

LBL-33997  
 UCB-PTH-93/15  
 hep-ph/9306309  
 June 1993  
 Rev.: March 1994

## The Top Quark Mass in Supersymmetric SO(10) Unification

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### Abstract

The successful prediction of the weak mixing angle suggests that the effective theory beneath the grand unification scale is the minimal supersymmetric standard model (MSSM) with just two Higgs doublets. If we further assume that the unified gauge group contains SO(10), that the two light Higgs doublets lie mostly in a single irreducible SO(10) representation, and that the  $t$ ,  $b$  and  $\tau$  masses originate in renormalizable Yukawa interactions of the form  $\underline{16}_3 \mathcal{O} \underline{16}_3$ , then also the top quark mass can be predicted in terms of the MSSM parameters. To compute  $m_t$  we present a precise analytic approximation to the solution of the 2-loop renormalization group equations, and study supersymmetric and GUT threshold corrections and the input value of the  $b$  quark mass. The large ratio of top to bottom quark masses derives from a large ratio,  $\tan\beta$ , of Higgs vacuum expectation values. We point out that when  $\tan\beta$  is large, so are certain corrections to the  $b$  quark mass prediction, unless a particular hierarchy exists in the parameters of the model. With such a hierarchy, which may result from approximate symmetries, the top mass prediction depends only weakly on the spectrum. Our results may be applied to any supersymmetric SO(10) model as long as  $\lambda_t \simeq \lambda_b \simeq \lambda_\tau$  at the GUT scale and there are no intermediate mass scales in the desert.

PACS numbers: 12.10.Dm,12.15.Ff,14.80.Dq,11.30.Pb

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## I. INTRODUCTION

The standard model of particle physics is extraordinarily successful, describing all known properties of the elementary particles in terms of just 18 free parameters. Nevertheless, the model leaves so many questions unanswered that numerous ideas and speculations leading toward a more fundamental theory have been developed. While there is no hard evidence to support any of these speculations, experiments have provided some hints. In particular, the only parameter of the standard model that has been successfully predicted with a high level of significance is the weak mixing angle, which is a prediction of supersymmetric grand unified theories (SUSY GUTs) [4].

While this is just a single parameter, the excellent agreement of data with the simplest SUSY GUT suggests that it is worthwhile to pursue other predictions of similar SUSY GUTs. This is harder than it sounds. The reason is that the weak mixing angle has a unique status within these theories: it is the only parameter which can be predicted by knowing only the gauge group structure of the model. In fact all one needs to know [5] is that the gauge group is, or breaks to,  $SU(5)$  [6]. All other predictions require additional, model-dependent information about the theory. A good example of this is the proton decay rate: it can only be computed after making an assumption about the spectrum of the superheavy colored states. It is a very interesting quantity, since the simplest possible structure for this superheavy spectrum gives a decay rate that may well be accessible to planned experimental searches. Nevertheless, only minor changes in the theory can lead to very large suppression factors in the rate.

A potentially copious source of predictions is the flavor sector, responsible for the quark and lepton masses and mixings. An early success of GUTs was the prediction of the bottom quark to tau lepton mass ratio,  $m_b/m_\tau$  [7]; however, several considerations make this less impressive than the weak mixing angle prediction. First and foremost is the low numerical significance of the  $m_b/m_\tau$  prediction: it cannot be made with much accuracy as long as the strong coupling  $\alpha_3$  and the top quark Yukawa coupling (which cannot be neglected for a heavy top) are not known very well, and it cannot be compared accurately with experiment without better knowledge of  $m_b$ . This low significance is especially troubling given that the theory is predicting only one of the 13 independent flavor parameters of the standard model. Nevertheless, with a heavy top quark, an acceptable value of  $m_b/m_\tau$  *requires* more than one light Higgs doublet—which provides another interesting hint pointing towards low-energy supersymmetry [8]. Finally, the simplest flavor sector which leads to this prediction immediately fails when extended to the lighter generations:  $m_s/m_d = m_\mu/m_e$  is unacceptable. To overcome these objections it is necessary to construct a more complicated flavor sector of the grand unified theory. Here there is a delicate balance: more structure requires further assumptions, but these

are perhaps justified if there are additional predictions. This approach was developed long ago [9–11] and has received considerable attention recently [12]. Using the full power of the grand unified group  $SO(10)$  [13] it is possible to obtain predictions for 7 of the 13 flavor parameters. Further development of the flavor sector can also lead to predictions for neutrino masses [11,14]. Despite these successes, one still has to admit that these schemes are based on the hope that the flavor sector at the grand unified scale is particularly simple: the quark and lepton masses must originate in just a few grand unified interactions. If there are many such interactions the predictions are lost. This is a particularly acute problem for the lighter generations. The smallness of these masses can be understood if they arise from higher dimensional operators. However in this case there is a very large number of operators that could be written down, and the restriction that just one or two of these operators contribute to the masses involves some strong assumptions.

We are unable to completely avoid this dilemma: to obtain quark and lepton mass predictions from grand unified theories, assumptions about the underlying flavor structure of the theory must be made. Of all the flavor predictions of GUTs those pertaining to the heaviest generation are most direct and subject to the fewest assumptions. In this paper we pursue a scheme for predicting the top mass which is unique in its simplicity. We attempt as complete and accurate analysis of this prediction as possible: the aim is to predict the top quark mass to within a few GeV.

There are many approaches in the literature which result in predictions for the top mass. We would like to emphasize the differences between two such approaches and a third one which we shall take. The first is the infrared fixed point behaviour of the renormalization group (RG) equation for the top quark Yukawa coupling [15]. This is an argument that certain values for the top mass are more probable than others if all GUT scale Yukawa couplings are equally probable. The second framework for predicting the top quark mass is that of textures for the generation structure of the Yukawa coupling matrices. The top mass is given in terms of lighter quark masses and entries of the Kobayashi-Maskawa mixing matrix [10,11]. This necessarily requires assumptions about the masses of the lighter generations. Finally, in the context of grand unified theories, it is sometimes possible to predict the top quark mass from a consideration only of the heaviest generation, with the  $b$  and  $\tau$  masses as inputs from experiment [16,17]. Consider the Yukawa interactions of a supersymmetric grand unified theory which lead to masses for the heaviest generation. There are three observable masses:  $m_t$ ,  $m_b$  and  $m_\tau$ . Since the Higgs doublet which leads to the top mass is forced by supersymmetry to be different from the one which gives mass to  $b$  and  $\tau$ , these three masses necessarily depend on the parameter  $\tan\beta$ , the ratio of the vacuum expectation values (VEVs) of these two doublets. If the grand unified theory has two or more independent Yukawa parameters

contributing to the heaviest generation masses, then there will be no prediction when the heavy generation is considered in isolation. The only possibility for a prediction resulting from consideration of the heaviest generation alone is that the  $t$ ,  $b$  and  $\tau$  masses originate predominantly from a single Yukawa interaction. This immediately excludes the grand unified gauge group  $SU(5)$  from consideration [18]. The simplest supersymmetric [19] grand unified gauge group which allows relations between masses in the up and down sectors is  $SO(10)$ .

In this paper we study the top quark mass prediction which results from the following three assumptions:

- (I) The masses of the third generation,  $m_t$ ,  $m_b$  and  $m_\tau$ , originate from renormalizable Yukawa couplings of the form  $\underline{16}_3 \mathcal{O} \underline{16}_3$  in a supersymmetric GUT with a gauge group containing (the conventional)  $SO(10)$ .
- (II) The evolution of the gauge and Yukawa couplings in the effective theory beneath the  $SO(10)$  breaking scale is described by the RG equations of the minimal supersymmetric standard model (MSSM).
- (III) The two Higgs doublets lie predominantly in a single irreducible multiplet of  $SO(10)$ .

We find it highly significant that such simple and mild assumptions are sufficient for predicting the top quark mass with an accuracy of a few GeV, in terms of a few parameters of the MSSM (which could be measured experimentally). In Secs. II–VII we in fact assume that the two light Higgs doublets lie completely in a single irreducible multiplet, while in Sec. IX we return to the effects of mixing with other multiplets. We will very rapidly be led to the result that the  $SO(10)$  multiplet containing the Higgs doublets is (almost) necessarily the  $\underline{10}_H$ , so that the relevant Yukawa interaction is  $\underline{16}_3 \underline{10}_H \underline{16}_3$  (where  $\underline{16}_3$  is the third-generation matter supermultiplet). The prediction of the top quark mass from this interaction was first considered by Ananthanarayan, Lazarides and Shafi [16]. We find the picture which emerges from such an interaction to be very elegant. While the three Yukawa couplings  $\lambda_{t,b,\tau}$  are different at low energies, they evolve according to RG equations to a common unified value at the large mass scale of the grand unified theory. The scenario is reminiscent of the evolution of the three gauge coupling constants to a common value at the unification scale. While the gauge coupling unification leads to a prediction for the weak mixing angle, the Yukawa coupling unification leads to a prediction for the top quark mass. The top-bottom mass hierarchy then originates in the Higgs sector, so another prediction is a large ratio of Higgs VEVs. The weak mixing angle prediction has undergone several refinements as higher order corrections and a variety of threshold corrections have been considered. An

aim of the present paper is to compute such corrections to the top mass prediction to a similar level of accuracy. In particular we study:

- the coupled two-loop RG equations for the three gauge couplings and the three Yukawa couplings. We give an analytic fit to the numerical results which is valid to better than 0.2%.
- some implications of generating the top-bottom mass hierarchy through a large ratio of the VEVs of the two light Higgs doublets. This source for up-down mass hierarchy is generic in models which unify all three Yukawa couplings of the third generation. Since *a priori* there is no symmetry protecting the down-type Higgs VEV and consequently the down-type fermion masses, large radiative corrections to these masses typically arise and change the top mass prediction considerably. Such corrections will be suppressed if the squarks are much heavier than the higgsinos and gauginos; indeed, we identify two symmetries which could then protect the down-type VEV and masses. Whether such suppression is favored in models with large  $\tan\beta$  is a question for future study [20], and whether it is the case in nature will be determined by future experiments.
- two consequences of these large corrections to the b quark mass. For a certain range of MSSM parameters the  $m_b/m_\tau$  prediction cannot be brought into agreement with experiment; for a separate, smaller range, different GUT-scale boundary conditions must be used.
- the supersymmetric threshold corrections to the three gauge couplings and the three Yukawa couplings. These are given for an arbitrary spectrum of the superpartners of the minimal supersymmetric standard model, ignoring only the electroweak breaking effects in the spectrum. We find in particular that, when the above symmetries hold approximately, raising any or all of the superpartner masses increases  $m_t$  for fixed  $\alpha_3(m_Z)$ .
- threshold corrections at the grand unified mass scale. We show that such corrections to the gauge couplings do not significantly affect the top mass prediction for a given  $\alpha_3(m_Z)$ . We calculate the corrections to the Yukawa couplings from superheavy splittings in the gauge  $\underline{45}$ , the  $\underline{10}_H$  and the  $\underline{16}_3$  multiplets (these corrections are not very large), and give general expressions for further possible superheavy threshold corrections.
- the extent to which the predicted value of the top quark mass depends on assumption (III). This assumption is, in our opinion, the weakest part of the theoretical picture which underlies the top mass prediction (with the possible exception of

the electroweak symmetry breaking sector). Even if the third generation only couples through a single SO(10) invariant Yukawa interaction, the relation between this coupling and the values of  $\lambda_t$ ,  $\lambda_b$  and  $\lambda_\tau$  renormalized at the unification scale may involve a set of mixing angles describing which two linear combinations of the doublets of the unified theory are light. An understanding of these mixing angles is in principle related to an understanding of why two doublets remain light (the doublet-triplet splitting problem). Our understanding of the resolution of this problem is at present not complete. We are however greatly encouraged by the following two facts: *i*) Due to the fixed point behavior of the RGE the prediction is somewhat insensitive to these mixing angles in a large class of models; *ii*) in very simple SO(10) models which provide a partial solution of the doublet-triplet splitting problem, both Higgs doublets are in fact contained within the same irreducible representation (namely a  $\underline{10}_H$ ) [21,20].

