak-scale value of B (achieved by fine-tuning its GUT-scale value) relative to the minimum value it would naturally have, namely the value $\sim M_{1/2}$ induced through the RG evolution. In the present case, using $\mu \sim M_{1/2} \sim M_S$ we obtain $B/m_{1/2} \sim (1/\tan\beta)\,\epsilon_c$.

8 D-terms

This highly unnatural state of affairs arises partly because of the degeneracy of the Higgs doublets and their subsequent parallel evolution. A possible remedy is actually generic in SO(10) unification, due to the rank of this group which exceeds by one the rank of SU(5) or the standard model. Thus we write $\mathrm{SO(10)} \supset \mathrm{SU(5)} \otimes \mathrm{U(1)_X}$, where $\mathrm{U(1)_X}$ is proportional to $3(\mathrm{B}-\mathrm{L}) + 4\mathrm{T_{3R}}$ [the generator of baryon- minus lepton-number symmetry and a generator of $\mathrm{SU(2)_R}$] and couples to the scalar fields according to the following table:

field:	H_U	H_D	Q	t	b	L	τ	$\langle \underline{16}_H \rangle$	$\langle \overline{\underline{16}}_H \rangle$
$\mathrm{U(1)_X}$ charge:	-2	2	1	1	-3	-3	1	5	-5

The $\underline{16}_H$ and $\overline{\underline{16}}_H$ are examples of extra superheavy Higgs representations which are typically added in order to break this $\mathrm{U(1)_X}$ (in this case by acquiring VEVs in the "ν_R" direction) and reduce the rank of the group. As usual, the spontaneous breakdown of a U(1) leads to a VEV for its D-term, which can induce masses for the various fields which appear in this D-term. In particular, *if we do not assume universal soft-breaking masses* for all scalars, then the soft-breaking masses of the $\underline{16}_H$ and $\overline{\underline{16}}_H$ need not be equal, and therefore their VEVs are also split, in proportion to their mass splitting. This splitting in turn generates a mass splitting in the low-energy MSSM Lagrangian through the cross-term:

$$\mathcal{L} \supset \frac{1}{2} D_X^2 = \frac{1}{2} \left(\langle |\underline{16}_H|^2 \rangle - \langle |\overline{\underline{16}}_H|^2 \rangle + 2|H_U|^2 - 2|H_D|^2 + \ldots \right)^2 . \tag{22}$$

In fact, this mechanism splits any fields which have different charges under $\mathrm{U(1)_X}$. Thus the boundary conditions for the scalar masses at the GUT scale become

$$\begin{aligned} M_U^2 &= M_H^2 + \mu^2 - 2M_X^2 \\ M_D^2 &= M_H^2 + \mu^2 + 2M_X^2 \\ M_{Q,t,\tau}^2 &= M_{sq}^2 \qquad + \ M_X^2 \\ M_{b,L}^2 &= M_{sq}^2 \qquad - 3M_X^2 \end{aligned} \tag{23}$$

where the capital letters on the left-hand side serve as reminders that these are the values at the GUT scale, and

$$M_X^2 = \frac{1}{10}(M_{16}^2 - M_{\overline{16}}^2) \tag{24}$$

is a new soft-breaking mass parameter in the low-energy theory. With this mass we no longer need rely on large gaugino masses to split the Higgs doublets: they can start out being different, and thus even with parallel evolution the symmetry-breaking conditions (5–6) can apparently be satisfied.

One problem with this mechanism is evident from the initial conditions in Eq. (23): not just the Higgs doublets but also the squarks and sleptons are split, so an excessively large M_X^2 could lower M_b^2 or M_L^2 sufficiently to make m_b^2 or m_L^2 negative at the electroweak scale, thereby spontaneously breaking the strong or electromagnetic gauge symmetries. If RG effects were irrelevant, namely for small λ_G, then M_{sq} could always be raised enough to prevent this without affecting $m^2_{U,D}$. However, for $\lambda_G \sim \mathcal{O}(1)$ the squark masses strongly affect the evolution of the Higgs doublets [see Eqs. (9-10)], and only for very constrained ranges of the initial parameters can the electroweak symmetry, and only that symmetry, be spontaneously broken at a reasonable scale. In fact, as we show in brief below, there is a focusing effect that is inherent in the MSSM RG equations when $\lambda_b \sim \lambda_t$, and which inevitably requires an adjustment of the GUT-scale parameters beyond the $1/\tan\beta$ level derived above. We first show this behavior of the RG equations for completely general initial conditions, and then return to discuss the specific case of Eqs. (23).

