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Inverse Hierarchy Approach to Fermion Masses

Zurab G. Berezhiani [1]

Sektion Physik, Universität München, D-8000 München 2, Germany

and

Institute of Physics, Georgian Academy of Sciences, SU-380077 Tbilisi, Georgia

Riccardo Rattazzi [2]

Theoretical Physics Group, Lawrence Berkeley Laboratory,

Berkeley, CA-94720, U.S.A.

Abstract

The first fermion family might play a special role in understanding the physics of flavour. This possibility is suggested by the observation that the up-down splitting within quark families increases with the family number: $m_u \sim m_d$, $m_c > m_s$, $m_t \gg m_b$. We construct a model that realizes this feature of the spectrum in a natural way. The inter-family hierarchy is first generated by radiative phenomena in a sector of heavy isosinglet fermions and then transferred to quarks by means of a universal seesaw. A crucial role is played by left-right parity and up-down isotopic symmetry. No family symmetry is introduced. The model implies $m_u/m_d > 0.5$ and the Cabibbo angle is forced to be $\sim \sqrt{m_d/m_s}$. The top quark is naturally in the 100 GeV range, but not too heavy: $m_t < 150$ GeV.

Inspired by the mass matrices obtained in the model for quarks, we suggest an ansatz also including charged leptons. The differences between u-, d- and e-type fermions are simply parametrized by three complex coefficients ϵ_u, ϵ_d and ϵ_e. Additional consistent predictions are obtained: m_s=100-150 MeV and $m_u/m_d < 0.75$.

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[1] Alexander von Humboldt fellow.
E-mail: zurab@hep.physik.uni-muenchen.de, 39967::berezhiani

[2] INFN fellow. E-mail: rattazzi%theorm.hepnet@csa.lbl.gov

1. Introduction

One of the major issues in modern particle physics is to understand the origin of the observed pattern of fermion masses and mixing. In the Standard Model all the flavour structure is determined by *uncalculable* Yukawa couplings. A hypothetical Theory of Flavour should allow a calculation of these couplings, or somehow constrain them. When thinking of such a theory, one should keep in mind the following qualitative features:

i) An inter-family mass hierarchy which is stronger for the up quarks than for the down quarks [1]. The plot in Fig. 1 suggests for the i-th family quark masses

$$m_i^u \sim \epsilon_u^{3-i} m_3^u \qquad m_i^d \sim \epsilon_d^{3-i} m_3^d \tag{1}$$

where $\epsilon_u^{-1} = 200 - 300$ and $\epsilon_d^{-1} = 20 - 30$.

ii) The CKM mixing matrix V is close to the unit matrix, and it has a hierarchy in its off-diagonal entries: $V_{ub} \sim \epsilon_d^2$, $V_{cb} \sim \epsilon_d$ and $V_{us} \sim \epsilon_d^{1/2}$.

The inter-family hierarchy makes the radiative picture of fermion mass generation [2-4] attractive. In particular, the quark mass pattern (1) can be interpreted in terms of a *charge diagonal* radiative cascade. In this view ϵ_u and ϵ_d are the loop expansion parameters respectively in the up and the down sector [4]. Leptons, on the other hand, seem to evade the simple rule $m_i \sim \epsilon^{3-i} m_3$, and this suggests that their masses may be more difficult to understand (see Fig. 1). We therefore focus first on the quark sector.

When discussing quarks, we can also exploit experimental information on the CKM matrix. The relation $V \simeq 1$ implies that the Standard Model Yukawa matrices Γ^u and Γ^d are somehow aligned. This property might hint that some "isotopic" symmetry interchanging u- and d-quarks has played a role in the mass generation phenomenon. (A horizontal family symmetry could also be responsible for the alignment of Γ's. However the idea of an up-down symmetry is simpler.) Let us suppose that such a symmetry indeed exists, and let us call it I_{ud}. In the low energy theory, i.e. the Standard Model, I_{ud} plays the role of the usual custodial symmetry [5]. In the limit of exact I_{ud} it would be $\Gamma_u = \Gamma_d$, so that we would have $V = 1$ and the up and down sectors unsplit. Within the radiative picture the breaking of I_{ud} implies the appearance of *small* mixing angles $V_{ub} \sim \epsilon_d^2$ and $V_{cb} \sim \epsilon_d$.[1)]

As far as the spectrum is concerned, Fig. 1 shows that I_{ud} is more badly broken for the heavier families. By focusing on the first family, we don't see a big violation of I_{ud}:

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1) However, also V_{us} has to be $O(\epsilon_d)$. This is a generic problem of the *direct* radiative scenarios [3-4]. As it was shown in Ref. [6], the correct value $V_{us} \approx 0.22$ implies a choice of parameters which spoils the validity of perturbation theory.

$m_u/m_d = O(1)$. What we consider interesting is the case in which I_{ud} becomes exact at very high energies, namely the case of spontaneous or soft breakdown. With this picture in mind, it is suggestive to think that the masses of u and d are somehow related to an energy scale Λ_1 at which I_{ud} is still good, while the masses of the second and third families are respectively related to lower scales Λ_2, Λ_3 at which I_{ud} is no longer as good. Thus, the first family seems to play a special role in mass generation. This is directly opposite to the common radiative picture [2-4], in which the third family (i.e. the heaviest) is the starting point. However, by taking the top and bottom masses as the *seeds* of mass generation, it is difficult to understand their large splitting.[2)] On the other hand, if we could start from u and d, we would have better chance to understand t-b splitting. Suppose that u and d are indeed the starting point, and that an expression like eq. (1) holds for $1/m_i$ instead of m_i, namely

$$1/m_i^{u,d} \sim \epsilon_{u,d}^{i-1}/m. \tag{2}$$

Then we have $m_u \sim m_d \sim m$ and $m_t/m_b \sim (\epsilon_d/\epsilon_u)^2 \sim (m_c/m_s)^2 \gg 1$. In this way, the splitting between up and down quark masses in Fig. 1 is understood by means of one parameter $(\epsilon_d/\epsilon_u) > 1$. We call the above formula for $1/m_i$ the *inverse hierarchy pattern.*

In this paper we explore the idea of inverse hierarchy for fermion masses. In Section 2, we discuss an illustrative model for the quark sector. The model is a particular case of the class of models discussed in Ref. [7]. These models naturally avoid the key problems of the previous models of radiative mass generation (see footnotes[1,2)]). A set of isosinglet heavy quarks, Q-fermions, in a one to one correspondence with the ordinary ones (q's), is introduced. The mass matrices of the q's $\hat{M}_u$ and $\hat{M}_d$ are induced by a seesaw mixing with the Q's [8]. In terms of the mass matrices $\hat{M}_{U,D}$ of the Q's we have essentially the inverse proportionality[3)] $\hat{M}_u \propto \hat{M}_U^{-1}$, $\hat{M}_d \propto \hat{M}_D^{-1}$. Thus the smallest masses m_u and m_d are indeed related to the highest scale: $m_u^{-1} \propto M_U$ and $m_d^{-1} \propto M_D$, where M_U and M_D are the largest eigenvalues respectively in $\hat{M}_U$ and $\hat{M}_D$. These matrices are generated by a radiative mechanism analogous to that in Ref. [4]: the largest eigenvalues $M_{U,D}$ appear at tree level, while the other ones are induced by charge diagonal loop effects. Moreover,

[2)] In Ref. [3-4] the large $t-b$ splitting is introduced at tree level by hand. A difficulty of *charge diagonal* radiative models is also the appearance of flavour-changing neutral currents (FCNC), generally contradicting the experimental bounds [4].