- the extraction of the  $b$  quark mass from experiment, using updated experimental information and including the dependence of the extracted value of this mass on the QCD coupling  $\alpha_3$ . This is of importance to us because the two crucial experimental inputs to the top mass prediction are  $m_b$  and  $\alpha_3$ .

In Sec. II we describe the basic framework for our calculation. A discussion of the implications of large  $\tan\beta$  is given in Sec. III, where we examine two potentially very large corrections to the  $b$  quark mass prediction; however, if the MSSM parameters exhibit a certain hierarchy (for large  $\tan\beta$ ) such corrections may be suppressed. In Sec. IV we give approximate analytic solutions to the two loop renormalization group equations, in the absence of threshold corrections. The extraction of the  $b$  quark mass from data is given in Sec. V. Armed with this experimental value of  $m_b$ , we return in Sec. VI to the large corrections to the predicted  $m_b$ , bounding these corrections (and consequently the MSSM parameters) and investigating the possibility of different GUT-scale initial conditions for the Yukawa couplings. The remaining threshold corrections are studied in Sec. VII, while in Sec. VIII the sensitivity of the predicted top quark mass to these threshold corrections is derived and discussed in a general way. In Sec. IX we give the prediction for the top quark pole mass and discuss its sensitivity to certain grand unified threshold corrections. In Sec. X we extend the discussion to include models in which the doublets are not completely contained in a single SO(10) irreducible multiplet, and to realistic models which include the other generations of matter. Sec. XI concludes.

## II. FRAMEWORK

The predicted value of the top mass depends on the renormalization group equations (RGE), the boundary conditions at the GUT and electroweak scales, and the threshold corrections at these two scales. The one-loop RGE in the minimal supersymmetric standard model (that is, with three generations, two Higgs doublets and no right-handed neutrinos) are given by

$$16\pi^2 \frac{d \ln \lambda_t}{dt} = \sum_{\nu=t,b,\tau} K_{t\nu} \lambda_\nu^2 + \sum_{i=1,2,3} L_{ti} g_i^2, \quad (1a)$$

$$16\pi^2 \frac{d \ln \lambda_b}{dt} = \sum_{\nu=t,b,\tau} K_{b\nu} \lambda_\nu^2 + \sum_{i=1,2,3} L_{bi} g_i^2, \quad (1b)$$

$$16\pi^2 \frac{d \ln \lambda_\tau}{dt} = \sum_{\nu=t,b,\tau} K_{\tau\nu} \lambda_\nu^2 + \sum_{i=1,2,3} L_{\tau i} g_i^2, \quad (1c)$$

and

$$16\pi^2 \frac{d \ln g_1}{dt} = b_1 g_1^2, \quad (2a)$$

$$16\pi^2 \frac{d \ln g_2}{dt} = b_2 g_2^2, \quad (2b)$$

$$16\pi^2 \frac{d \ln g_3}{dt} = b_3 g_3^2, \quad (2c)$$

where  $t = \ln \mu$  and [22]  $K_t = (6, 1, 0)$ ,  $K_b = (1, 6, 1)$ ,  $K_\tau = (0, 3, 4)$ ,  $L_t = (-\frac{13}{15}, -3, -\frac{16}{3})$ ,  $L_b = (-\frac{7}{15}, -3, -\frac{16}{3})$ ,  $L_\tau = (-\frac{9}{5}, -3, 0)$ ,  $b_1 = \frac{33}{5}$ ,  $b_2 = 1$ , and  $b_3 = -3$ . In our analysis, however, we employ two-loop evolution equations for both the Yukawa and the gauge couplings. The gauge couplings are related to experimental observables by  $g_1^2 = \frac{5}{3}e^2/(1 - \sin^2 \theta_W)$ ,  $g_2^2 = e^2/\sin^2 \theta_W$ , and  $g_3^2 = 4\pi\alpha_3$ , whereas the Yukawa couplings are related to the running quark masses via  $\lambda_t = \sqrt{2}m_t/v_U$ ,  $\lambda_{b,\tau} = \sqrt{2}m_{b,\tau}/v_D$  and  $v_U^2 + v_D^2 = v^2 = (247 \text{ GeV})^2$ . The VEVs of the two light Higgs doublets  $H_U$  and  $H_D$  are denoted, respectively, by  $v_U$  and  $v_D$ , and their ratio is denoted by  $v_U/v_D \equiv \tan \beta$  as usual. The boundary conditions in the gauge sector at the electroweak scale will be [23,24] the  $\overline{\text{MS}}$  values  $\sin^2 \theta_W = 0.2314$  (appropriate in anticipation of a heavy top quark) and  $4\pi/e^2 \equiv 1/\alpha_{\text{em}} = 127.9$ ; these are extracted from data using the 6-flavor standard model as the effective theory at the scale  $m_Z$ . In the Yukawa sector we have [25]  $m_\tau(m_\tau) = 1.777 \text{ GeV} \Rightarrow m_\tau(m_Z) = 1.749 \text{ GeV}$ , and the range of  $m_b$  values derived below. The remaining uncertainties in  $\sin^2 \theta_W$  and  $\alpha_{\text{em}}$ , contribute negligibly to the prediction of  $m_t$ .

The tree-level initial conditions at the GUT scale  $M_G$  are  $g_1 = g_2 = g_3 \equiv g_G$  for the gauge sector, but in the Yukawa sector they depend on the source of the low-energy Yukawa couplings. In SO(10) unification they typically arise from terms of the form



$\underline{16}_3 \mathcal{O} \underline{16}_3$ , where  $\underline{16}_3$  is the chiral supermultiplet for the third generation and the Higgs multiplet  $\mathcal{O}$  may be a  $\underline{10}_H$  or  $\underline{126}_H$  of  $\text{SO}(10)$ . (The  $\underline{120}_H$  is antisymmetric and therefore makes no contribution.) To go beyond the  $\text{SU}(5)$  predictions and exploit the larger  $\text{SO}(10)$  symmetries, we must *assume* that both light Higgs doublets lie predominantly in a *single*  $\text{SO}(10)$  multiplet, rather than being arbitrary mixtures of doublets in several representations. As stated above, our understanding of which doublets remain light, and why they do so, is not complete. Thus to make progress we simply assume the mixing is negligible. Note that if the mixing is with other doublets which do not couple directly to the  $\underline{16}_3$ , then the effect is only to split  $\lambda_t^G$  from  $\lambda_{b,\tau}^G$ , and even a 30% splitting of this sort will have less than a 3% effect on the low-energy top mass, as we show below. Furthermore, a supersymmetric theory without mixing terms in the Lagrangian is technically natural, due to the nonrenormalization theorems. Then either  $\mathcal{O} \sim \underline{10}_H$ , in which case the initial conditions for the Yukawa couplings are

$$\lambda_t = \lambda_b = \lambda_\tau \equiv \lambda_G, \quad (3)$$

or else  $\mathcal{O} \sim \underline{126}_H$  from whence  $3\lambda_t = 3\lambda_b = \lambda_\tau \equiv \lambda_G$ . In the second case, a numerical investigation shows that the prediction of  $m_b/m_\tau$  is too low unless  $\alpha_3(m_Z) > 0.13$  and  $m_b(m_b) < 4.0 \text{ GeV}$ , as well as  $\lambda_G > 4$ . This last requirement, however, when combined with the full  $\text{SO}(10)$  RGE, implies a Landau pole in the Yukawa coupling less than 20% above the unification scale, which in general allows  $\mathcal{O}(1)$  GUT-scale threshold corrections and makes any predictions impossible. In fact, the only option for perturbative unification with  $\mathcal{O} \sim \underline{126}_H$  is for very large electroweak-scale threshold corrections to arise, which are of just the right magnitude and sign to restore the agreement of  $m_b/m_\tau$  with experiment. Only for a very limited range of  $\lambda_G$  values and MSSM parameters can such a scenario be successful, as we show in Sec. VI. For most of the parameter range, therefore, we need only consider the consequences of  $\underline{16}_3 \underline{10}_H \underline{16}_3$ . In this case, we must restrict  $\lambda_G < 2$  to make sure the Landau pole is at least a factor of 4 above the unification mass; for higher values of  $\lambda_G$ , we simply cannot make any reliable predictions, although these values are by no means ruled out.

### III. LARGE $\tan\beta$

We discuss in this section some of the implications of the large value of  $\tan\beta \equiv v_U/v_D \simeq 50-60$  necessitated by the boundary condition  $\lambda_t^G = \lambda_b^G = \lambda_\tau^G$ . In the standard model and most of its extensions, the hierarchy  $m_b \ll m_t$  (and similarly for the  $\tau$  mass) is imposed through  $\lambda_b \ll \lambda_t$ ;  $\lambda_b$  is the small parameter quantifying the breakdown of the chiral symmetry which protects the bottom quark mass. In multi-Higgs models such as the MSSM, there is also the option of explaining  $m_b \ll m_t$  by having the down-type Higgs

acquire a much smaller VEV than that of the up-type Higgs,  $v_D \ll v_U$ . We are *forced* to take this second route by our GUT boundary condition, which implies  $\lambda_b \sim \lambda_t \sim 1$ . We will assume as usual that the electroweak symmetry is broken by an instability of the scalar potential  $m_U^2 |H_U|^2 + m_D^2 |H_D|^2 + B\mu(H_U H_D + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_U|^2 - |H_D|^2)^2$  in the  $H_U$  direction. This breaking is then communicated to  $H_D$  by the soft SUSY-breaking term  $\mu B H_U H_D$ :

$$\frac{-\mu B}{m_U^2 + m_D^2} = \frac{1}{2} \sin 2\beta \simeq \frac{1}{\tan \beta} = \frac{v_D}{v_U} \sim \frac{1}{50}. \quad (4)$$

So the magnitude of the hierarchy between the up and down sectors is determined by  $\mu B / (m_U^2 + m_D^2)$ , while its direction is set by the direction in which the instability develops in the scalar potential. The denominator is given by  $m_U^2 + m_D^2 = m_A^2$ , the squared mass of the pseudoscalar neutral Higgs boson; evidently, either the  $\mu$  or the  $B$  parameter or both must be much smaller than  $m_A$  in order to generate the top-bottom mass hierarchy. How, and indeed whether, the various parameter values may arise will be discussed in a future paper [20].