9 Solutions of the RG equations (II)

Consider the RG equations of the MSSM in the limit of exact $\mathcal{PQ}$ and $\mathcal{R}$ symmetries, in which $\mu = M_{1/2} = A = B = 0$ at all scales. For future reference, we call this scenario *the maximally symmetric case*. This limit is interesting for two reasons: First, no large corrections arise to the b quark mass, and the $R = m_b/m_\tau$ prediction for all values of $\lambda_G \sim \mathcal{O}(1)$ falls nicely within the range allowed by experiment (see Fig. 1); in other words, a heavy top quark near its fixed-point mass favors small δm_b. Second, as we saw above, having a large μ and $m_{1/2}$ calls for fine-tuning B (or some equivalent adjustment), so we'd like to explore the opposite limit to see whether a more natural scenario can be achieved. Of course, eventually we must relax this limit to agree with LEP bounds, but the qualitative behavior we shall discover will persist. If we further approximate $\lambda_b \simeq \lambda_t \equiv \lambda$ and neglect the sleptonic contributions (thereby restoring custodial symmetry), the RG solutions simplify considerably. There are now five relevant parameters. In terms of their initial conditions at the GUT scale, M^2_U, M^2_D, M^2_Q, M^2_t and M^2_b, the solutions at the electroweak scale are:

$$\begin{aligned}
-2m^2_U &= -\tfrac{3}{7}\epsilon_\lambda X - \tfrac{3}{5}\epsilon'_\lambda X' - I - I' \\
m^2_A &= \tfrac{3}{7}\epsilon_\lambda X + I - I' \\
m^2_Q &= \tfrac{1}{7}\epsilon_\lambda X - \tfrac{1}{4}I + \tfrac{1}{4}I'' \\
m^2_t &= \tfrac{1}{7}\epsilon_\lambda X + \tfrac{1}{5}\epsilon'_\lambda X' - \tfrac{1}{4}I - \tfrac{1}{2}I' - \tfrac{1}{4}I'' \\
m^2_b &= \tfrac{1}{7}\epsilon_\lambda X - \tfrac{1}{5}\epsilon'_\lambda X' - \tfrac{1}{4}I + \tfrac{1}{2}I' - \tfrac{1}{4}I''
\end{aligned} \tag{25}$$

where

$$\begin{aligned}
\epsilon_\lambda &= \exp\left(-\frac{7}{8}\int_{\ln m_Z}^{\ln M_G} \frac{\lambda^2}{\pi^2}\, d\ln\mu\right) \\
&\sim 0.085 \qquad (\text{for } \lambda_G \simeq 1)\,,
\end{aligned} \tag{26}$$

$$\epsilon'_\lambda = \epsilon_\lambda^{5/7}\,, \tag{27}$$

and

$$X = M^2_U + M^2_D + 2M^2_Q + M^2_t + M^2_b$$

$$\begin{array}{rcl} X' &=& M_U^2 - M_D^2 \qquad\qquad + M_t^2 - M_b^2 \\ I &=& \frac{4}{7}(M_U^2 + M_D^2) - \frac{3}{7}(2M_Q^2 + M_t^2 + M_b^2) \\ I' &=& \frac{2}{5}(M_U^2 - M_D^2) - \frac{3}{5}(M_t^2 - M_b^2) \\ I'' &=& \qquad\qquad 2M_Q^2 - M_t^2 - M_b^2\,. \end{array} \tag{28}$$

(Note again the use of capital letters to denote GUT-scale initial parameters, and recall that U, D, Q, t and b refer to the up-type Higgs, the down-type Higgs, the SU(2)-doublet third-generation squarks, the SU(2)-singlet stop and the SU(2)-singlet sbottom, respectively.)

Evidently, two linear combinations of masses, labeled by X and X' at the GUT scale, renormalize multiplicatively and exponentially contract at low energies for $\lambda_G \sim \mathcal{O}(1)$. The three other linear combinations, I, I' and I'', are invariant. The important observation here is that in the first contraction—the sum rule $m_A^2 + 2m_Q^2 + m_t^2 + m_b^2 = \epsilon_\lambda X$—the coefficient of every term is positive, while we already know that each mass-squared itself must be positive for a proper electroweak-breaking scenario. Therefore each term by itself must be small, less than $\epsilon_\lambda X$. This can only happen if the various combinations of invariants (and possibly also $\epsilon'_\lambda X'$) in the expressions (25) for these terms are adjusted to be small relative to X. The exact constraints that follow from this requirement are given explicitly elsewhere [3]. They are of the form I, I', $I'' \lesssim \max(\epsilon_\lambda X, \epsilon'_\lambda X')$. We learn that, for $\lambda_G \sim \mathcal{O}(1)$ where this focusing effect is important, any given model for the GUT-scale soft-breaking masses must either provide an explanation of why each invariant should be small relative to the sum $X = M_U^2 + M_D^2 + 2M_Q^2 + M_t^2 + M_b^2$, or else that invariant must be tuned by hand to be small. We also learn that the conditions for successful symmetry-breaking are sensitive to any other small effects. One such effect is custodial symmetry violation, which is parametrized above by ϵ_c and results from hypercharge and λ_τ (or the absence of ν_R). In the running of the Yukawas, both of these cause λ_t to slightly exceed λ_b, and thus drive m_U^2 below m_D^2, as in the conventional scenario of electroweak symmetry breaking. In the running of the masses, the contributions of the τ Yukawa have an opposite and numerically more relevant impact. Therefore the custodial-breaking effects make it harder to break the symmetry correctly—significantly harder in the specialized case discussed below. In any realistic scenario there are also contributions from the gauginos and μ, so in the end the sum rule, and therefore the general limit which must be set on I, I' and I'', takes the form