[3)] The seesaw mechanism [9], universally extended to quarks and charged leptons was also explored in the papers in Ref. [10,11]. The inverse hierarchy, however, corresponds to the spirit of the original paper [8], where it was first suggested, though in the context of horizontal symmetry.

the relation $M_U \sim M_D$ arises naturally as a consequence of an up-down symmetry. Then $m_u \sim m_d$, so that the pattern of eq. (2) is realized.

In the model we discuss below, the field content and the I_{ud} symmetry imply strictly $M_U = M_D$. The model also has P_{LR} and CP symmetries. These, together with I_{ud}, are softly (or spontaneously) broken in the scalar potential. The effects of this breaking are reflected in the fermion sector via loop corrections *only*. As a consequence, the mass matrices are more constrained than those in Ref. [7]. In particular, the splitting between u and d is related to the enhancement of the Cabibbo angle above its natural radiative value $O(\epsilon_d)$. The consequences, including formulae for the top quark mass and the Cabibbo angle, are discussed in Section 3.

The quark mass matrices obtained in the model are a very attractive realization of the inverse hierarchy. Inspired by the form of those matrices, in Section 4 we simply postulate an inverse hierarchy ansatz, which includes charged leptons. It turns out that the first family (u, d, e) really plays the role of a *mass unification point*, with its splittings understood by the same mechanism that enhances V_{us}. This ansatz provides remarkable predictions for the light quark masses, consistent with the present knowledge on these quantities.

Finally, in Section 5 we discuss our results.

2. A model

Let us consider the simple left-right symmetric model, based on the gauge group $G_{LR} = [SU(2)_L \times U(1)_L] \times [SU(2)_R \times U(1)_R] \times U(1)_{\overline{B}-\overline{L}}$, suggested in [7]. The left- and right-handed components of usual quarks $q_i = (u_i,\, d_i)$ and their heavy partners $Q_i = U_i,\, D_i$ are taken in the following representations

$$
\begin{array}{ll}
q_{Li} \sim (1/2, 0, 0, 0, 1/3) & q_{Ri} \sim (0, 0, 1/2, 0, 1/3) \\
U_{Li} \sim (0, 1, 0, 0, 1/3) & U_{Ri} \sim (0, 0, 0, 1, 1/3) \\
D_{Li} \sim (0, -1, 0, 0, 1/3) & D_{Ri} \sim (0, 0, 0, -1, 1/3)
\end{array}
\tag{3}
$$

where $i = 1, 2, 3$ is the family index (the indices of the colour $SU(3)_c$ are omitted). We also introduce an additional family of quarks [4)]

$$
\begin{array}{ll}
p_L \sim (0, -1/2, 0, 3/2, 1/3) & p_R \sim (0, 3/2, 0, -1/2, 1/3) \\
n_L \sim (0, 1/2, 0, -3/2, 1/3) & n_R \sim (0, -3/2, 0, 1/2, 1/3).
\end{array}
\tag{4}
$$

4) Notice that, by adding the obvious lepton fields (with $\overline{B} - \overline{L} = -1$) to the quarks of eqs. (3-4), the theory is free of gauge anomalies.

The scalar sector of the theory is given by

$$
\begin{array}{rlrl}
H_L \sim & (1/2,0,0,1,0) & H_R \sim & (0,1,1/2,0,0) \\
T_{uL} \sim & (0,-2,0,0,-2/3) & T_{uR} \sim & (0,0,0,-2,-2/3) \\
T_{dL} \sim & (0,2,0,0,-2/3) & T_{dR} \sim & (0,0,0,2,-2/3) \\
\phi \sim & (0,1/2,0,-1/2,0) & \Phi \sim & (0,2,0,-2,0) \\
\Omega_u \sim & (0,1/2,0,1/2,-1) & \Omega_d \sim & (0,-1/2,0,-1/2,-1)
\end{array}
\tag{5}
$$

where the T-scalars are colour triplets or sextets. We impose CP, P_{LR} and I_{ud} discrete symmetries. The left-right symmetry P_{LR} [12], which is essentially parity, and CP act in the usual way. The "up-down" symmetry I_{ud} is defined by

$$
\begin{array}{lllll}
U_{L,R} \leftrightarrow D_{L,R}, & p_{L,R} \leftrightarrow n_{L,R}, & A^\mu_{L,R} \to -A^\mu_{L,R}, & & \\
H_{L,R} \leftrightarrow \tilde{H}_{L,R}, & T^u_{L,R} \leftrightarrow T^d_{L,R}, & \phi \leftrightarrow \phi^*, & \Phi \leftrightarrow \Phi^*, & \Omega_u \leftrightarrow \Omega_d
\end{array}
\tag{6}
$$

where $A^\mu_{L,R}$ are the gauge bosons of $U(1)_{L,R}$. Then the most general Yukawa interactions, consistent with gauge invariance, I_{ud}, P_{LR} and CP are

$$
\begin{aligned}
\mathcal{L}_1 &= \Gamma^{ij}\big\{\bar{q}_{Li}U_{Rj}\tilde{H}_L + \bar{q}_{Li}D_{Rj}H_L + (L\leftrightarrow R)\big\} + \text{h.c.} \\
\mathcal{L}_2 &= \frac{\lambda_{ij}}{2}\big\{T_{uL}U_{Li}CU_{Lj} + T_{dL}D_{Li}CD_{Lj} + (L\leftrightarrow R)\big\} + \text{h.c.} \\
\mathcal{L}_3 &= \big\{h_i(\phi^*\bar{U}_{Li}p_R + \phi\bar{D}_{Li}n_R) + h(\Phi^*\bar{p}_Lp_R + \Phi\bar{n}_Ln_R) + (L\leftrightarrow R)\big\} + \text{h.c.}
\end{aligned}
\tag{7}
$$

where C is the charge conjugation matrix. The coupling constants h, h_i, λ_{ij}, Γ_{ij} $(i,j = 1,2,3)$ are real, due to CP-invariance. In what follows we do not make any particular assumption on their flavour structure. We only suppose that they are typically $O(1)$, just like the gauge coupling constants. Notice that λ is antisymmetric (symmetric) when the T-scalars are colour triplets (sextets). Without loss of generality, by a suitable redefinition of the fermion basis, we can always take

$$
h_{2,3} = 0, \qquad \lambda_{13} = 0, \qquad \Gamma_{ij} = 0 \;\text{ if }\; i<j \;. \tag{8}
$$