At tree level, an appropriate choice of parameters in the scalar potential leads to  $v_D \ll v_U$  and hence to  $m_{b,\tau} \ll m_t$ . However, no symmetry has (yet) been imposed on the Lagrangian to protect such a hierarchy, and therefore we expect large radiative corrections. In fact corrections arise from the gluino- and higgsino-exchange diagrams of Fig. 1 (the analogue of Fig. 1a, where a bino is exchanged, is suppressed by the small hypercharge coupling). Such corrections have been overlooked in past work on large  $\tan \beta$  scenarios, though they were discussed in the context of radiative mass generation [26]. Thus the MSSM prediction for  $m_b$  becomes, after replacing  $H_U \rightarrow v_U / \sqrt{2}$ ,

$$m_b = \lambda_b \frac{v_D}{\sqrt{2}} + \delta m_b^{(\tilde{g})} + \delta m_b^{(\tilde{h})} \quad (5)$$

where

$$\delta m_b^{(\tilde{g})} = \frac{2\alpha_3}{3\pi} m_{\tilde{g}} \mu \lambda_b \frac{v_U}{\sqrt{2}} I(m_{\tilde{b},+}^2, m_{\tilde{b},-}^2, m_{\tilde{g}}^2) \quad (6)$$

and

$$\delta m_b^{(\tilde{h})} = \frac{\lambda_t \lambda_b}{16\pi^2} \mu A_t \lambda_t \frac{v_U}{\sqrt{2}} I(m_{\tilde{t},+}^2, m_{\tilde{t},-}^2, \mu^2), \quad (7)$$

$I$  is given in the appendix,  $m_{\tilde{g}}$  is the gluino mass,  $\mu$  is the supersymmetric coupling of the two Higgs doublets,  $A_t$  is the trilinear soft SUSY-breaking coupling of the stop fields to the up-type Higgs, and  $m_{\tilde{b},\pm}$  and  $m_{\tilde{t},\pm}$  are the squark mass eigenstates propagating in the loop. Numerically  $m_{\tilde{g}} \simeq 3m_{1/2}$  where  $m_{1/2}$  is the mass of either the gauginos at the GUT scale or of the wino at the electroweak scale. To appreciate the significance of

FIG. 1. The leading (finite) 1-loop MSSM contributions to the  $b$  quark mass.

such corrections, consider the limit in which the squarks have roughly equal masses  $m_0$ , and  $\mu$  or  $m_{\tilde{g}}$  are either much less than or equal to  $m_0$ . Then  $I(m_0^2, m_0^2, 0) = 1/m_0^2$  and  $I(m_0^2, m_0^2, m_0^2) = 1/(2m_0^2)$ , and we find:

$$\begin{aligned}
 m_b &= \lambda_b \frac{v_D}{\sqrt{2}} \left[ 1 + \frac{\tan \beta}{16\pi^2} \left( \frac{8}{3} g^2 \frac{m_{\tilde{g}} \mu}{(2)m_0^2} + \lambda_t^2 \frac{\mu A_t}{(2)m_0^2} \right) \right] \\
 &\simeq \lambda_b \frac{v_D}{\sqrt{2}} \left[ 1 + 0.35 \left( 4 \frac{m_{\tilde{g}} \mu}{(2)m_0^2} + \frac{\mu A_t}{(2)m_0^2} \right) \right]
 \end{aligned} \tag{8}$$

(Note that in the second line above, we have approximated  $\tan \beta \simeq 50$ , which is inaccurate if  $\delta m_b$  becomes large and lowers the top mass prediction significantly; we will return to this point once we have extracted the experimental bounds on  $m_b$ .) We see that the radiative corrections may in general be comparable to the tree-level mass; when we equate this prediction to the experimental value of  $m_b$  to extract  $\lambda_b$ , we would find  $\mathcal{O}(1)$  corrections and hence large changes in  $\lambda_G$  and in the prediction for  $m_t$ . Our final  $m_t$  prediction would then be very sensitive to the exact values of the squark, higgsino and gaugino masses (and perhaps  $A_t$  as well)—far more sensitive than expected from ordinary threshold corrections. On the other hand, the squarks may turn out to be relatively heavy, namely  $m_0^2 \gg \mu m_{\tilde{g}}, \mu A_t$ , in which case these corrections would be suppressed. For example, if  $m_0 \simeq 1$  TeV but  $\mu \simeq m_{\tilde{g}} \simeq A_t \simeq 200$  GeV then  $m_b$  changes by only  $\sim 6\%$ , which in turn corrects the top mass prediction by  $\sim 4\%$ . Of course, the sign of this correction is determined by the sign of the parameters which enter  $\delta m_b$ .

We should mention at this point that there is a diagram in the 2-Higgs standard model analogous to the higgsino-exchange diagram of Fig. 1, in which the stop propagator is replaced with a top propagator, the higgsino with the Higgs, and the couplings are replaced by  $A_t \lambda_t \rightarrow \lambda_t$  and  $\mu \rightarrow \mu B$ . For this diagram the large  $\tan \beta$  enhancement gained by coupling the  $b$  to  $H_U^*$  is manifestly and exactly cancelled by the  $\mu B/m_A^2$  factor from the propagators, independent of any symmetries. This threshold contribution is included in the function  $f_R$  defined in Sec. V.

We have seen that when  $m_0^2 \gg \mu m_{\tilde{g}}, \mu A_t$  the large radiative corrections are suppressed. We also know that for large  $\tan \beta$ ,  $m_A^2 \gg \mu B$ . These hierarchies can be related by imposing two approximate symmetries. The symmetry which sets  $\mu$  to zero is a Peccei-Quinn ( $\mathcal{PQ}$ ) symmetry under which the superfield  $H_D$  and the  $SU(2)$ -singlet

bottom antiquark superfield  $b^c$  have equal and opposite charges while all other fields are invariant. It is explicitly broken only by the soft SUSY-breaking term  $\mu B H_U H_D$  and by the term  $\mu H_U H_D$  in the superpotential, so when treated as a spurion  $\mu$  should be assigned a  $\mathcal{PQ}$  charge opposite to that of  $H_D$ . The SUSY-breaking term contributes at tree level to the VEV of  $H_D$ , while the supersymmetric one enters into (finite) loop diagrams which correct the  $b$  mass. We will quantify the degree to which this symmetry is broken by the dimensionless parameter  $\epsilon_{\mathcal{PQ}} \equiv \mu/m_0$ .

The symmetry which sets  $B$  to zero must also set the gaugino mass  $m_{1/2}$  and the trilinear scalar coupling  $A$  (and in particular  $A_t$ ) to zero; note that  $m_{1/2}$  and  $A$  generate  $B$  through the RG evolution. The desired symmetry is in fact a continuous  $\mathcal{R}$  symmetry, and it is convenient to choose an  $\mathcal{R}$  such that the superpotential is invariant while the soft SUSY-breaking terms (except for the common scalar mass) is not. Furthermore, we choose an  $\mathcal{R}$  under which the scalar  $H_U$  is invariant but the scalar  $H_D$  and the  $b$  quark mass operator  $Qb^c$  are not. We will assign the superspace coordinate  $\theta$  a charge of  $+1$ , the superfield  $H_U$  a charge  $0$ , and the quark doublet superfield  $Q$  a charge  $0$ ; then the superfield  $H_D$  carries a charge of  $+2$ , the superfield  $b^c$  carries a charge of  $0$ , and the superfield  $t^c$  carries a charge of  $+2$ . Thus the left-handed quarks have charge  $-1$ , the SU(2)-singlet  $b^c$  antiquark has charge  $-1$  and the SU(2)-singlet  $t^c$  antiquark has charge  $+1$ . In a spurion analysis, the soft SUSY-breaking parameters  $m_{1/2}$ ,  $A$ , and  $B$  carry a charge of  $-2$ . Once again, we define a dimensionless symmetry-breaking measure  $\epsilon_{\mathcal{R}} \equiv B/m_0 \lesssim A/m_0 \sim m_{1/2}/m_0$ .

When the  $\mathcal{PQ}$  or  $\mathcal{R}$  symmetries are approximately valid, we find

$$\delta m_b^{(\tilde{g})} \sim \frac{2\alpha_3}{3\pi} (\epsilon_{\mathcal{PQ}}\epsilon_{\mathcal{R}} \tan\beta) \left(\frac{m_{\tilde{g}}}{B}\right) m_b = \frac{2\alpha_3}{3\pi} \left(\frac{m_{\tilde{g}}}{B}\right) \left(\frac{m_A^2}{m_0^2}\right) m_b \quad (9)$$

and

$$\delta m_b^{(\tilde{h})} \sim \frac{\lambda_t \lambda_b}{16\pi^2} (\epsilon_{\mathcal{PQ}}\epsilon_{\mathcal{R}} \tan\beta) \left(\frac{A_t}{B}\right) \left(\frac{\lambda_t}{\lambda_b}\right) m_b = \frac{\lambda_t^2}{16\pi^2} \left(\frac{A_t}{B}\right) \left(\frac{m_A^2}{m_0^2}\right) m_b. \quad (10)$$

As expected, the large  $\tan\beta$  enhancement was cancelled by the  $\epsilon_{\mathcal{PQ}}\epsilon_{\mathcal{R}}$  factor, provided  $m_{\tilde{g}} \sim A_t \sim B$  and  $m_A^2 \sim m_0^2$ .

If either of these symmetries is to hold even approximately, there must be a certain hierarchy in the supersymmetric spectrum. Notice, however, that  $\mu$  and  $m_{1/2}$  are bounded below by LEP data [27], at least for large  $\tan\beta$ : numerically, they read roughly  $(|\mu| - m_Z/2)(|m_{1/2}| - m_Z/2) > m_Z/2$ , so we expect both  $\mu$  and  $m_{1/2}$  to be at least as large as  $m_Z$ . Thus the  $\mathcal{PQ}$  and  $\mathcal{R}$  symmetries can be meaningful only if the scalar superpartners are significantly more massive than the  $Z$ . This typically implies a degree of fine-tuning to get the proper  $Z$  mass, so it remains to be seen [20] whether it is more natural to expect  $m_0 \gg m_Z \sim \mu \sim m_{1/2} \sim A_t \sim B$  and fine-tune the  $Z$  mass, or to

expect  $m_0 \sim m_Z \sim \mu \sim m_{1/2} \sim A_t \gg B$  and fine-tune  $B$ . Perhaps electroweak symmetry breaking does not arise from the usual running of the parameters in the Higgs sector, but rather from some other mechanism (which does not significantly alter the RG evolution of the gauge and Yukawa couplings). Ultimately, experiment will decide what if any hierarchy exists in these parameters. Some experimental information already exists: attaching a photon in all possible ways to the diagrams of Fig. 1 and inserting a flavor-changing vertex leads to an amplitude for  $b \rightarrow s\gamma$  which again is proportional to  $\epsilon_{\mathcal{P}\mathcal{Q}}\epsilon_{\mathcal{R}}\tan\beta$ . To reconcile this amplitude with the CLEO data on  $\Gamma(b \rightarrow s\gamma)$ , we must either impose the  $\mathcal{P}\mathcal{Q}$  and  $\mathcal{R}$  symmetries or raise all the superpartner masses (since the operator for  $b \rightarrow s\gamma$  is of dimension higher than 4). We will leave these constraints to future work [20]. (Analogous considerations can be extended to other observables such as  $B^0\bar{B}^0$ ,  $D^0\bar{D}^0$  and  $K^0\bar{K}^0$  mixing, the electric dipole moment of the neutron and proton decay.) For now, we can only make definite predictions of the top mass in the case  $m_0^2 \gg \mu m_{\tilde{g}}, \mu A_t$ .