$$\left\{I, I', I'', m_A^2 + 2m_Q^2 + m_t^2 + m_b^2\right\} \lesssim \mathcal{O}\left[\max\left(\epsilon_\lambda, \epsilon_c, \frac{\mu^2}{M_S^2}, \frac{m_{1/2}^2}{M_S^2}\right)\right] M_S^2 \tag{29}$$

where M_S is the largest mass parameter in the initial conditions at the GUT scale.

If we now return to the more specialized boundary conditions of Eq. (23), we find (after setting $\mu = 0$):

$$\begin{array}{rcl} X &=& 2M_H^2 + 4M_{sq}^2 \\ X' &=& 0 \\ I &=& \frac{4}{7}(2M_H^2 - 3M_{sq}^2) \end{array} \tag{30}$$

$$I' = -4M_X^2$$
$$I'' = +4M_X^2\,.$$

For this choice of boundary conditions, keeping the invariants small imposes only two requirements:

$$M_H^2 - \tfrac{3}{2}M_{sq}^2 = \epsilon_{\lambda c} M_S^2 \ll M_S^2 \qquad (31)$$

and small M_X. The upper bound on the invariants involves both ϵ_λ and ϵ_c, and as we hinted above they partially cancel in the combination $\epsilon_{\lambda c}$ which enters into the requirements: $|\epsilon_{\lambda c}| < |\epsilon_\lambda|, |\epsilon_c|$. (One sum rule which can be formed under these boundary conditions, $-2m_U^2 + \frac{4}{3}m_A^2 + \frac{4}{3}m_b^2$, is particularly sensitive to this cancellation[3].) The first requirement, Eq. (31), entails a definite tuning of parameters to a precision of $\epsilon_{\lambda c}$, which does not apparently follow from any symmetry. Note that without this requirement, either color breaks when $M_H^2 > \frac{3}{2}M_{sq}^2$ or a Coleman-Weinberg mechanism operates when $M_H^2 < \frac{3}{2}M_{sq}^2$. The requirement of small M_X may on the other hand be natural, since [see Eq. (24)] the value of M_X^2 is smaller by an order of magnitude than the soft-breaking masses whose splitting generates the D-terms, and those masses may be expected to be comparable to M_{sq} and M_H. In any case, we see that because of the focusing effect of the RG equations, the D-terms cannot be *allowed* to induce splittings bigger than those we already had through custodial SU(2)-breaking effects. Hence they do not eliminate the criticism that the electroweak symmetry is hard to break when the Yukawas are comparable. But there is still a significant advantage in using these D-terms, since they can now substitute for large values of $m_{1/2}$ and μ, and with light gauginos and μ it is much easier to obtain a large $\tan\beta$, according to Eqs. (8) and (21).

10 Radiative bottom decay (II)

Before putting the various observations to use in examining specific scenarios and their merits, we point out another feature of the solutions to the RG equations which will further constrain the scenarios. As is evident from Eqs. (16-18) (or directly from the RG equations), the initial value A_G hardly affects the low-energy values of $A_{t,b,\tau}$; they are instead largely determined in magnitude and sign by the gaugino mass $M_{1/2}^G$, which also fixes the low-energy gluino mass. (It is difficult, though perhaps not impossible for sufficiently small λ_G, to construct models in which $A_G \gg M_{1/2}$ and yet the electroweak symmetry but neither color nor charge breaks spontaneously and correctly, so we shall disregard this possibility in these proceedings. The implications of tuning A_G to cancel the gaugino mass at low energies in the expression for A_t will be considered elsewhere [3].) This observation, which was also emphasized by Carena et al. [9], directly relates the δm_b corrections of Eq. (2) to the large $b \to s\gamma$ graphs discussed above. (More precisely, the gluino- and higgsino-exchange diagrams for each process are directly related.) The sign of this correlation [9] is such that when $\delta m_b < 0$ (i.e. the predicted R is lowered, and therefore so is the top mass) then the large $b \to s\gamma$ graphs interfere *constructively* with the usual 2-Higgs standard model amplitude, and vice-versa. On one hand, we see from Eq. (3) or from Fig. 1, that the bounds on δm_b are more severe when $\delta m_b > 0$. On the other hand, as noted above, when $\delta m_b < 0$ the interference is constructive and the bounds on the large $b \to s\gamma$ graphs are stricter. Thus these two bounds are much stronger when taken together, and translate into the following statement: either (a) the gauginos or μ or both are significantly lighter than the squarks, or (b) the superpartners are much heavier than the Z.