We assume that the discrete symmetries CP, P_{LR} and I_{ud} are *softly* broken by bilinear and trilinear terms in the Higgs potential [5]. The trilinear terms are given by

$$
\mathcal{L}_4 = \Lambda_u T^*_{uL}T_{uR}\Phi + \Lambda_d T^*_{dL}T_{dR}\Phi^* + \text{h.c.} \tag{9}
$$

[5] Actually, these symmetries can be broken spontaneously by suitable scalars [7].

where $\Lambda_{u,d}$ are mass dimensional coupling constants. We suppose that these couplings are complex, and also that $\Lambda_u \neq \Lambda_d$. In this way, CP, P_{LR} and I_{ud} are broken.

The v.e.v.'s $\langle\Phi\rangle = v_\Phi$, $\langle\phi\rangle = v_\phi$ and $\langle\Omega_u\rangle$ break the $U(1)_L \times U(1)_R \times U(1)_{\overline{B}-\overline{L}}$ symmetry down to the usual $U(1)_{B-L}$: $B-L = Y_L + Y_R + \overline{B} - \overline{L}$. (Notice that we must have $\langle\Omega_d\rangle = 0$.) Then, interactions $\mathcal{L}_3$ in eq. (7) give rise to mass terms for p and n, and also for the Q-fermions of the "first family" U_1 and D_1. It is interesting to consider the limiting case $v_\Phi \gg v_\phi$. In this case, the tree-level mass eigenvalues are given by the seesaw formula: $M_p = M_n \simeq hv_\Phi$ and $M_{U_1,D_1} = M \simeq h_1^2 v_\phi^2/hv_\Phi$. Because of the relation $M \ll M_{p,n}$, the mass matrices obtained after including radiative effects depend, in general, on a smaller number of parameters. In fact, we shall see below that a *soft* hierarchy $hv_\Phi/h_1 v_\phi \sim 3$ is already enough for our purposes.

The coloured scalars T_{uL},T_{uR} and T_{dL},T_{dR} get mixed due to interaction terms in eq. (9). At this point the interactions in $\mathcal{L}_2$ trigger the radiative mass generation chains $U_1 \to U_2 \to U_3$ and $D_1 \to D_2 \to D_3$ (see Fig. 2). The two-loop corrected mass matrices for the Q-fermions can be written as

$$\begin{aligned}\hat{M}_{U,D} =& M\Big\{\hat{P}_1 + \epsilon_{u,d}\ \lambda^{\mathrm{T}}\,\hat{P}_1\lambda + C_{u,d}|\epsilon_{u,d}|^2\ \lambda^{\mathrm{T}2}\,\hat{P}_1\,\lambda^2 \\ &+ \epsilon_{u,d}\lambda^{\mathrm{T}}\big(\beta^L_{u,d}\lambda^{\mathrm{T}2} + \alpha^L_{u,d}\Gamma^{\mathrm{T}}\Gamma\big)\hat{P}_1\lambda + \epsilon_{u,d}\lambda^{\mathrm{T}}\hat{P}_1\big(\beta^R_{u,d}\lambda^2 + \alpha^R_{u,d}\Gamma^{\mathrm{T}}\Gamma\big)\lambda\Big\}\end{aligned} \tag{10}$$

where $\hat{P}_1 = \mathrm{diag}(1,0,0)$ is the tree-level term. In terms of the mixing angle $\xi_{u,d}$ and mass eigenvalues $M_+^{u,d}$, $M_-^{u,d}$ of the scalars $T_L^{u,d}$-$T_R^{u,d}$, the expansion parameters are

$$\begin{aligned}\epsilon_{u,d} &= C_T\frac{e^{i\omega_{u,d}}}{16\pi^2}\sin(2\xi_{u,d})\ln r_{u,d}, \qquad\qquad r_{u,d} = (M_+^{u,d}/M_-^{u,d})^2, \\ C_{u,d} &= \frac{1}{2}\big(1 + H(r_{u,d})\big)\ .\end{aligned} \tag{11}$$

Here $\omega_{u,d}$ are the arguments of the mixing terms $\Lambda_{u,d}$ in eq. (9), while H is a real function given in the Appendix. C_T is a colour factor equal to 1/2 for a triplet T and to 1 for a sextet T. The first three addenda in eq. (10) represent respectively the contributions of graphs (a), (b) and (c) of Fig. 2. They are rank 1 matrices determining the eigenvalues of $\hat{M}$ to be $O(M)$, $O(\epsilon M)$ and $O(\epsilon^2 M)$. The terms proportional to $\beta_{u,d}$ and $\alpha_{u,d}$ in eq. (10) arise from the graphs in Figs. 2d-e. Apart from pieces which are formally 3-loop, i.e. $O(\epsilon\alpha^2, \epsilon\beta^2, \epsilon\alpha\beta)$, these terms simply determine a multiplicative redefinition of the 1-loop contribution (Fig. 2b). (We remind that α's and β's are loop coefficients $\sim 1/16\pi^2$.) This means that in 2-loop order their effect on the eigenvalues is only multiplicative, i.e. $M_i \to M_i(1 + O(\alpha,\beta))$ [3]. For our purposes, these terms are therefore negligible.

Some comments on eqs. (10-11) are in order. Eq. (11) is valid in the regime $M^2 \ll M_+^{u,d^2}, M_-^{u,d^2} \ll M_p^2$. In this regime the loop-diagrams of Fig. 2b-c are dominated by loop momenta between $M_-^{u,d}$ and $M_+^{u,d}$. Thus, the result is completely determined by the ratios $r_{u,d}$, and the predictivity is increased. The choice of the intermediate range for $M_\pm^{u,d}$ is also motivated by the fact that both for $M_\pm^{u,d} < M$ and for $M_\pm^{u,d} > M_p$ the loop integrals are suppressed. In the first case the suppression factor is given by $\sim (M_+^{u,d}/M)^2$, in the second by $\sim (M_p/M_-^{u,d})^2$. (In fact M_p acts as a cut-off of the mass insertions in Fig. 2 [4].) Notice also that $(M/M_p)^2 \simeq (h_1 v_\phi / h v_\Phi)^4 \ll 1$ is already implied by our assumption $v_\Phi > v_\phi$. For instance, $h_1 v_\Phi / h v_\Phi \sim 3$ already gives $(M/M_p)^2 \sim 10^{-2}$.