#### IV. RUNNING AND MATCHING

If we temporarily ignore all threshold corrections (the “unperturbed scenario”), the solution of the RG equations proceeds schematically as follows. By requiring that the two gauge couplings  $g_1$  and  $g_2$  meet, we solve Eqs. (2a,2b) to obtain  $M_G$  and  $g_G$ . (At this stage there is only a weak dependence on the Yukawa couplings in the 2-loop RGE, and we may use representative values for them.) Then by running back down with Eq. (2c) we predict  $g_3(m_Z)$  and hence  $\alpha_3(m_Z)$ . Next, we solve the two equations (1b,1c) for the two unknowns  $\lambda_G$  and  $\tan\beta$ , using  $M_G$  and  $g_G$  as well as  $m_b(m_Z)$  and  $m_\tau(m_Z)$  and the initial condition (3). In practice, this step is simplified because  $\tan\beta$  will always be  $\sim m_t(m_Z)/m_b(m_Z) \sim 50 - 60$ , so we may set  $\sin\beta$  to unity from now on. We are then left with a single equation, namely  $m_b(m_Z)/m_\tau(m_Z) \equiv R = \lambda_b(m_Z)/\lambda_\tau(m_Z)$ , which we solve for the single unknown  $\lambda_G$ . Finally, we use  $\lambda_G$  to run down with Eq. (1a) and obtain  $\lambda_t(m_Z)$ , which is then used to determine the top quark mass.

More precisely, we will adopt the following procedure to correctly incorporate 2-loop RG evolution with 1-loop matching conditions. We choose to match the MSSM with the broken-electroweak standard model using as an intermediate step the 2-Higgs standard model (2HSM). This two-step procedure has two advantages: the presentation is clearer, and the various matching contributions are easy to isolate.

- We first treat the gauge sector: we match the experimentally-determined gauge couplings  $g_i(m_Z)$  of the standard model, which are essentially those in the 2HSM, to the couplings of the MSSM by integrating in the superpartners. The conversion

from the  $\overline{\text{MS}}$  to the  $\overline{\text{DR}}$  scheme is numerically insignificant. We then use 2-loop MSSM RGEs to run from  $m_Z$  to some GUT scale  $\mu_G$  which we will *fix* to be some convenient value near  $10^{16}$  GeV; in this running we employ approximate values of the Yukawa couplings. We thus calculate the gauge coupling boundary values at this GUT scale.

- Starting with these gauge couplings and with a given set of Yukawa boundary conditions (collectively denoted by  $\lambda_G$  for the moment) at the GUT scale  $\mu_G$ , we evolve the gauge and Yukawa couplings with 2-loop MSSM RGEs in the  $\overline{\text{DR}}$  scheme to an *arbitrary* electroweak scale  $\mu_Z$  and obtain  $\lambda_{t,b,\tau} \equiv \lambda_{t,b,\tau}^{\text{MSSM},\overline{\text{DR}}}(\mu_Z; \lambda_G)$ . These are then matched to the 2-Higgs standard model, in which the superpartners have been integrated out, to yield  $\lambda_{t,b,\tau}^{2\text{HSM},\overline{\text{DR}}}(\mu_Z; \lambda_G, \{m_a\}) = \lambda_{t,b,\tau}^{\text{MSSM},\overline{\text{DR}}}(\mu_Z; \lambda_G)[1 + k_{t,b,\tau}(\mu_Z; \{m_a\})]$ , where  $\{m_a\}$  are the superpartner masses.
- We also start with the running  $\overline{\text{MS}}$  value of the  $b$  quark mass  $m_b^{\overline{\text{MS}}}(4.1 \text{ GeV})$ , and evolve it with 2-loop QCD running [that is, in the no-Higgs low-energy standard model (0HSM) having only strong and electromagnetic interactions] to obtain  $m_b^{0\text{HSM},\overline{\text{MS}}}(\mu_Z) = m_b^{\overline{\text{MS}}}(4.1 \text{ GeV})/\eta_b$ . Similarly we run up the  $\tau$  mass to obtain  $m_\tau^{0\text{HSM},\overline{\text{MS}}}(\mu_Z) = m_\tau/\eta_\tau$ . Their ratio is defined to be  $R^{0\text{HSM},\overline{\text{MS}}}(\mu_Z)$ , which may be translated into the  $\overline{\text{DR}}$  scheme:  $R^{0\text{HSM},\overline{\text{DR}}}(\mu_Z) \simeq R^{0\text{HSM},\overline{\text{MS}}}(\mu_Z)[1 - \alpha_3/3\pi]$ . To match the 0HSM to the 2HSM requires some knowledge of the Higgs sector masses. At tree-level and in the limit of large  $\tan\beta$ , these are all given [28] in terms of  $m_Z$ ,  $m_W$ , and the mass  $m_A$  of the physical neutral pseudoscalar Higgs: the (mostly up-type) scalar  $m_h \simeq m_Z$ , the other (mostly down-type) scalar  $m_H \simeq m_A$ , and the charged (also mostly down-type) scalars  $m_{H^\pm} = \sqrt{m_W^2 + m_A^2}$ . We then obtain  $R \equiv R^{2\text{HSM},\overline{\text{DR}}}(\mu_Z; m_A) = R^{0\text{HSM},\overline{\text{DR}}}(\mu_Z)[1 + f_R(\mu_Z; m_A)]$ .
- Next we demand  $R = \lambda_b^{2\text{HSM},\overline{\text{DR}}}(\mu_Z; \lambda_G, \{m_a\})/\lambda_\tau^{2\text{HSM},\overline{\text{DR}}}(\mu_Z; \lambda_G, \{m_a\})$  which we may solve for  $\lambda_G$ .
- Finally, we use the  $\lambda_t^{2\text{HSM},\overline{\text{DR}}}(\mu_Z; \lambda_G, \{m_a\})$  corresponding to this  $\lambda_G$  and calculate the top pole mass  $m_t^{\text{pole}} = (v/\sqrt{2})\lambda_t^{2\text{HSM}}(\mu_Z; \lambda_G, \{m_a\})[1 + f_t(\mu_Z; m_A)]$ , defined as the position of the pole in the 2HSM with perturbative QCD. The function  $f_t$  contains the well-known contribution from perturbative QCD radiative corrections as well as often-neglected contributions from Yukawa radiative corrections. Note that the observable  $m_t^{\text{pole}}$  must be independent of  $\mu_Z$  to 1-loop order; we have indeed checked that our final values do not change by more than a GeV or so when we vary  $\mu_Z$  around the electroweak scale. To be specific, we will use the value  $\mu_Z = m_Z$  in all the explicit values we present below.

We will make the following approximations when appropriate. First, we use the full 1-loop threshold expressions involving  $\lambda_t^2$ ,  $\lambda_b^2$ ,  $\lambda_\tau^2$ , and  $g_3^2$ , with the following exception: when integrating out the superpartners and matching to the 2HSM, we are neglecting operators of dimension  $> 4$  which are suppressed by the superpartner masses; when calculating the top (rather than bottom or tau) mass, this amounts to neglecting finite terms of order  $m_t^2/m_{\text{superpartner}}^2$ , which is not valid in the case of a light superpartner but will numerically be sufficiently accurate. Second, corrections proportional to electroweak gauge couplings have only been included in leading  $\log(m_{\text{SUSY}})$  approximation; this means that we have neglected finite parts from SUSY loops and all electroweak gauge-boson loops. Third, we keep only the dominant diagrams (namely the ones in Fig. 1) from the class of finite diagrams proportional to  $\epsilon_{\mathcal{P}\mathcal{Q}}\epsilon_{\mathcal{R}}$ ,  $\epsilon_{\mathcal{P}\mathcal{Q}}^2$  and  $\epsilon_{\mathcal{R}}^2$ ; the rest contribute at most  $\sim 1\%$  effects, or much less if  $\epsilon_{\mathcal{P}\mathcal{Q}} < 1$  and  $\epsilon_{\mathcal{R}} < 1$ . Fourth, when integrating out the Higgs sector we neglect the effects of various  $1/\tan\beta$  mixings. Finally, in the numerical results we have grouped together and assigned common average masses to the squarks ( $m_{\bar{q}}$ ), the sleptons ( $m_{\bar{\ell}}$ ), and the higgsinos ( $m_{\bar{h}}$ ).

The evolution of the Yukawa and gauge couplings from  $\mu_G$  to  $\mu_Z$  in the MSSM, even to 1-loop accuracy, cannot in general be calculated analytically. Numerical solutions are straightforward, and show that, in order to obtain the correct  $m_b/m_\tau$  ratio, the GUT-scale Yukawa couplings must be  $\mathcal{O}(1)$ , so the top mass is typically predicted near its fixed point value [16,17,24]. With this in mind, it will be useful and illuminating to obtain simple, explicit approximate solutions by fitting  $\lambda_{t,b,\tau}(\mu_Z)$  numerically to a quadratic polynomial in  $1/\lambda_G$ . The constant term reflects the independence of the low-energy Yukawa couplings on the initial conditions in the large- $\lambda_G$  limit, a consequence of the fixed point behavior. When varying  $\lambda_G$  between 0.5 and 2, we find in the unperturbed scenario

$$\lambda_{t,b,\tau}(\mu_Z) \simeq A_{t,b,\tau} + \frac{B_{t,b,\tau}}{\lambda_G} + \frac{C_{t,b,\tau}}{\lambda_G^2} \quad (11)$$

where  $A_{t,b,\tau} = (1.099, 1.014, 0.673)$ ,  $B_{t,b,\tau} = (-0.045, -0.012, -0.107)$ , and  $C_{t,b,\tau} = (-0.019, -0.025, 0.001)$ ; these values correspond to  $\mu_Z = 90 \text{ GeV}$  and to  $g_3^G = g_G$  (that is, no superheavy threshold corrections) which leads to  $\alpha_3(m_Z) = 0.125$ . We have checked that the errors we make in this fit are smaller than 0.2% over the entire range  $0.5 \leq \lambda_G \leq 2.0$ . We may then solve the quadratic equation  $R = (A_b + B_b/\lambda_G + C_b/\lambda_G^2)/(A_\tau + B_\tau/\lambda_G + C_\tau/\lambda_G^2)$  to obtain  $\lambda_G$  as a function of  $R$ , which in turn gives an explicit solution for  $\lambda_t(\mu_Z)$  as a function  $F(R)$  of the experimentally-determined ratio  $R$ . The functions (11), as well as the ratio  $R = \lambda_b(\mu_Z)/\lambda_\tau(\mu_Z)$ , are plotted versus  $\lambda_G$  in Fig. 2a, while  $F(R)$  is shown in Fig. 3a.