11 Case studies

With these remarks in mind, we first examine the popular [1] case of universal soft-breaking masses. This scenario has also been recently studied in some detail by Carena et al. [9]. If all soft-breaking scalar masses are equal then the D-term contributions vanish ($M_X \equiv 0$), and we are left with the three parameters μ, $M_{1/2}$ and $M_0 \equiv M_{sq} = M_H$, in addition to B_G which is adjusted at the end to obtain the correct $\tan\beta$ [see Eq. (8)]. We have already mentioned that in this case we need μ comparable to $M_{1/2}$, and both at least as big as M_0, to break electroweak symmetry correctly. We have also seen that these three parameters must be tuned in order to obtain a positive m_Z^2 and m_A^2:

$$m_Z^2 \sim m_A^2 \sim \epsilon_c M_{1/2}^2 \,. \tag{32}$$

Next, to achieve a hierarchy of Higgs VEVs, B_G must be adjusted very precisely such that, at low energies,

$$\frac{B}{m_{1/2}} = \frac{1}{\tan\beta} \frac{m_A^2}{\mu \, m_{1/2}} \sim \frac{\epsilon_c}{\tan\beta} \,. \tag{33}$$

Finally, since μ and the gauginos are not lighter than the squarks, δm_b is rather large, and so must be negative, as can be seen from Eq. (3) or from Fig. 1. Hence the $b \to s\gamma$ constraint is strong, necessitating large superpartner masses of at least $\mathcal{O}(\text{TeV})$ and therefore a *further* tuning (by roughly another order of magnitude) of the three parameters to achieve correct electroweak breaking. Of course, such a highly-tuned scenario is also highly predictive: for example, the spectrum is highly constrained, and the top mass is predicted by the large δm_b corrections to be light.

In search of a more natural scenario, we next relax the assumption of universal soft-breaking masses, which was perhaps arbitrary to begin with. We begin with the maximally symmetric scenario studied previously (see also scenario C above), in which $\mu \sim M_{1/2} \sim A_G \sim B_G \, (\sim m_Z) \ll m_A, \ldots, m_b \, (\ll M_{sq} \sim M_H)$ so that the $\mathcal{PQ}$ and $\mathcal{R}$ symmetries are approximately obeyed. Since this hierarchy directly implies a small δm_b and therefore (from Fig. 1) a relatively large λ_G, the focusing effect of Eq. (26) takes its toll, and once again the initial parameters—this time M_{sq}, M_H, and M_X—must be adjusted to at least $\mathcal{O}\,[\max(\epsilon_\lambda, \epsilon_c)]$. Then, to truly get a maximally symmetric scenario, we choose to obtain a large $\tan\beta$ not by tuning to get a small B but rather by (equivalently) tuning to get a small $m_Z^2 \sim m_A^2/\tan\beta$, which then allows small μ and $m_{1/2}$ relative to the typical low-energy soft-breaking masses and therefore establishes approximate $\mathcal{PQ}$ and $\mathcal{R}$ symmetries even at the electroweak scale:

$$-2m_U^2 = m_Z^2 \sim \frac{\max(\epsilon_\lambda, \epsilon_c)}{\tan\beta} M_S^2 \tag{34}$$

where $M_S \equiv M_H \sim M_{sq}$. After this adjustment, large $\tan\beta$ is automatic:

$$\frac{B}{m_{1/2}} = \frac{1}{\tan\beta}\frac{m_A^2}{\mu\, m_{1/2}} \sim \mathcal{O}(1)\,. \tag{35}$$

And no further adjustment is necessary to suppress $b \to s\gamma$ since μ and $M_{1/2}$ are small. So this scenario requires much less adjusting than the universal case, although more than just the inevitable $1/\tan\beta$ tuning. (The actual tuning needed is in reality slightly more than indicated, due to the squark- and slepton-splittings induced by the D-terms and due to the effects of custodial symmetry violation, as mentioned above; qualitatively, though, the picture we described remains.) It is also predictive: the superspectrum is hierarchical with light charginos and neutralinos but heavy squarks, and since λ_G is large so is the top mass.