The function $H(r) = H(1/r)$ has the maximum at $r = 1$ and goes to zero monotonically when $r \to \infty$ (see Appendix). However, in a reasonable range ($1 < r < 10$) it is $H(r) \simeq 0.3$ with good accuracy. Thus, in what follows we shall set $C_{u,d} = C \simeq 0.65$. Notice that the two loop term proportional to $C_{u,d}$ in eq. (10) is real. This is because the mass insertions on the two scalar lines in Fig. 2c are complex conjugate of each other.

As a final comment, notice that, in the basis of eq. (8), we have

$$\hat{M} \propto \begin{pmatrix} O(1) & O(\epsilon) & O(\epsilon^2) \\ O(\epsilon) & O(\epsilon) & O(\epsilon^2) \\ O(\epsilon^2) & O(\epsilon^2) & O(\epsilon^2) \end{pmatrix} \tag{12}$$

Then, in lowest order, the eigenvalues are given by the diagonal entries of $\hat{M}$, and their ratios are $O(1) : O(\epsilon) : O(\epsilon^2)$.

3. The quark mass matrices

We now discuss the masses of the ordinary quarks q. The v.e.v's $\langle H_L \rangle = (0, v_L)$ and $\langle H_L \rangle = (0, v_R)$, with $v_R \gg v_L = 174$ GeV, break the intermediate $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry down to $U(1)_{\rm em}$, and at the same time mix the q's to the Q's via the interactions $\mathcal{L}_1$ in eq. (7). The mass matrix of the u-type quarks is written in block form as

$$(\bar{u}, \bar{U})_L \begin{pmatrix} 0 & v_L \Gamma \\ v_R \Gamma^{\rm T} & \hat{M}_U \end{pmatrix} \begin{pmatrix} u \\ U \end{pmatrix}_R \tag{13}$$

and analogously for the d-type ones. When $\hat{M}_{U,D} \gg v_R$, the resulting mass matrix $\hat{M}_{u,d}$ for the ordinary quarks is given by the seesaw formula [8-10]

$$\hat{M}_{u,d} = v_L v_R \Gamma \hat{M}_{U,D}^{-1} \Gamma^{\rm T} \tag{14a}$$

In this way the inverse proportionality of eq. (2) is realized [7,8]. The seesaw limit $M_{U,D} \gg v_R$ is certainly very good for light quarks, since their masses must be much smaller than v_L. However, since $m_t = O(v_L)$, we expect the mass of its Q-partner M_{T} to be of the order of v_R (remember that the Γ^{ij} are considered to be $O(1)$). Thus, to evaluate m_t, we need the mass matrix without the restriction $\hat{M}_U \gg v_R$. This is given by

$$(\hat{M}_{u,d}{}^{\dagger}\hat{M}_{u,d})^{-1} = v_L^{-2}\Gamma^{-1}\left\{v_R^{-2}\hat{M}_{U,D}{}^{\dagger}(\Gamma\Gamma^{\mathrm{T}})^{-1}\hat{M}_{U,D} \;+\; 1\right\}(\Gamma^{\mathrm{T}})^{-1}. \tag{14b}$$

Notice that, when $v_R \gg \hat{M}_{U,D}$, this equation gives the obvious result $\hat{M}_{u,d} = v_L\Gamma$. On the other hand, when $v_R \ll \hat{M}_{U,D}$, eq. (14b) reduces to the seesaw formula.

The v.e.v. $v_R \sim M_{\mathrm{T}}$ plays the role of the Flavour scale Λ_F. Below the scale v_R the effective theory is the minimal standard model. In fact $v_R\Gamma\hat{M}_{U,D}^{-1}\,\Gamma^{\mathrm{T}} = \hat{M}_{u,d}/v_L$ are the standard model Yukawa matrices $\Gamma_{u,d}$ arising at the decoupling of the heavy Q states. The Q-fermion masses are such that $M_{\mathrm{T}} \ll M_B \ll ... \ll M_U = M_D = M$. Then the ratio M/v_R is given by $v_L/m_u \sim 10^5$. Obviously, FCNC's are well controlled in such a theory [8] due to natural flavour conservation within the standard model [13]. Flavour changing effects are suppressed by inverse powers of the Q masses. Phenomenologically, the most stringent constraints come from $K^0 - \bar{K}^0$ mixing and $K_L \to \mu^+\mu^-$. It turns out that the typical scale of suppression for these processes is given by M. This is readily seen in basis (8): the light quarks u, d mix only with the heaviest Q-fermions U, D. Consistency with the data is obtained for $M > 100\,\mathrm{TeV}$. Since $M/v_R \sim 10^5$, we have that $v_R \sim 1 - 10$ TeV is already consistent with the experimental data. Therefore, it is in principle possible to have the W_R bosons and the lightest Q-fermion, T, in the TeV range, with implications at SSC/LHC.

It is useful to first study $\hat{M}_{u,d}$ in the seesaw limit (14a). The exact formula (14b) will only be relevant to evaluate m_t. Once again, the inverse matrices are easier to analyse. Neglecting α and β terms, as explained above, eq. (10) gives

$$(\hat{M}_{u,d})^{-1} = m^{-1}\left\{\hat{P}_1 + \epsilon_{u,d}\;\gamma^{\mathrm{T}}\,\hat{P}_1\,\gamma + C|\epsilon_{u,d}|^2\;\gamma^{\mathrm{T}2}\,\hat{P}_1\,\gamma^2 + \cdots\right\} \tag{15}$$

where $\gamma = \Gamma\lambda\Gamma^{-1}$ and $m = \Gamma_{11}^2 v_L v_R/M$. We also used the relation $(\Gamma^{\mathrm{T}})^{-1}\hat{P}_1\Gamma^{-1} = \Gamma_{11}^{-2}\hat{P}_1$, true in the basis of eq. (8). Notice that, in general, γ is no longer symmetric or antisymmetric. However $\mathrm{Tr}\,\gamma = \mathrm{Tr}\,\lambda$, Det$\gamma$=Det$\lambda$ and, because of eq. (8), $\gamma_{13} = \lambda_{13} = 0$. Eq. (15) is then written in the same form as eq. (12), which is easily diagonalized to lowest order in ϵ.