FIG. 3. The induced dependence  $\lambda_t(m_Z) = F(R)$  derived from Fig. 2, as well as the sensitivity functions  $\{F_n^{(\epsilon)}(R)\}$  and  $\{F_\nu^{(k)}(R)\}$ .



## V. THE MASS OF THE BOTTOM QUARK

What is the experimental value of  $R$ ? Using updated values from the Particle Data Group compilation [29], we have reanalyzed the well-known extraction of the  $b$  mass from  $e^+e^-$  collisions via QCD sum rules [30]. The idea is to use a dispersion relation to relate the experimental spectral distribution of  $e^+e^- \rightarrow b\bar{b}$  to the expectation value of the product of two vector  $b$ -quark currents. This product is then rewritten as an operator product expansion; perturbative QCD is used to calculate the coefficients of the identity and other operators in the expansion, while the nonperturbative information is assumed to be contained in the condensates of these other operators. For the  $b$  system, one expects the nonperturbative terms to be at most  $\mathcal{O}(\langle\bar{\psi}\psi\rangle/m_b^3 \sim \Lambda_{\text{QCD}}^3/m_b^3)$  and therefore negligible. The remaining calculation is purely perturbative QCD, so the uncertainty in  $m_b$  will be dominated by our ignorance of the  $\mathcal{O}(\alpha_3^2)$  terms in the calculation of the coefficient of the identity operator. This coefficient can be calculated reliably only for highly off-shell momenta, for instance  $q^2 \ll m_b^2$ . Expanding the coefficient in powers of  $q^2/m_b^2$  results in a relation between the moments of the spectral distribution and derivatives of the coefficient at  $q^2 = 0$ :

$$\mathcal{M}_n^{\text{expt}} = \frac{27}{4\pi\alpha_{\text{em}}^2} \left[ \sum_V \frac{\Gamma_V}{M_V^{2n+1}} + \frac{\alpha_{\text{em}}^2}{27\pi} \left( 1 + \frac{\alpha_3}{\pi} \right) \frac{1}{nE_T^{2n}} \right] \quad (12)$$

$$\mathcal{M}_n^{\text{theor}} = \frac{\mathcal{M}_{0n}}{m_{b,E}^{2n}} \left( 1 + A_n\alpha_3 + \mathcal{O}(\alpha_3^2) \right) \quad (13)$$

where the sum approximating the spectral integral is over the various resonances  $V$  characterized by a mass  $M_V$  and a width  $\Gamma_V$ ,  $E_T \simeq 10.56$  GeV is the estimated threshold energy where the continuum  $b\bar{b}$  production begins, and  $\mathcal{M}_{0n}$  and  $A_n$  are numerical constants given in reference [30].

The mass is extracted from the ratios of the first few successive moments,

$$r_n \equiv \frac{\mathcal{M}_n}{\mathcal{M}_{n-1}} \simeq \frac{r_{0,n}}{m_{b,E}^2} \left( 1 + a_n\alpha_3 + \kappa\alpha_3^2 \right) \quad (14)$$

where  $r_{0,2} = -0.00452$ ,  $r_{0,3} = -0.00462$ ,  $a_1 = -0.0286$ ,  $a_2 = -0.197$ , and  $\kappa$  has not yet been calculated but is expected to be at most  $\mathcal{O}(1)$  for the first few moments. We will make a very rough estimate of our uncertainty by allowing  $\kappa$  to vary between  $-2$  and  $2$ . The strong coupling  $\alpha_3$  in these expressions must be run down from its given value at  $m_Z$ . The parameter  $m_{b,E}$  is the Euclidean pole mass of Georgi and Politzer [31], which is related to the running  $\overline{\text{MS}}$  mass we need via

$$m_{b,E} = m_b(\mu) \left[ 1 + \frac{\alpha_3}{\pi} \left( \frac{4}{3} - 2 \ln 2 - 2 \ln \frac{m_{b,E}}{\mu} \right) \right]. \quad (15)$$

Using the first three moments, we obtain  $3.93 \text{ GeV} < m_b(4.1 \text{ GeV}) < 4.36 \text{ GeV}$  if  $\alpha_3(m_Z) = 0.11$ , and  $3.86 \text{ GeV} < m_b(4.1 \text{ GeV}) < 4.42 \text{ GeV}$  if  $\alpha_3(m_Z) = 0.12$ . The central values of  $m_b(4.1 \text{ GeV})$  extracted from the next few moments fall well within these ranges, providing some confidence that our error bars are not too small. Thus we estimate

$$m_b(m_b) = \begin{cases} 4.15 \text{ GeV} \pm 0.22 \text{ GeV}, & \alpha_3(m_Z) = 0.11 \\ 4.14 \text{ GeV} \pm 0.28 \text{ GeV}, & \alpha_3(m_Z) = 0.12 \end{cases}. \quad (16)$$

The central values we extract for  $m_b$  are not very sensitive to  $\alpha_3$  since the  $a_n \alpha_3 \ll 1$ ; they are somewhat lower than in the older analyses mainly because the more precise experimental value we use for the electronic partial width of the  $\Upsilon(9460)$  is higher than the older value. Our error bars in  $m_b$  are larger than those of Gasser and Leutwyler [32] because of the different ways we estimate the error from the  $\mathcal{O}(\alpha_3^2)$  terms.

The 2-loop QCD evolution between  $4.1 \text{ GeV}$  and  $\mu_Z$  reduces the mass of the  $b$  by a factor  $\eta_b \simeq 1.437 + 0.075[\alpha_3(m_Z) - 0.115]/0.01$  for  $\mu_Z = 90 \text{ GeV}$ ; the corresponding electromagnetic reduction factor for  $m_\tau$  is  $\eta_\tau \simeq 1.016$ . The translation from  $\overline{\text{MS}}$  to  $\overline{\text{DR}}$  increases  $m_b$  by roughly half a percent and has virtually no effect on  $m_\tau$ . Finally, we match the 0HSM model to the 2HSM by including the radiative corrections of the Yukawa couplings via the function  $f_R(m_A)$ ,

$$\begin{aligned} f_R(m_A) &\equiv R^{2\text{HSM}}/R^{0\text{HSM}} - 1 \\ &\simeq -0.014 \ln(m_A/m_Z) \quad \text{when } \mu_Z = m_Z \quad \text{and } m_A > m_Z \end{aligned} \quad (17)$$

and arrive at the final value of  $R$ :

$$\begin{aligned} R &= \frac{m_b^{\overline{\text{MS}}}(4.1 \text{ GeV})}{m_\tau} \frac{\eta_\tau}{\eta_b} \left[ 1 - \frac{\alpha_3(\mu_Z)}{3\pi} \right] [1 + f_R(m_A)] \\ &= \begin{cases} 1.67 \pm 0.09, & \alpha_3(m_Z) = 0.11, & m_A = 90 \text{ GeV} \\ 1.58 \pm 0.11, & \alpha_3(m_Z) = 0.12, & m_A = 90 \text{ GeV} \\ 1.62 \pm 0.09, & \alpha_3(m_Z) = 0.11, & m_A = 1000 \text{ GeV} \\ 1.53 \pm 0.11, & \alpha_3(m_Z) = 0.12, & m_A = 1000 \text{ GeV}. \end{cases} \end{aligned} \quad (18)$$

The exact expression for  $f_R$  may be found in the appendix. Notice that a heavy second Higgs decreases the apparent experimental value of  $R$ , or more intuitively increases the GUT prediction of  $R$ —hence to agree with the experimental value, we need to lower the prediction of  $R$ , which entails raising  $\lambda_G$  and with it  $m_t$ . The lightest top masses result from a light Higgs sector.

## VI. IMPLICATIONS OF THE LARGE CORRECTIONS

With the experimental value of  $m_b$  in hand, we can return to the threshold corrections of Eq. (8) to bound the allowed range of MSSM parameters and to possibly allow different initial conditions for the Yukawa couplings at the GUT scale.

First, let us see how large can the corrections to  $m_b$  become before  $m_b$  as predicted from Yukawa unification disagrees with the experimental range given above. We focus on the (usually dominant) gluino contribution; similar considerations will bound the higgsino diagram. It is convenient to first divide Eq. (8) by  $m_\tau$ , and substitute  $\tan\beta = (m_t/m_\tau)(\lambda_\tau/\lambda_t) = (m_t/m_\tau)(\lambda_b/\lambda_t)R_{\text{MSSM}}^{-1} \simeq 0.95R_{\text{MSSM}}^{-1}(m_t/m_\tau)$  and  $I(m_{b,+}^2, m_{b,-}^2, m_g^2) \equiv 1/m_{\text{eff}}^2$ , to obtain:

$$\begin{aligned} R_{\text{exp}} = \frac{m_b}{m_\tau} &= R_{\text{MSSM}} + \left(0.95 \frac{8}{3} \frac{\alpha_3}{4\pi}\right) \frac{m_t(R_{\text{MSSM}})}{m_\tau} \left(\frac{m_{\tilde{g}}\mu}{m_{\text{eff}}^2}\right) \\ &\simeq R_{\text{MSSM}} + \frac{m_t(R_{\text{MSSM}})}{73 \text{ GeV}} \left(\frac{m_{\tilde{g}}\mu}{m_{\text{eff}}^2}\right). \end{aligned} \quad (19)$$

Note that  $m_t$  depends on  $\lambda_G$  which is in turn determined by  $R_{\text{MSSM}}$ . The  $R_{\text{exp}}$  on the left-hand side is the experimental value  $R$  extracted above, while  $R_{\text{MSSM}}$  on the right-hand side denotes the value of  $\lambda_b/\lambda_\tau$  obtained by running down in the MSSM from the GUT scale. Also, to first approximation  $m_t(R_{\text{MSSM}}) \simeq (174 \text{ GeV}) \lambda_t(R_{\text{MSSM}})$ . As we will establish from Eq. (29b) below, and as illustrated in Fig. 2a,  $R_{\text{MSSM}}$  is bounded from below by roughly 1.6, corresponding to  $\lambda_G \rightarrow \infty$ ; and from above by roughly 2.4, corresponding to  $\lambda_G \rightarrow 0$ . Since the latter limit also corresponds to  $m_t \rightarrow 0$ , it can be improved by enforcing the experimental bound  $m_t > 130 \text{ GeV}$ , so we conclude that  $1.6 < R_{\text{MSSM}} < 2.15$ . We use these bounds and those of Eq. (18) to set limits on  $(m_{\tilde{g}}\mu/m_{\text{eff}}^2)$ : if this quantity is positive, an upper bound results from taking the smallest  $R_{\text{MSSM}}$  possible and the largest  $R_{\text{exp}}$  allowed, in which case  $m_t$  is fixed at its maximal value; if that quantity is negative, the largest  $R_{\text{MSSM}}$  and smallest  $R_{\text{exp}}$  are needed, in which case  $m_t \simeq 130 \text{ GeV}$ . In this way we find the surprisingly stringent limits

$$-0.37 \lesssim \frac{m_{\tilde{g}}\mu}{m_{\text{eff}}^2} \lesssim 0.08 \quad (20)$$

which are obviously phenomenologically interesting signatures, but are also relevant to the electroweak symmetry-breaking sector of this model [20].