We can continuously retreat from this maximally symmetric case by increasing μ or $M_{1/2}$ (or the related parameters), thereby losing $\mathcal{PQ}$ or $\mathcal{R}$, first at low energies when μ or $m_{1/2}$ become comparable to $\epsilon_\lambda^{1/2} M_S$, and then at all energies when μ or $m_{1/2}$ become comparable to M_S itself. Both may be interesting for model-building (for example, if μ is radiatively generated by A terms at the GUT scale, then $\mathcal{PQ}$ rather than $\mathcal{R}$ symmetry should be evident) and for comparison with experiments once the superspectrum is measured. In either case, the tuning is comparable to the maximally symmetric case, though less tuning is needed in Eq. (34) and more in Eq. (35). We will however defer their discussion to our more complete study [3], and instead consider the case where *both* $\mathcal{PQ}$ and $\mathcal{R}$ symmetries are abandoned in favor of a smaller λ_G. We will call this scenario, unimaginatively, the asymmetric case. It alleviates the focusing effect of Eq. (26) since now $\epsilon_\lambda \sim \mathcal{O}(1)$, which in turn allows the initial conditions (specifically the D-terms) to split the Higgs doublets and the other multiplets by a large amount, so now $M_X \sim M_S$. Small λ_G also entails larger (negative) δm_b corrections to correctly predict m_b/m_τ, and to this end we take $\mu \sim M_{1/2} \sim M_{sq} \sim M_H \equiv M_S$. We then see, however, that what we gain by eliminating the focusing effect we lose by restoring the $b \to s\gamma$ problem: since μ and $m_{1/2}$ are no longer small, we are forced to raise the SUSY scale to $\sim \mathcal{O}(\text{TeV})$, and therefore again to tune the initial parameters to make the Z light:

$$-2m_U^2 = m_Z^2 \sim \left(\frac{1}{10} M_S\right)^2 . \tag{36}$$

Since the D-term splitting of the Higgs is now large, $m_A^2 \sim M_S^2$, we find that B_G requires only the typical tuning

$$\frac{B}{m_{1/2}} = \frac{1}{\tan\beta}\frac{m_A^2}{\mu\, m_{1/2}} \sim \frac{1}{\tan\beta}\,. \tag{37}$$

This scenario is comparable in its naturalness (or lack thereof) to the symmetric case. The superspectrum is uniformly heavy rather than hierarchical, and the top is light since λ_G is small.

12 Conclusions

These last two scenarios are qualitatively the best one can hope for from an SO(10) model with Yukawa unification, for two reasons. First, obtaining a large $\tan\beta$ and thereby the top-bottom hierarchy is never natural in the MSSM, due to the LEP bounds on μ and $m_{1/2}$ which force either m_A^2 to be much heavier than the Z or B to be much lighter than the gauginos. Second, the last two scenarios illustrate how, for large λ_G, the *inherent* focusing property of the RG equations in the symmetric limit necessitates a further tuning of the initial parameters, while for small λ_G a similar tuning is mandated by bounds on the rate of $b \to s\gamma$. There are also intermediate scenarios, with only one of the symmetries, but they are apparently no more natural. These various possibilities are considered in more detail elsewhere [3]. The universal case is generically much more tuned than most of these scenarios, as we have shown. Hence departures from universality, although possibly dangerous for the flavor-changing neutral current interactions they can induce in some models, are strongly favored in achieving a large $\tan\beta$.

What is the status of the predictions? Perhaps surprisingly, the top mass is not an independent prediction of Yukawa unification, but rather depends strongly (i.e. non-logarithmically) on a certain ratio of superpartner masses appearing in δm_b. Since the large- and small-λ_G cases are equally fine-tuned, naturalness arguments do not single out any particular top mass within this SO(10) framework. Instead, information about the top mass can be combined with Fig. 1 and Eq. (2) to restrict the mass parameters which can be consistent with Yukawa unification, and to determine a favored superspectrum. It must be admitted that, with Yukawa unification, the attractive conventional picture of *radiative* electroweak symmetry breaking due to a large λ_t but small λ_b is largely lost: the symmetry is either broken radiatively using small custodial isospin-violating effects and extraordinarily fine-tuned initial conditions, or else the Higgs doublets are already split at the GUT scale. Furthermore, we have seen that all the large $\tan\beta$ scenarios are technically unnatural. On the other hand (and to some extent because of the necessary fine-tuning), they are certainly predictive and so will be tested in future accelerators: for example, if the charginos are light but the squarks are heavy, the top should also be heavy; if the $SU(2)_L$-singlet bottom squark or the doublet sleptons were lighter than the other squarks and sleptons, this would be a sign that the D-term splittings were large [10]; and finally, $\tan\beta$ can itself eventually be measured to decisively confirm or dismiss the large $\tan\beta$ hypothesis.