There is, however, a subtlety involving the Cabibbo angle. Let us consider eq. (15) restricted to the first two families at $O(\epsilon)$. We have

$$(\hat{M})^{-1} = m^{-1}\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \epsilon\begin{pmatrix} {\gamma_{11}}^2 & \gamma_{11}\gamma_{12} \\ \gamma_{11}\gamma_{12} & {\gamma_{12}}^2 \end{pmatrix}\right\}. \tag{16}$$

This gives, in lowest order, $\sin\theta_{\rm C} \simeq \epsilon_d\gamma_{11}\gamma_{12}$ and $d/s \simeq \epsilon_d{\gamma_{12}}^2$. These relations imply $\epsilon_d{\gamma_{11}}^2 \simeq \sin^2\theta_{\rm C}(s/d) \simeq 1$, which looks like trouble for perturbation theory [6-7]. However, $\gamma_{11} = \lambda_{12}\Gamma_{21}/\Gamma_{22}$ is not a coupling constant: its value can be enhanced by the ratio Γ_{21}/Γ_{22}, without trouble for perturbativity [7]. This is an advantage of the seesaw mechanism: the "sandwiching" between Γ's in eq. (14a) allows the mass matrix in eq. (15) to deviate a little bit from the *rigid* radiative form of eq. (12). In particular, when Γ_{21} is larger than Γ_{11} and Γ_{22}, the 1-2 entry gets enhanced. This effect is reflected in eq. (15-16) by the fact that γ_{11} is *large*. Notice that, by supposing $\epsilon_d{\lambda_{12}}^2 \sim d/s = 1/20$, we only need $\Gamma_{21}/\Gamma_{22} = 4-5$.

To accommodate a large Cabibbo angle, the matrix (15) must be diagonalized considering that $\epsilon_d{\gamma_{11}}^2 = O(1)$. With an obvious notation, we indicate by $u, d, ...$ the quark mass eigenvalues at the flavour scale Λ_F. Then we get

$$\begin{array}{lll} (m/u) = |1+\epsilon_u{\gamma_{11}}^2| & (m/c) = \dfrac{|\epsilon_u|{\gamma_{12}}^2}{|1+\epsilon_u{\gamma_{11}}^2|} & (m/t) = C|\epsilon_u|^2{\gamma_{12}}^2{\gamma_{23}}^2 \\ (m/d) = |1+\epsilon_d{\gamma_{11}}^2| & (m/s) = \dfrac{|\epsilon_d|{\gamma_{12}}^2}{|1+\epsilon_d{\gamma_{11}}^2|} & (m/b) = C|\epsilon_d|^2{\gamma_{12}}^2{\gamma_{23}}^2 \end{array} \tag{17a}$$

and the Cabibbo angle is

$$\begin{array}{l} V_{us} = \gamma_{11}\gamma_{12}\left|(\dfrac{d}{m})\epsilon_d e^{i\delta_d} - (\dfrac{u}{m})\epsilon_u e^{i\delta_u}\right| = \gamma_{11}\sqrt{|\epsilon_d|}\sqrt{\dfrac{d}{s}}\left|1-\dfrac{s}{c}e^{i(\omega_u+\delta_u-\omega_d-\delta_d)}\right| \\ \delta_{u,d} = \arg(1+\epsilon^*_{u,d}{\gamma_{11}}^2). \end{array} \tag{17b}$$

By dropping terms proportional to $\epsilon_{u,d}{\gamma_{11}}^2$ in the above equations, we would get the naive lowest order result. (Notice that we have kept also $\epsilon_u{\gamma_{11}}^2$ terms. However, we shall see that $\epsilon_u \ll \epsilon_d$, so that those terms are not relevant). Notice also that the third family masses are simply given by the 3-3 entry in eq. (15), i.e. they do not get relevant contributions from off-diagonal terms. This is because their mixings V_{cb} and V_{ub} must be of the natural *radiative* size, namely $V_{cb} \sim \epsilon_d \sim d/s, s/b$ and $V_{ub} \sim \epsilon_d^2 \sim d/b$.

The following comment is in order. In contrast to the quantities in eqs. (17), V_{cb} and V_{ub} get lowest order contributions which are not expressed in terms of ϵ. One of these comes from Fig. 2d-e: these graphs contribute in lowest order to V_{cb} and V_{ub}, but not to the quantities in eq. (17). In particular, V_{cb} gets contributions $O(\alpha,\beta)$, whereas V_{ub} gets $O(\epsilon\alpha,\epsilon\beta)$. This is consistent with the radiative picture of mixings, since α,β are loop parameters $\sim \epsilon$.

Another contribution comes from the renormalization of Γ's. These effects are given at 1-loop by triangle graphs involving three Yukawa vertices in $\mathcal{L}_1$ in eq. (7). The mass splittings between up and down Q-fermions determine this renormalization to differ by a finite amount in the two sectors. This is an additional source of mixing. It is remarkable, however, that a contribution appears at 1-loop *only* in the mixing between the second and third family, namely in V_{cb}. This is a consequence of the fact that the first family (u, d) is only connected to the first heavy family (U, D), which is unsplit in lowest order. Then the vertex graphs involving u and d are equal and do not contribute to the mixing in lowest order: the relation $M_U = M_D$ protects V_{us} and V_{ub} from 1-loop corrections and thereby maintains V_{ub} as a two loop effect.

The above observation selects the present model from the wide class discussed in Ref. [7], not only as a predictive one (see below), but also as the *only* one in which the mixing angles V_{cb} and V_{ub} follow the natural *radiative* pattern: $V_{cb} \sim$ 1-loop, $V_{ub} \sim$ 2-loop.[6)]

Let us now discuss the implications of eqs. (17). We have seen that V_{us} is enhanced by seesaw effects above the *radiative* value $O(\epsilon_d)$. Eqs. (17) show the connection between the *large* value of the Cabibbo angle and the u-d splitting. Neglecting terms of order $|\epsilon_u/\epsilon_d|$, we get

$$V_{us} \simeq \sqrt{\frac{d}{s}\left|1 - \frac{u}{d}e^{i\delta_d}\right|} \tag{18}$$

Thus, without any hypothesis on δ_d, we have V_{us} in the right range: $|V_{us}| \sim \epsilon_d^{1/2}$. For instance, for $u/d \leq 0.6$ and $s/d = 20$ [1], we obtain $0.14 \leq |V_{us}| \leq 0.26$. Notice that the experimental value of $|V_{us}|$ requires large values $\delta_d \sim 1$. On the other hand, δ_d contributes appreciably to the CP phase in the CKM matrix, even though there are other independent contributions. These are related to the effects on V_{ub} and V_{cb} that we discussed above. However, barring accidental cancellations, we can conclude that the model favours a large CP-phase, in accord with experiments [14].