Second, recall that if we impose the GUT-scale initial condition  $3\lambda_t = 3\lambda_b = \lambda_\tau \equiv \lambda_G$  corresponding to mass generation through a  $\underline{126}_H$ , then after evolving down to the electroweak scale the prediction for  $R$  is too small. Large  $\delta m_b$  corrections can restore agreement with experiment. Fig. 2b shows  $R$  as well as  $\lambda_t$ ,  $\lambda_b$  and  $\lambda_\tau$  at the electroweak scale as functions of  $\lambda_G$ . In the presence of the  $\underline{126}_H$ , the coupling  $\lambda_G$  must be kept below

$\sim 1.5$  (rather than  $\sim 2$  for the  $\underline{10}_H$ ) to raise the Landau pole by a factor of 4 above the unification mass. Also, since  $m_t \sim (174 \text{ GeV})\lambda_t$ , the lower bound of  $m_t > 130 \text{ GeV}$  implies  $\lambda_t > 0.75$  and hence from the figure  $\lambda_G > 0.82$ . Within this restricted range,  $R_{\text{MSSM}}$  varies between 1.0 and 1.1, so to reconcile this with  $R_{\text{exp}}$  (now using  $\lambda_b/\lambda_t \simeq 0.89$  and again focusing on the gluino diagram) requires

$$0.24 \lesssim \frac{m_{\tilde{g}}\mu}{m_{\text{eff}}^2} \lesssim 0.42. \quad (21)$$

We learn that *if* the gluino, higgsino and squark mass parameters satisfy  $m_{\tilde{g}}\mu/m_{\text{eff}}^2 = 0.33 \pm 0.09$  and *if*  $\lambda_G \simeq 1.1 \pm 0.3$  then the mass may originate perturbatively from the coupling  $\underline{16}_3 \underline{126}_H \underline{16}_3$ . Notice that in such a scenario, even if  $m_{\tilde{g}}\mu/m_{\text{eff}}^2$  is known precisely, then the uncertainty in  $R_{\text{exp}}$  is large enough that the top mass prediction will usually be very imprecise.

Thus for Yukawa unification in the MSSM it is useful to distinguish four regions of parameter space:

1. If  $|m_{\tilde{g}}\mu/m_{\text{eff}}^2| \ll 1$  then the  $\delta m_b$  corrections may be ignored, the mass must originate from a  $\underline{16}_3 \underline{10}_H \underline{16}_3$  interaction, and we can predict  $m_t$  with little further dependence on the MSSM parameters, as shown below;
2. If  $-0.37 \lesssim m_{\tilde{g}}\mu/m_{\text{eff}}^2 \lesssim 0.08$  then the mass must still arise from a  $\underline{10}_H$ , but now the prediction for  $m_t$  depends very sensitively upon  $m_{\tilde{g}}\mu/m_{\text{eff}}^2$  and can vary over the full experimentally-allowed range;
3. If  $0.24 \lesssim m_{\tilde{g}}\mu/m_{\text{eff}}^2 \lesssim 0.42$  then the  $\underline{126}_H$  must be used, with  $\lambda_G \simeq 1.1 \pm 0.3$ . In this case the prediction, while imprecise, tends to be in the lower half of the experimentally-allowed range;
4. If  $m_{\tilde{g}}\mu/m_{\text{eff}}^2$  lies outside of these ranges then perturbative Yukawa unification under our assumptions cannot be reconciled with experiment.

## VII. THRESHOLD CORRECTIONS

Next, we investigate the deviations in the top mass prediction induced by threshold corrections at the GUT and SUSY scales. For convenience, we choose to *always* match the full SO(10) theory with the MSSM at the scale  $M_G$  of the unperturbed scenario, where  $g_1$  and  $g_2$  met: we *define* from now on  $\mu_G = 2.7 \times 10^{16} \text{ GeV}$ . The top mass prediction as a function of  $R$  is completely determined once we know the initial conditions  $\{\lambda_t^G, \lambda_b^G, \lambda_\tau^G, g_1^G, g_2^G, g_3^G\}$  at the fixed scale  $\mu_G$ , as well as the functions  $k_{t,b,\tau}(\{m_a\})$  of the superpartner masses which match between the MSSM (in which we run with the RGE)

and the 2-Higgs standard model (in which we calculate the top pole mass). So we first calculate the perturbations  $\epsilon_{t,b,\tau,1,2,3}$  to the initial conditions and the matching functions  $k_{t,b,\tau}$  in terms of the various mass thresholds and  $\alpha_3(m_Z)$ . Then we study the sensitivity of the top mass prediction to these  $\epsilon$ 's and  $k$ 's. A linear analysis of these perturbations is sufficient for our purposes.

Define for convenience

$$t_a \equiv \ln(m_a/m_Z), \quad T_\alpha \equiv \ln(M_\alpha/\mu_G), \quad (22a)$$

$$t_{a,b} \equiv G_1(m_a^2/\mu_Z^2, m_b^2/\mu_Z^2) \simeq \begin{cases} \ln(m_a/\mu_Z) - \frac{1}{4}, & m_a \gg m_b, \\ \ln(m_{a=b}/\mu_Z), & m_a = m_b, \\ \ln(m_b/\mu_Z) - \frac{3}{4}, & m_a \ll m_b, \end{cases} \quad (22b)$$

$$t'_{a,b,c} \equiv \ln \frac{\max(m_a, m_b, m_c)}{\mu_Z}, \quad (22c)$$

where  $G_1$  is given in the appendix,  $\{m_a\}$  are the masses of the various superpartners and  $\{M_\alpha\}$  are those of the superheavy particles. The functions  $t_a$  and  $t_{a,b}$  yield the exact threshold corrections and will be used in the dominant terms in  $k_{t,b,\tau}$  below. The function  $t'_{a,b,c}$  only yields threshold corrections in the leading  $\log(m_{\text{SUSY}})$  approximation and will be used in the subdominant terms. The contributions of the superpartners to the gauge  $\beta$ -functions will be denoted by  $b_i^a$ . Superheavy threshold corrections can arise from couplings dressing the  $\underline{16}_3 \underline{10}_H \underline{16}_3$  vertex and having strength  $g_G$  (the gauge 45),  $\lambda_G$  (the  $\underline{16}_3$  and  $\underline{10}_H$ ), or some other  $\lambda'_A$ . We write the corresponding contributions of any superheavy particle with mass  $M_\alpha$  to the Yukawa RGE by  $L_\nu^\alpha g_G^2$ ,  $K_\nu^\alpha \lambda_G^2$  and  $\sum_A K_{\nu A}^{\prime\alpha} \lambda_A^{\prime 2}$ . We denote squarks, sleptons, higgsinos, winos, binos and gluinos by  $\tilde{q}$ ,  $\tilde{\ell}$ ,  $\tilde{h}$ ,  $\tilde{w}$ ,  $\tilde{b}$ , and  $\tilde{g}$ , and define  $\alpha_G \equiv g_G^2/4\pi$  and  $y_x \equiv \lambda_x^2/4\pi$  for any  $x$ . We will neglect all electroweak-breaking effects in the mass splittings, and the few subdominant corrections mentioned below. We expect all such effects to alter  $m_t$  by less than a GeV or so.

The gauge couplings at  $\mu_G$  may be completely determined by their low-energy values  $\alpha_{1,2,3}(m_Z)$  and by the SUSY spectrum simply by running them up from the Z mass to  $\mu_G$ . When the superpartner masses are changed away from the 90 GeV value of the unperturbed scenario, the couplings  $g_1^G$  and  $g_2^G$  at  $\mu_G$  deviate from the common value  $g_G = 0.730$  of the unperturbed scenario in an easily-calculable way. Similarly,  $g_3^G$  deviates from this  $g_G$  value when the masses of the colored superpartners changes and also when we vary [33]  $\alpha_3(m_Z)$  away from 0.125. While these changes to  $g_i^G$  are calculated using the experimentally-accessible quantities  $\{m_a\}$  and  $\alpha_3(m_Z)$ , from a top-down viewpoint they should be regarded as the net threshold corrections resulting from integrating out the superheavy (or Planck-scale [34]) degrees of freedom in the SO(10) theory to arrive at the MSSM. The initial conditions for the gauge couplings become

$$g_{1G} \equiv g_G(1 + \epsilon_1) = g_G \left[ 1 - \frac{\alpha_G}{4\pi} \left( \frac{11}{10}t_{\tilde{q}} + \frac{9}{10}t_{\tilde{\ell}} + \frac{2}{5}t_{\sqrt{2}\tilde{h}} \right) \right], \quad (23a)$$

$$g_{2G} \equiv g_G(1 + \epsilon_2) = g_G \left[ 1 - \frac{\alpha_G}{4\pi} \left( \frac{3}{2}t_{\tilde{q}} + \frac{1}{2}t_{\tilde{\ell}} + \frac{4}{3}t_{\sqrt{2}\tilde{w}} + \frac{2}{3}t_{\sqrt{2}\tilde{h}} \right) \right], \quad (23b)$$

$$g_{3G} \equiv g_G(1 + \epsilon_3) = g_G \left[ 1 - \frac{\alpha_G}{4\pi} (2t_{\tilde{q}} + 2t_{\sqrt{2}\tilde{g}}) + \frac{1}{2} \frac{\alpha_G}{\alpha_3(m_Z)} \left( \frac{\delta\alpha_3}{\alpha_3} \right) \right] \quad (23c)$$

where  $\delta\alpha_3/\alpha_3 \equiv [\alpha_3(m_Z) - 0.125]/0.125$ . (Note that the fermionic superpartners  $\tilde{w}$ ,  $\tilde{h}$  and  $\tilde{g}$  must be integrated out at  $\sqrt{2}$  times their mass.) The Yukawa couplings at  $M_G$  differ from  $\lambda_G$  due to GUT thresholds effects, and hence

$$\lambda_{tG} \equiv \lambda_G(1 + \epsilon_t) = \lambda_G \left[ 1 + \frac{1}{4\pi} \sum_{\alpha} (K_t^{\alpha} y_G + L_t^{\alpha} \alpha_G + \sum_A K_{tA}^{\prime\alpha} y'_A) T_{\alpha} \right], \quad (24a)$$

$$\lambda_{bG} \equiv \lambda_G(1 + \epsilon_b) = \lambda_G \left[ 1 + \frac{1}{4\pi} \sum_{\alpha} (K_b^{\alpha} y_G + L_b^{\alpha} \alpha_G + \sum_A K_{bA}^{\prime\alpha} y'_A) T_{\alpha} \right], \quad (24b)$$

$$\lambda_{\tau G} \equiv \lambda_G(1 + \epsilon_{\tau}) = \lambda_G \left[ 1 + \frac{1}{4\pi} \sum_{\alpha} (K_{\tau}^{\alpha} y_G + L_{\tau}^{\alpha} \alpha_G + \sum_A K_{\tau A}^{\prime\alpha} y'_A) T_{\alpha} \right]. \quad (24c)$$

Two remarks should be made at this point. First, the SUSY threshold corrections actually enter Eqs. (24) indirectly, even if all superheavy particles are degenerate, since these corrections generically change the scale at which  $g_1$  and  $g_2$  meet and therefore change the predicted average mass of the superheavy particles. The result is a shift in all the  $T_{\alpha}$  that is independent of  $\alpha$ . Second, any effect that contributes equally to  $\epsilon_t$ ,  $\epsilon_b$  and  $\epsilon_{\tau}$  makes no contribution to our final result, since this only amounts to a redefinition of  $\lambda_G$ .