Even before any further experimental input, there are a few theoretical avenues worth pursuing which would make Yukawa unification much more attractive. First, as discussed also by J. Lykken in these proceedings [11], string models which lead to true grand-unified models as their low-energy effective Lagrangians can exhibit higher symmetries in certain sectors of the GUT than in other sectors. In particular, a string theory with an SO(10) gauge symmetry might break to an effective low-energy SU(5) GUT in a stringy way, leaving *all three* Yukawa couplings unified at the Planck scale à la SO(10) but splitting the $\underline{5}_H$ from the $\overline{\underline{5}}_H$ soft masses, namely M_U^2 from M_D^2. Note that unlike the D-term splittings we have considered before, M_U^2 and M_D^2 could now be split without necessarily decreasing some squark or slepton masses. This approach can thus provide more freedom in the choice of boundary conditions—although as we have shown, some features of the RG equations, in particular the focusing effect, are inherent in the equations themselves in certain limits and apply to any boundary conditions, so that freedom could either be the key to a more natural scenario (see the following remark) or could necessitate still more arbitrary fine-tunings at the GUT scale. Second, we have shown that in the symmetric case with large λ_G, the ratio of the soft-breaking masses for the $\underline{10}_H$ and the $\underline{16}_3$ needs to have a certain value if the electroweak symmetry is to break correctly. While such a value is arbitrary in the context of an SO(10) model and therefore apparently requires a fine-tuning, perhaps this value could be explained as a ratio of integral conformal weights in the context of a string theory into which the GUT is embedded. (Notice that this value is favored simply by the unification of Yukawas at some large scale, and does not depend on an SO(10) symmetry.) If such an explanation could be found, then the symmetric large-λ_G case would now be strongly favored: it would only require the minimal $1/\tan\beta$ tuning (and would predict a heavy top!). In fact, if the squarks were to be *experimentally determined* to be heavy while the gauginos were light, then this case would be no more fine-tuned than the conventional small $\tan\beta$ scenario, but would have the advantage of explaining the top-bottom mass hierarchy (through the $\mathcal{PQ}$ and $\mathcal{R}$ symmetries)—which, after all, was historically the motivation for studying Yukawa unification.

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Figure Captions

Fig. 1: The dependence of the *low-energy* values $\lambda_{t,b,\tau}$ and of the ratio $R = \lambda_b/\lambda_\tau$ on the initial condition $\lambda^G_{t,b,\tau} = \lambda_G$, without any threshold corrections. The allowed range of $R_{\rm expt}$ is shown shaded, and the minimal value of λ_G allowed by this range in the absence of any corrections is indicated by the solid dot; lower values of λ_G require finite, negative δm_b (see the text). The corresponding value of λ_t is marked by the shaded dot. The vertical scale on the right indicates the approximate tree-level top mass $\sim 174\lambda_t$ GeV which would result from the values of λ_t on the left vertical scale; for example, the shaded dot predicts a heavy top, above 170 GeV or so.

Fig. 2: The leading (finite) 1-loop MSSM corrections to the bottom quark mass, namely δm_b.

Fig. 3: Our predictions [2] for the pole mass of the top quark, without superheavy corrections and using two qualitatively-different superpartner spectra, specifically $m_{\rm higgsino} \sim \mu = 100\,{\rm GeV}$, $m_{\rm gluino} = 300\,{\rm GeV}$, $m_{\rm wino} = 100\,{\rm GeV}$, $m_{\rm squark} = m_{\rm slepton} = 1000\,{\rm GeV}$ and $m_A = 1000\,{\rm GeV}$ for which the δm_b corrections are small, and $m_{\rm higgsino} \sim \mu = 250\,{\rm GeV}$, $m_{\rm gluino} = 300\,{\rm GeV}$, $m_{\rm wino} = 100\,{\rm GeV}$, $m_{\rm squark} = m_{\rm slepton} = 400\,{\rm GeV}$ and $m_A = 400\,{\rm GeV}$, for which $|\delta m_b/m_b| \sim 0.25$. The upper or lower horizontal axes should be used for these two spectra, respectively. The "cloud" indicates the region where the theory becomes nonperturbative at the GUT scale. Also shown are the estimated allowed mass ranges for the running parameter m_b as extracted in our previous work [2].