Let us now discuss the mass eigenvalues. From the masses of the first two families in eq. (17a) we get

$$\left|\frac{\epsilon_u}{\epsilon_d}\right| = \frac{sd}{cu}. \tag{19}$$

On the other hand $b/t = |\epsilon_u/\epsilon_d|^2$, so that we have a mass formula

$$\frac{t}{b} = \left(\frac{uc}{ds}\right)^2. \tag{20}$$

6) The relation $M_U = M_D$ does not generally hold in the models of Ref. [7]. It was considered to be broken at tree level. Consequently, in order to have a small enough V_{ub}, some Yukawa coupling in $\mathcal{L}_1$ (namely Γ_{31}) must be taken about one order of magnitude smaller then the others. In Ref. [7] the role of the renormalization of Γ's had been overlooked.

This equation, valid in the seesaw limit of eq. (15), is an appealing expression for the top mass. However, even when exact, eq. (20) does not translate into a sharp top mass prediction, because of our poor experimental knowledge of $R = (uc/ds)^2$. By using the correct mass matrix of eq. (14), eq. (20) is reduced to an inequality

$$\left(1/\lambda_t\right)^2 = \left(1/\lambda_b R\right)^2 + \left(1/\Gamma_{33}\right)^2 \geq \left(1/\lambda_b R\right)^2 \tag{21}$$

where $\lambda_{t,b} = m_{t,b}/v_L$ are the standard model Yukawa couplings. Eq. (21) is valid at the "Flavour scale" v_R, and, to discuss its implications, the running of masses needs to be considered. For our purposes, we only need to consider the combined effects of QCD and of the top Yukawa coupling [15] (electroweak gauge couplings modify our conclusions by less than 5%). In what follows we assume $\alpha_s(M_Z) = 0.11$ and use the estimate of the QCD running of masses given in ref. [16] at 2-loop accuracy. There are two implications from eq. (21).

i) An upper bound on the top mass. This value is determined by the maximal allowed values for R and Γ_{33}. Taking $m_u/m_d \leq 0.8$ and $m_c/m_s \leq 10.7$ from the values given in Ref. [1,17] we have $R < R_{\max} = 74$ (Notice that R is not affected by the running of masses, when electroweak couplings are neglected [15].) On the other hand, Γ_{33} is bounded by requiring the perturbativity of the model: $\Gamma_{33} < 0.5 - 1$. This upper bound requires some explanation. From the definition of γ, and from the basis choice eq. (8), we have[7)] $\Gamma_{21}/\Gamma_{33} = (\epsilon_d \gamma_{11} \gamma_{23})/(\epsilon_d \lambda_{12} \lambda_{23})$. Then, from the expressions for d/s and $\sin\theta_{\mathrm{C}}$ in eq. (17), we have $\epsilon_d {\gamma_{11}}^2 \simeq 1$, while from the b-mass we have $\epsilon_d {\gamma_{23}}^2 \sim 0.06$. On the other hand, $\epsilon_d \lambda_{12} \lambda_{23}$ is a true loop expansion parameter in eq. (10), so that we demand that it be smaller than $0.1 - 0.05 \sim d/s$. Putting all these constraints together we get $\Gamma_{21}/\Gamma_{33} > 2-4$. Finally, by demanding the reasonable perturbative upper bound $\Gamma_{21} < 2$ we obtain the bound $\Gamma_{33} \lesssim 0.5 - 1.0$.

The results are shown in Table 1a for different values of Λ_F. Notice that for $\Lambda_F = 10^{12}$ GeV the heaviest states of the model lay just below the Planck mass. However, for the sake of demonstration we also kept $\Lambda_F = 10^{16}$ GeV. As we see the top quark cannot reasonably be heavier than 150 GeV. For instance, for $\Lambda_F = 10^8$ GeV, in order to push m_t to 170 GeV we should take $\Gamma_{33} \sim 2.2$. Such a value, as discussed above, would lead to a large $\Gamma_{21} > 4$ and the loss of perturbative control. In fact, at the same value of Λ_F, even in the limiting seesaw case, i.e. $\Gamma_{33} \to \infty$ in eq. (21), the top quark cannot weigh more than 176 GeV.

[7)] Actually this relation is true only in the case of a triplet T. However, the consequences in the case of a sextet do not differ appreciably.

ii) A lower bound on $\sqrt{R}$. This is a consequence of the CDF bound $m_t > 91\,\mathrm{GeV}$ [18] (see Table 1b). Notice that the lower bound on $\sqrt{R} = (m_u m_c / m_d m_s)$ is given, in practice, by $R > [m_t/m_b(m_t)] \gtrsim 30 = R_{\min}$. Then, by using the maximal value $c/s = 10.7$ [1], we get

$$u/d > \sqrt{R}_{\min}(s/c) > 0.5 \tag{22}$$

The determination of u/d in chiral perturbation theory is still controversial, essentially because the axial $U(1)_A$ is broken by the anomaly and instanton effects may mimic a substantial portion of the u quark mass [19]. The range of u/d allowed in our model is consistent with the values reported by Leutwyler in Ref. [17]. On the other hand, Donoghue and Wyler [20] give the estimate $u/d = 0.3 \pm 0.1$. If this last estimate turns out to be the correct one, then our model is ruled out, unless $c/s \gtrsim 14$. By taking $m_c(1\,\mathrm{GeV}) \leq 1.4\,\mathrm{GeV}$ [1], this bound translates into $m_s \lesssim 100\,\mathrm{MeV}$, which seems too small a value for m_s. (Even though it is very difficult to determine the absolute scale of the strange quark mass [1]).

4. Inverse Hierarchy Ansatz with charged leptons

Lepton masses have a *mixed* behaviour, $m_\mu/m_e \sim \epsilon_u$ and $m_\tau/m_\mu \sim \epsilon_d$, which seems hard to explain by means of only one expansion parameter (see Fig. 1). On the other hand, we saw that the large value of the Cabibbo angle requires a deviation from the genuine radiative pattern in the quark sector. In fact the off-diagonal entries in the mass matrix affect sizably the masses of the first two families. Here we want to show that the same mechanism allows to explain charged lepton masses by means of one complex expansion parameter ϵ_e.

We simply postulate the inverse hierarchy form of eq. (15) for all mass matrices[8)]

$$(\hat{M}_k)^{-1} = m^{-1}\Big\{\hat{P}_1 + \epsilon_k\ \gamma^{\mathrm{T}}\,\hat{P}_1\,\gamma + |\epsilon_k|^2\ \gamma^{\mathrm{T}2}\,\hat{P}_1\,\gamma^2\Big\} \tag{23}$$

where $k = u, d, e$. This ansatz is simple: the difference between u-, d- and e-type fermions is parametrized by the ϵ_k only. The first family plays the role of a *mass unification* point.