Finally, the superpartner masses induce threshold corrections to the Yukawa couplings when they are integrated out of the MSSM to yield the 2-Higgs standard model. We find

$$4\pi k_t = y_t \left( \frac{1}{2}t_{\tilde{t}_R, \tilde{h}} + t_{\tilde{t}_L, \tilde{h}} \right) + \frac{1}{2}y_b t_{\tilde{b}_R, \tilde{h}} + \frac{4}{3}\alpha_3 \left( t_{\tilde{t}_L, \tilde{g}} + t_{\tilde{t}_R, \tilde{g}} \right) + 3\alpha_2 \left( \frac{1}{4}t_{\tilde{t}_L, \tilde{w}} + \frac{1}{2}t_{\tilde{h}, \tilde{w}} - t'_{\tilde{w}, \tilde{t}_L, \tilde{h}} \right) + \frac{3}{20}\alpha_1 \left( \frac{1}{9}t_{\tilde{t}_L, \tilde{b}} + \frac{16}{9}t_{\tilde{t}_R, \tilde{b}} + 2t_{\tilde{h}, \tilde{b}} - \frac{16}{3}t'_{\tilde{b}, \tilde{h}, \tilde{t}_R} + \frac{4}{3}t'_{\tilde{b}, \tilde{h}, \tilde{t}_L} \right), \quad (25a)$$

$$4\pi k_b = y_b \left( \frac{1}{2}t_{\tilde{b}_R, \tilde{h}} + t_{\tilde{b}_L, \tilde{h}} \right) + \frac{1}{2}y_t t_{\tilde{t}_R, \tilde{h}} + \frac{4}{3}\alpha_3 \left( t_{\tilde{b}_L, \tilde{g}} + t_{\tilde{b}_R, \tilde{g}} \right) + 3\alpha_2 \left( \frac{1}{4}t_{\tilde{b}_L, \tilde{w}} + \frac{1}{2}t_{\tilde{h}, \tilde{w}} - t'_{\tilde{w}, \tilde{b}_L, \tilde{h}} \right) + \frac{3}{20}\alpha_1 \left( \frac{1}{9}t_{\tilde{b}_L, \tilde{b}} + \frac{4}{9}t_{\tilde{b}_R, \tilde{b}} + 2t_{\tilde{h}, \tilde{b}} - \frac{8}{3}t'_{\tilde{b}, \tilde{h}, \tilde{b}_R} - \frac{4}{3}t'_{\tilde{b}, \tilde{h}, \tilde{b}_L} \right) + 4\pi k'_b, \quad (25b)$$

$$4\pi k_{\tau} = y_{\tau} \left( \frac{1}{2}t_{\tilde{\tau}_R, \tilde{h}} + t_{\tilde{\tau}_L, \tilde{h}} \right) + 3\alpha_2 \left( \frac{1}{4}t_{\tilde{\tau}_L, \tilde{w}} + \frac{1}{2}t_{\tilde{h}, \tilde{w}} - t'_{\tilde{w}, \tilde{\tau}_L, \tilde{h}} \right) + \frac{3}{20}\alpha_1 \left( t_{\tilde{\tau}_L, \tilde{b}} + 4t_{\tilde{\tau}_R, \tilde{b}} + 2t_{\tilde{h}, \tilde{b}} - 8t'_{\tilde{b}, \tilde{h}, \tilde{\tau}_R} + 4t'_{\tilde{b}, \tilde{h}, \tilde{\tau}_L} \right) \quad (25c)$$

where the  $y_{\nu}$  and  $\alpha_i$  are evaluated at  $\mu_Z$ . The non-logarithmic threshold correction of Eq. (8) must be included if the squarks are not much heavier than  $\mu$ ,  $m_{\tilde{g}}$  and  $A_t$ :

$$k'_b = \frac{\tan\beta}{4\pi} \left( \frac{8}{3}\alpha_3 \frac{m_{\tilde{g}}\mu}{m_{\text{eff}}^2} + \frac{\lambda_t^2}{4\pi} \frac{\mu A_t}{m_{\text{eff}}^2} \right) \quad (26)$$

where  $m_{\text{eff}}^2 \equiv 1/I(m_{\tilde{b},+}^2, m_{\tilde{b},-}^2, m_{\tilde{g}}^2)$  and  $m_{\text{eff}'}^2 \equiv 1/I(m_{\tilde{t},+}^2, m_{\tilde{t},-}^2, \mu^2)$ .

Note that the majority of the terms in  $k_\nu$  are due to wavefunction renormalization from scalar-fermion interactions and hence increase with the superpartner masses; the eventual conclusion will be that the smallest  $m_t$  is predicted when all superpartners are as light as possible (if  $k'_b$  may be neglected, as discussed in Sec. III).

The mass of the Higgs bosons (which are determined by  $m_W$  and  $m_A$ ) enter the top prediction through the matching of  $R$  between the 0HSM and the 2HSM, and through the calculation of the top quark pole mass in the 2HSM. The former was included in the previous section as a correction to  $R$ , while the latter is studied in Sec. IX.

### VIII. SENSITIVITY FUNCTIONS

When the initial conditions are perturbed, so are the coefficients of the fit in Eq. (11). We expect the fit parameters to vary linearly with the perturbations as long as these are sufficiently small, and so we write

$$\lambda_\nu(\mu_Z) = \left[ \left( A_\nu + \sum_n a_{\nu n} \epsilon_n \right) + \frac{B_\nu + \sum_n b_{\nu n} \epsilon_n}{\lambda_G} + \frac{C_\nu + \sum_n c_{\nu n} \epsilon_n}{\lambda_G^2} \right] (1 + k_\nu) \quad (27)$$

where the sum ranges over  $n = t, b, \tau, 1, 2, 3$ . The new fit coefficients  $\{a_{\nu n}, b_{\nu n}, c_{\nu n}\}$  must be computed numerically. We then solve  $R = \lambda_b(m_t)/\lambda_\tau(m_t)$  for  $\lambda_G$  as before, substitute back into Eq. (27) and expand to first order in  $\epsilon_n$  and  $k_\nu$  to find

$$\lambda_t(\mu_Z) = F(R) + \sum_n F_n^{(\epsilon)}(R) \epsilon_n + \sum_\nu F_\nu^{(k)}(R) k_\nu. \quad (28)$$

$F(R)$  and the five ‘‘sensitivity functions’’  $F_{t,b,\tau,1,2}^{(\epsilon)}(R)$  are plotted in Fig. 3a while the four sensitivity functions  $F_3^{(\epsilon)}(R)$  and  $F_{t,b,\tau}^{(k)}(R)$  appear in Fig. 3b. We have checked that, for the entire range  $0.105 \leq \alpha_3(m_Z) \leq 0.13$  and  $\{m_a\} \leq 3 \text{ TeV}$ , our approximation is off by at most  $\sim 1\%$ .

We can understand the behavior of  $\lambda_t(\mu_Z)$  shown in Figs. 2a and 3a,b as follows.

- First, as is well known [15],  $\lambda_t$  and  $\lambda_b$  (and to a lesser extent  $\lambda_\tau$ ) are quite insensitive to  $\lambda_G$  for large  $\lambda_G$ , since they both tend towards a fixed point as  $\lambda_G \rightarrow \infty$ . The fixed-point behavior of  $\lambda_t$  is manifested in the smallness of  $F_t^{(\epsilon)}(R)$  — changing the initial  $\lambda_t^G$  by 10% changes its final value  $\lambda_t(\mu_Z)$  by at most 1%. The sensitivity of  $\lambda_t(\mu_Z)$  to  $\lambda_t^G$  is even less for small  $R$ , since small  $R$  implies large  $\lambda_G$  and hence a stronger fixed-point behavior for  $\lambda_t$ .
- The value at the fixed point is determined predominantly by  $\alpha_3$  at low energies, and hence  $F_3^{(\epsilon)}(R)$  is large; conversely,  $\alpha_{1,2}$  are smaller at low energies and hence have little influence, so  $F_{1,2}^{(\epsilon)}(R) \ll 1$ .

- Next, we establish the fixed-point nature of  $\lambda_b(t)/\lambda_\tau(t) \equiv R(t)$  by examining its RGE:

$$16\pi^2 \frac{d \ln R(t)}{dt} = \lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2 + \frac{4}{3}g_1^2 - \frac{16}{3}g_3^2 \quad (29a)$$

$$\sim \lambda_t^2 \left(4 - \frac{3}{R^2}\right) - \frac{16}{3}g_3^2 \quad (29b)$$

where in the last line we have made the rough approximations  $\lambda_t \simeq \lambda_b$  and  $g_3^2 \gg g_1^2$ . For very small  $\lambda_G$  the first term is negligible, and  $R(t)$  is driven purely by the gauge coupling evolution from  $R^G = 1$  to  $R(\mu_Z) \simeq 2.4$ , independent of  $\lambda_G$ . For very large  $\lambda_G$ , the first term dominates at the beginning, but now  $R$  decreases almost instantly until  $R \simeq \sqrt{3/4}$  while  $\lambda_t$  decreases quickly and becomes independent of  $\lambda_G$ ; the subsequent evolution from these effective initial conditions to  $R(\mu_Z) \simeq 1.6$  is therefore also independent of  $\lambda_G$ . Thus  $R(t)$  has a fixed-point behavior as a function of  $\lambda_G$  for both small and large  $\lambda_G$ . The values of interest to us,  $0.5 \leq \lambda_G \leq 2.0$ , lie in the intermediate- to large- $\lambda_G$  range, which is why  $R(\mu_Z)$  becomes less sensitive to  $\lambda_G$  as  $\lambda_G \rightarrow 2$ .