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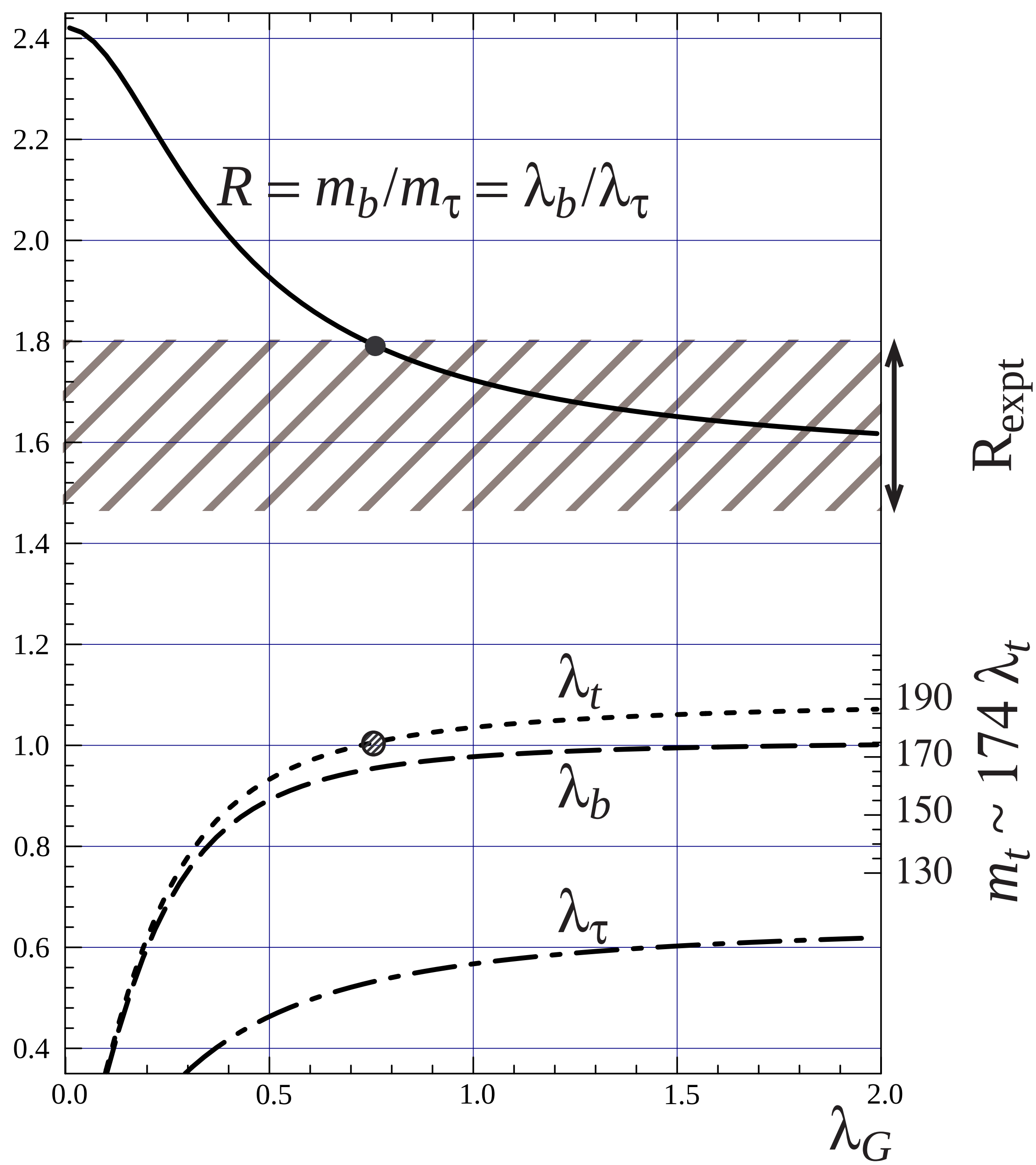

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$R = m_b/m_\tau = \lambda_b/\lambda_\tau$
R_{expt}
λ_t
λ_b
λ_τ
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170
150
130
$m_t \sim 174\,\lambda_t$
λ_G

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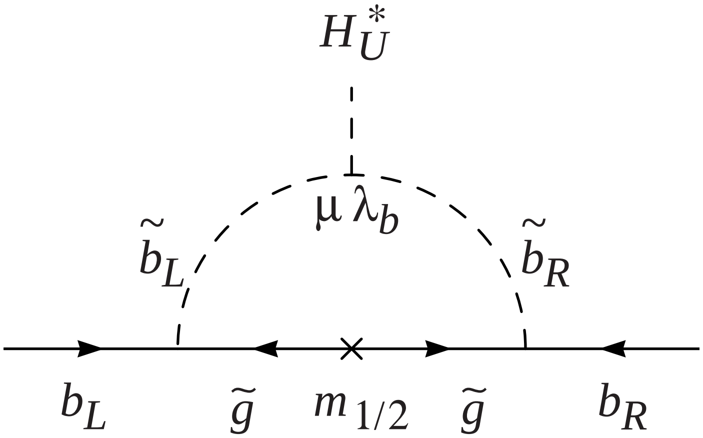
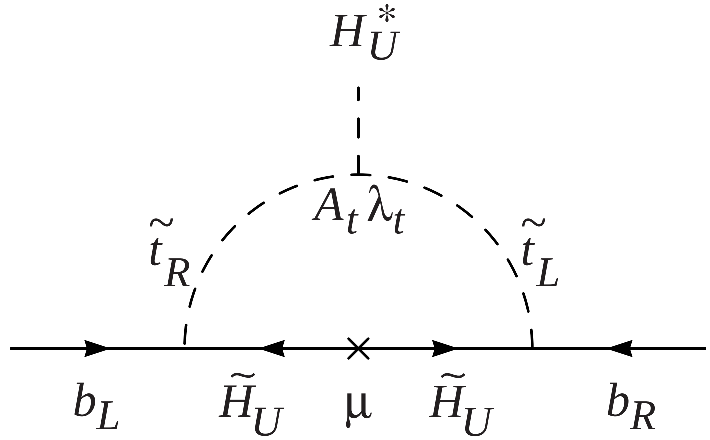