The eigenvalues of quarks at the Flavour scale Λ_F are the same as those in eq. (17) (with $C = 1$) while for leptons we have the analogous formulae

$$(m/e) = |1 + \epsilon_e {\gamma_{11}}^2| \qquad (m/\mu) = \frac{|\epsilon_e| {\gamma_{12}}^2}{|1 + \epsilon_e {\gamma_{11}}^2|} \qquad (m/\tau) = |\epsilon_e|^2 {\gamma_{12}}^2 {\gamma_{23}}^2. \tag{24}$$

8) We set $C = 1$ by rescaling γ_{23}. This does not affect the generality of the ansatz.

Comparing the above equation with eq. (17a), we see that the experimental relation $\tau \simeq b$ implies $|\epsilon_d| \simeq |\epsilon_e|$. Then $|\epsilon_e|{\gamma_{11}}^2 \simeq 1$ as in the down sector, so that e and μ receive in general sizable corrections from this term. In particular, when the phase of ϵ_e is close to zero, we have that μ is enhanced and e is decreased as required by the experimental data (Fig. 1). Eq. (24) gives a mass formula involving leptons and analogous to eq. (20) for quarks. We can write it in terms of down-type quark and lepton masses

$$\frac{\tau}{b} = \left|\frac{\epsilon_d}{\epsilon_e}\right|^2 = \left(\frac{e\mu}{ds}\right)^2 . \tag{25}$$

Running down this relation from the Flavour scale Λ_F, we get a prediction for $m_s m_d$

$$m_s m_d = m_e m_\mu \sqrt{\frac{m_b}{m_\tau}} z_t^{1/4} \eta_s^2 \eta_b^{-1/2} \eta_F^{3/2} \simeq 277\, z_t^{1/4} \eta_F^{3/2}\, (\,\mathrm{MeV})^2. \tag{26}$$

Where η_s is the QCD running of masses from $m_t = 130\,\mathrm{GeV}$ to 1 GeV, and η_b is the analogous quantity from m_t to m_b. The remaining z_t and η_F account for the running from Λ_F to m_t: $\eta_F = (\alpha_s(m_t)/\alpha_s(\Lambda_F))^{4/7}$ represents the effects of pure QCD. While z_t gives the effects of the top quark Yukawa coupling [15]. For $m_t < 180\,\mathrm{GeV}$ the effects of z_t in eq. (26) are always $< 5\%$, so that we neglect it. The quantity $m_s m_d$ for different values of Λ_F and the corresponding results for the light quark masses are displayed in Table 2. We have let Λ_F go up to 10^{16} GeV.[9)] The values reported correspond to $\alpha_s(M_Z) = 0.11$, and the η's have been deduced from Ref. [16]. Notice that the prediction for m_s (and hence m_d) is increased by about 30% when $\alpha_s(M_Z) = 0.13$. The lower bounds on m_u/m_d correspond to the limit $R = R_{\min} = 30$. We also get an upper bound for m_u/m_d. Indeed, from eqs. (17a) and (24) we obtain

$$|1+\epsilon_d {\gamma_{11}}^2| = |1+\epsilon_e {\gamma_{11}}^2| \left(\frac{es}{\mu d}\right)^{1/2} \left(\frac{\tau}{b}\right)^{1/4} < \left(x + \frac{1}{x}|\epsilon_d|{\gamma_{11}}^2\right) \left(\frac{m_e m_s}{m_\mu m_d}\right)^{1/2} \tag{27}$$

where $x = (\eta_b \eta_F m_\tau/m_b)^{1/4} = 0.9 - 1.1$, for Λ_F varying between 10^4 GeV and 10^{16} GeV. To derive the inequality in eq. (27) we have used eq. (25) for $|\epsilon_e/\epsilon_d|$. Then rather independently on Λ_F we obtain $u/d = |1+\epsilon_d{\gamma_{11}}^2|/|1+\epsilon_u{\gamma_{11}}^2| < 0.75$, by considering that $|\epsilon_d|{\gamma_{11}}^2 \simeq |V_{us}|^2(s/d) \simeq 1$ due to eq. (17b) and $|\epsilon_u/\epsilon_d| < 1/6$ due to eq. (19).

The ansatz (23) gives correct predictions by taking the first family as a *mass unification* point (namely, by taking the same normalizing factor $1/m$ for u, d, and e type fermions).

---

9) Notice that even in a model like the one of the previous section it is meaningful to run up to such a scale. The fact is that, in comparing quarks to leptons, QCD running becomes effective above v_R.

We can also reverse the argument, by testing different factors $1/M_i$ in eq. (23), instead of the same $1/m$. It is then interesting to see what values for M_i's are implied by our experimental knowledge on fermion masses. We have $(M_d/M_u)^3 = (t/b)(ds/uc)^2$ and $(M_d/M_e)^3 = (\tau/b)(ds/e\mu)^2$. For instance for $\Lambda_F = 10^{12}$, after rescaling masses, we get

$$
\begin{aligned}
\frac{M_d}{M_u} &\simeq z_t^{-1/3}\left(\frac{m_t}{120\,\mathrm{GeV}}\frac{4.25\,\mathrm{GeV}}{m_b}\right)^{1/3}\left(\frac{m_s}{130\,\mathrm{MeV}}\frac{1.35\,\mathrm{GeV}}{m_c}\right)^{2/3}\left(0.6\frac{m_d}{m_u}\right)^{2/3} \\
\frac{M_d}{M_e} &\simeq z_t^{-1/6}\left(\frac{4.25\,\mathrm{GeV}}{m_b}\right)^{1/3}\left(\frac{m_s}{130\,\mathrm{MeV}}\right)^{4/3}\left(20\frac{m_d}{m_s}\right)^{2/3}
\end{aligned}
\tag{28}
$$

where we have put the lepton masses to their values. It is remarkable to see that, by varying the masses of eq. (28) in their allowed experimental ranges, all M_i's remain equal within a factor of 2.

5. Discussion

Motivated by the observation of the *inverse hierarchy pattern* $1/m_i^{u,d} \sim \epsilon_{u,d}^{i-1}/m$ in the quark spectrum and by the fact that an up-down symmetry could be responsible for the smallness of the CKM mixing angles, we have constructed a realistic model for quark masses. The key features of this model are:

i) A universal seesaw mechanism [8,10]: the masses of the ordinary quarks q are induced via their mixing with ultraheavy families of weak isosinglet fermions Q.

ii) The mass matrices of the Q's are generated by a *charge diagonal* radiative cascade [4,7]: the 1^{st} family gets its mass at tree level, while the 2^{nd} and 3^{rd} get their masses at 1- and 2-loops, respectively.

iii) The left-right P_{LR} and up-down I_{ud} [10)] discrete symmetries. These are *softly* or *spontaneously* broken in the Higgs potential.

The heaviest Q-family (U, D) is unsplit because of the I_{ud} symmetry. The lighter Q's get their masses from loop-diagrams involving the breaking of I_{ud} and are thereby split. We considered a particular, but reasonable range for the masses of the scalar fields involved in mass generation (see the discussion below eq. (11)). In this range, the Q mass matrices are simply determined by two expansion parameters ϵ_u and ϵ_d. These are in general complex

[10)] In fact, we used a discrete up-down symmetry just for simplicity. I_{ud} can be easily extended to a local "isotopic" invariance via the embedding $U(1)_L \times U(1)_R \times I_{ud} \subset SU(2)_L^Q \times SU(2)_R^Q$. Again, for simplicity we have imposed CP-invariance. Our results are unchanged in the case of complex Yukawa coupling constants.

and $\epsilon_u \neq \epsilon_d$, due to the breaking of CP, P_{LR} and I_{ud}. The $SU(2)_R$ breaking scale plays the role of the Flavour scale. The Q masses are such that $v_R \sim M_{U_3} < M_{D_3} < ... < M$. The ratio v_R/M is determined by $m_u/v_L \sim 10^{-5}$. As a result, the q's mass matrices, given by a seesaw mixing with the Q's, have the *inverse hierarchy* form displayed in eq. (15).

Once again we would like to stress that in our approach the ordinary light quarks are just spectators of the phenomena that determine the flavour structure. This structure arises in a sector of heavy fermions and is transferred to the light ones at their decoupling. This explains why flavour changing effects are suppressed in low energy phenomena.

The mass matrices of eq. (15) reproduce the quark spectrum (see Fig. 1) and mixing angles in a very economical way. The big differences between u- and d-type quark masses are essentially determined by one parameter $|\epsilon_d/\epsilon_u| = \sqrt{R} \sim 6-8$. Upper bounds for m_t and m_d/m_u are then obtained. Rather independently on the Flavour scale $\Lambda_F = v_R$, by invoking reasonable perturbativity requirements, we have $m_t < 150\,\mathrm{GeV}$. On the other hand, without further assumptions, we obtain $(m_u m_c/m_d m_s) > 6$. Then the relation $m_u/m_d > 0.5$ is obtained by using the strange quark mass range given in Ref. [1]. The experimental input $m_u/m_d < 1$ forces the the Cabibbo angle to be *large*: $V_{us} \sim (m_d/m_s)^{1/2}$ instead of $V_{us} \sim m_d/m_s$ as one generally expects within the radiative scenario. Moreover, the correct value $V_{us} \simeq 0.22$ does not imply the loss of perturbativity, in contrast to the previous models of radiative mass generation.

The model shows that the inverse hierarchy pattern can be obtained in an ordinary field theory. Having in mind the idea of some symmetry between quarks and leptons,[11)] eq. (15) can be generalized to charged leptons, in the inverse hierarchy ansatz of eq. (23). In this ansatz the differences between u-, d- and e-type fermions are simply parametrized by three complex numbers ϵ_u, ϵ_d and ϵ_e. In fact, in eq. (23) we have kept the same notation of eq. (15) just for convention: actually $\hat{P}_1$, $\hat{P}_2 = \gamma^{\mathrm{T}}\hat{P}_1\gamma$ and $\hat{P}_3 = \gamma^{\mathrm{T}2}\hat{P}_1\gamma^2$ parametrize the most general set of three symmetric and real matrices of rank 1. Then, eq. (23) contains 11 relevant parameters to calculate 13 observables (9 masses + 4 angles). Hence, we have the two relations in eq. (20,25). (Notice, however, that even by taking $\hat{P}_i$ hermitean these relations remain; this simply means that CP-invariance is not essential for our results.) The predictions can be phrased as $m_s = 100-150\,\mathrm{MeV}$ and $m_u/m_d = 0.35-0.75$.

[11)] The question of whether the $SU(4)$ quark-lepton symmetry of Pati-Salam [21] could provide this ansatz requires further speculations. The T-scalars of $SU(4)$ contain lepto-quarks, that violate in general the *charge diagonality* of the radiative cascade. Also, $SU(4)$ or any GUT extension brings neutrinos into the game. The related physical consequences are under study.

We find it amusing that the radiative idea, implemented with the simplest symmetries (P, CP, "isotopic" and "lepton-quark" symmetries), can explain the key features of the fermion mass spectrum and weak mixing. Notice, that we did not exploit any horizontal structure, in contrast to all known predictive frameworks for fermion masses (see e.g. [11,22]). Clearly, a *clever* horizontal structure would only enhance the predictive power of our approach.

Last but not least, we wish to remark that our approach can automatically solve the strong CP-problem, without introducing an axion. In fact, due to P/CP-symmetries the coefficient Θ_{QCD} of $G\tilde{G}$ is zero. In addition, the contribution to Θ, that arises from the determinant of the whole mass matrix of the fermions q, Q and p, n, is vanishing at tree level. This is a consequence of P/CP and of the seesaw structure [23]. Higher order corrections to the mass matrix can provide a Θ-term in the range $10^{-9} - 10^{-10}$ which is of interest for the search of the the neutron electric dipole moment.

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Appendix

We report the expression of the function $H(r)$. Let us define:

$$f(r) = \int_0^\infty \frac{dx}{x(x+r)} \ln(x+1) = \frac{1}{r} \left\{ \ln r \ln |1-r| + \Re \mathrm{Li}_2(r) \right\} \qquad A.1$$

where $\Re\mathrm{Li}_2$ is the real part of the Euler dilogarithm. Then H is given by:

$$H(r) = \frac{2}{\ln^2 r} \left[f(r) + f(1/r) - \frac{\pi^2}{3} \right]. \qquad A.2$$

H is a positive function which reaches its maximum at $r = 1$: $H(1) = \pi^2/3 - 3 \simeq 0.290$. In the range $1 < r < \infty$, H decreases monotonically and very slowly to zero (see Fig. 3).

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Figure captions

Fig. 1 Logarithmic plot of fermion masses as a function of the family number. Points corresponding to fermions with the same electric charge are joined. Leptons are given by the dashed line. The value $m_t = 130\,\mathrm{GeV}$ has been assumed. The masses correspond to a renormalization scale $\mu = 10^6\,\mathrm{GeV}$.

Fig. 2 a) tree level diagram giving mass to U via mixing with p. b) one-loop correction to the mass matrix of the Q-fermions. c,d,e) two-loop corrections. The analogues of d) and e), where a T_R and a H_L are respectively exchanged, are not displayed.

Fig. 3 Plot of $H(r)$.