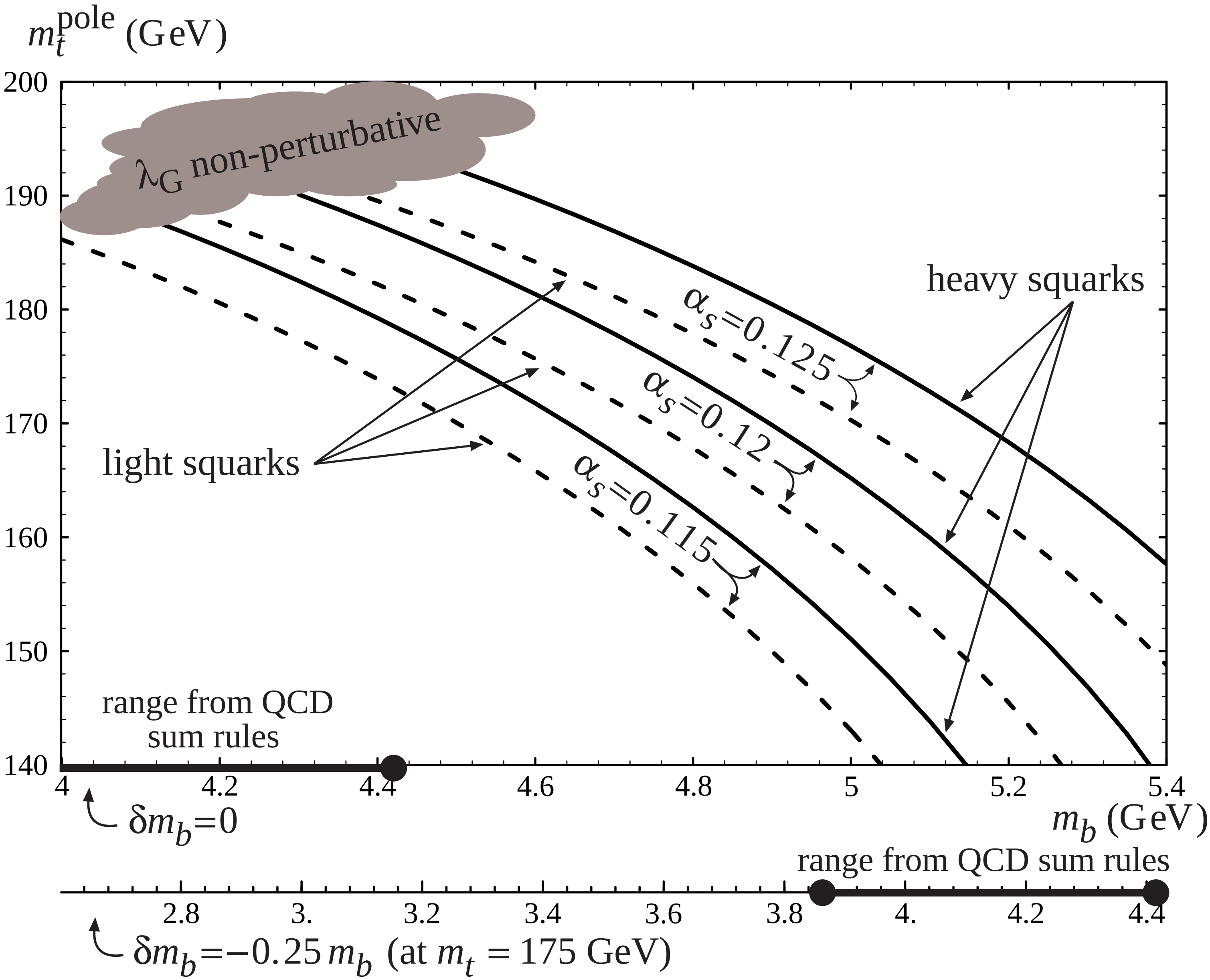
H_U^*
$\mu\,\lambda_b$
$\tilde{b}_L$
$\tilde{b}_R$
b_L
$\tilde{g}$
$m_{1/2}$
$\tilde{g}$
b_R

(a)



H_U^*
$A_t\,\lambda_t$
$\tilde{t}_R$
$\tilde{t}_L$
b_L
$\tilde{H}_U$
μ
$\tilde{H}_U$
b_R

(b)



$m_t^{\rm pole}$ (GeV)
200
190
180
170
160
150
140
λ_G non-perturbative
heavy squarks
light squarks
α_s=0.125
α_s=0.12
α_s=0.115
range from QCD
sum rules
4
4.2
4.4
4.6
4.8
5
5.2
5.4
m_b (GeV)
δm_b=0
range from QCD sum rules
2.8
3.
3.2
3.4
3.6
3.8
4.
4.2
4.4
$\delta m_b = -0.25\, m_b$ (at m_t = 175 GeV)