- To understand  $F(R)$ , observe that both  $R$  and  $\lambda_t$  display similar fixed-point dependence on  $\lambda_G$ , so when we eliminate  $\lambda_G$  to obtain  $\lambda_t(\mu_Z) = F(R)$  we arrive at a roughly linear dependence of  $\lambda_t$  on  $R$ .
- The behavior of  $F_{b,\tau}^{(\epsilon)}(R)$  follows from similar reasoning. For small  $\lambda_G$ , changing  $R^G$  via  $\epsilon_b - \epsilon_\tau$  has a reasonably large effect on  $R(m_t)$ , which requires (in order to match the experimental value) a moderate change in  $\lambda_G$  — see Fig. 2a. For large  $\lambda_G$ , the same change in  $R^G$  has only a small effect on  $R(\mu_Z)$  due to the fixed-point behavior, but now this small effect gets magnified back into a moderate change required in  $\lambda_G$ . Thus a fixed  $\epsilon_b - \epsilon_\tau$  always necessitates roughly the same change in  $\lambda_G$ . But the resulting change in  $\lambda_t(\mu_Z)$  is tiny for large  $\lambda_G$  or equivalently for small  $R$ , and that is why  $F_{b,\tau}^{(\epsilon)}(R)$  become small for small  $R$ . Now  $F_t^{(\epsilon)}(R) + F_b^{(\epsilon)}(R) + F_\tau^{(\epsilon)}(R) = 0$  because when  $\epsilon_t = \epsilon_b = \epsilon_\tau \equiv \epsilon$  the final value of  $m_t$  must not depend on  $\epsilon$  (this just amounts to a redefinition of  $\lambda_G$ ). Hence  $F_b^{(\epsilon)}(R) = -F_\tau^{(\epsilon)}(R) - F_t^{(\epsilon)}(R) \simeq -F_\tau^{(\epsilon)}(R)$ .
- Finally,  $F_t^{(k)}$  is by definition equal to  $F(R)$ , while  $F_b^{(k)}(R)$  and  $F_\tau^{(k)}(R)$  measure the changes in  $\lambda_t(\mu_Z)$  induced by changing  $R$  directly at the top mass scale, so their behavior follows immediately from the dependence of  $\lambda_t$  on  $R$  described above.

## IX. PREDICTIONS FOR THE TOP MASS

The final step is the calculation of the position of the pole in the propagator of the top quark within the 2-Higgs standard model. (Note that the pole mass is only a parameter



in the calculation of experimental observables; we leave the study of the relation between  $m_t^{\text{pole}}$  and the actual “top mass” extracted from collider data to future work.) There are two important radiative corrections to the pole mass: the usual QCD correction from gluon dressing, and Yukawa interaction corrections to the top quark propagator and to the Fermi constant. The result may be written as

$$\begin{aligned} m_t^{\text{pole}} &= \lambda_t^{2\text{HSM}} \left[ \frac{1}{\sqrt{2^{3/2} G_F}} \left( 1 + \frac{\Sigma_W(0)}{2m_W^2} \right) \right] \left( 1 + \frac{\delta m_t}{m_t} + \frac{1}{2} \delta_L^t + \frac{1}{2} \delta_R^t \right) (1 + \delta_{\text{QCD}}) \\ &= \lambda_t 177 \text{ GeV} [1 + f_t(m_A)]. \end{aligned} \quad (30)$$

In the last line we have substituted  $\lambda_t^{2\text{HSM}} \equiv \lambda_t$  and  $1 + \delta_{\text{QCD}} \equiv 1 + 5\alpha_3/3\pi - (8\alpha_3/4\pi) \ln(m_t/\mu_Z) \simeq 1.015$  when  $\mu_Z = m_Z$ , and defined  $f_t(m_A) \equiv \Sigma_W(0)/2m_W^2 + \delta m_t/m_t + \frac{1}{2} \delta_L^t + \frac{1}{2} \delta_R^t$ . One should not confuse the pole mass in the above equation and the Euclidean pole mass defined in Eq. (15): the one just above represents the real pole of the propagator at timelike momenta calculated in perturbative QCD, while the Euclidean pole mass does not actually correspond to any pole in the propagator (for obvious reasons). Notice also that the above top pole mass, which is scheme independent, has been written in terms of  $\overline{\text{DR}}$  quantities. The dominant 1-loop correction to the muon decay constant (used to define  $G_F$ ) is taken into account via  $247 \text{ GeV} = 1/\sqrt{2^{1/2} G_F} = v [1 - \Sigma_W(0)/2m_W^2]$  where  $\Sigma_W(0)$  is the top contribution to the self energy of the  $W$  at zero momentum. The wavefunction renormalization of the top quark propagator is given by  $\frac{1}{2} \delta_L^t$  and  $\frac{1}{2} \delta_R^t$ , while the mass renormalization  $\delta m_t$  is  $\mu_Z$ -independent and vanishes in the 't Hooft-Feynman gauge we employ. The complete expression for  $f_t$  is given in the appendix; to a good approximation,  $f_t \simeq 0.04 + 0.003 \ln m_A/\mu_Z$ .

The complete form of our prediction for  $m_t$  may be obtained by substituting the expressions for  $R$ ,  $\epsilon_n$ , and  $k_\nu$  into Eq. (28) and inserting the result into Eq. (30). To untangle the various dependences of the prediction on the SUSY- and GUT-scale parameters, we divide the various contributions into three classes, and discuss each separately before combining them into a prediction.

First, let us *ignore* both the (potentially large) finite  $\delta m_b$  corrections discussed in Secs. III and VI as well as GUT-scale thresholds, and concentrate on the logarithmic SUSY-scale threshold corrections. Varying only the superpartner masses affects the predictions in three ways: through  $k_\nu$ , through  $\epsilon_i$ , and (as remarked above) through the  $\alpha$ -independent shift in all the  $T_\alpha$ . The latter effect (which actually shifts all GUT-scale masses together) is shown below to be small. The first two can be significant, but *only* when they serve to *increase*  $m_t$ . For example, if  $\alpha_3(m_Z) = 0.11$ ,  $m_b(m_b) = 4.2 \text{ GeV}$  and  $m_A = 90 \text{ GeV}$ , then when all superpartners have a mass equal to  $m_Z$  we predict  $m_t = 167 \text{ GeV}$ . By allowing the various  $\{m_a\}$  to vary between  $m_Z$  and  $3 \text{ TeV}$ , we obtain

$m_t$  as high as 195 GeV but only as low as 163 GeV. To understand this fact, recall that  $F_3^{(\epsilon)}(R) \gg F_2^{(\epsilon)}(R) \sim F_1^{(\epsilon)}(R)$  so we may neglect  $\epsilon_{1,2}$  relative to  $\epsilon_3$ . In the three  $k_\nu$  the dominant terms are those proportional to  $y_t$ ,  $y_b$  and  $\alpha_3$ . Therefore, for fixed  $\alpha_3(m_Z)$  the top mass prediction depends predominantly (to within a few GeV) on the squark, gluino and higgsino masses rather than the slepton, wino and bino masses. Keeping only these terms, we find the approximate formula

$$\lambda_t \simeq F + \frac{\delta\alpha_3}{\alpha_3} \left[ \frac{1}{2} F_3^{(\epsilon)} \frac{\alpha_G}{\alpha_3} \right] - (t_{\tilde{q}} + t_{\sqrt{2}\tilde{g}}) \left[ 2F_3^{(\epsilon)} \frac{\alpha_G}{4\pi} \right] + t_{\tilde{q},\tilde{g}} \left[ \frac{8}{3} \frac{\alpha_3}{4\pi} (F + F_b^{(k)}) \right] + t_{\tilde{q},\tilde{h}} \left[ \frac{1}{2} \frac{y_t}{4\pi} (3F + F_b^{(k)}) + \frac{1}{2} \frac{y_b}{4\pi} (F + 3F_b^{(k)}) \right], \quad (31)$$

where the  $R$  dependence is implicit and we have used  $F_t^{(k)} = F$ . As it turns out, the positive terms proportional to  $t_{\tilde{q},\tilde{g}}$  and  $t_{\tilde{q},\tilde{h}}$  (from wavefunction renormalization in the  $k_\nu$  SUSY threshold corrections) are always larger in magnitude than the negative terms proportional to  $t_{\tilde{q}}$  and  $t_{\sqrt{2}\tilde{g}}$  (from  $\epsilon_3$  SUSY threshold corrections to the QCD gauge coupling). Thus—if the  $\delta m_b$  corrections are ignored—heavy superpartners inevitably lead to a heavier top.

The top mass prediction is further influenced [33] by  $\alpha_3(m_Z)$ , which changes not only the value of  $g_3$  used in the running of the Yukawa couplings but also the value of  $R$  as a function of  $m_b$ . Finally, the Higgs mass parameter  $m_A$  enters into the matching between  $R$  in the low-energy no-Higgs standard model and the  $R$  in the 2-Higgs standard model, and into the expression for the pole mass of the top quark. The resulting dependence of  $m_t^{\text{pole}}$  on  $m_{\tilde{q}}$ ,  $m_{\tilde{g}}$ ,  $\mu \sim m_{\tilde{h}}$ ,  $m_A$ ,  $m_b$  and  $\alpha_3(m_Z)$  is somewhat complicated, so we display it in two complementary ways. First, we list in Table I the predictions of our complete expressions for various choices of the parameters, again omitting the  $\delta m_b$  corrections of Sec. III. (Such an omission is clearly unjustified for many of the parameters chosen for this table—it is only intended to illustrate the conclusions of the previous paragraph.) In this table we have used as average values  $m_{\tilde{w}} = m_{\tilde{b}} = m_{\tilde{\tau}} = 500$  GeV. Second, we can approximate  $F$ ,  $F_b^{(k)}$  and  $F_3^{(\epsilon)}$  as linear functions of  $R$ , and further approximate  $y_t \simeq F^2/4\pi$  and  $y_b \simeq 0.87y_t$ . For a given set of values for  $\{m_A, \alpha_3(m_Z), m_b\}$  we obtain an expression

$$m_t = m_t^0 - c_3(t_{\tilde{q}} + t_{\sqrt{2}\tilde{g}}) + c_{\tilde{q},\tilde{g}} t_{\tilde{q},\tilde{g}} + c_{\tilde{q},\tilde{h}} t_{\tilde{q},\tilde{h}}. \quad (32)$$

The values of the  $c_i$  are given in Table II, and the resulting top mass values are accurate to within  $\sim \pm 5$  GeV when the various masses are varied between  $m_Z$  and 3 TeV.

Next, we consider the finite corrections to the  $b$  quark mass as discussed in Sec. III. As we saw, these are small only when the squared squark masses are much greater than  $m_{\tilde{g}}\mu$  and  $\mu A_t$ . In this case, choosing for concreteness  $m_{\tilde{h}} \sim \mu = 100$  GeV,  $m_{\tilde{g}} = 300$  GeV,  $m_{\tilde{w}} = 100$  GeV,  $m_{\tilde{q}} = m_{\tilde{\tau}} = 1000$  GeV and  $m_A = 1000$  GeV, and considering several

TABLE I. The top quark pole mass predictions for a range of values of the parameters. All masses are in GeV, and we have used set  $m_{\tilde{w}} = m_{\tilde{b}} = m_{\tilde{\ell}} = 500$  GeV, though the results are almost independent of these masses. We have not included the  $\delta m_b$  corrections which are large in some cases considered in this table.

$m_A$	90 GeV	1 TeV	90 GeV	1 TeV	90 GeV	1 TeV	90 GeV	1 TeV		
$\alpha_3(m_Z)$	0.11	0.11	0.12	0.12	0.11	0.11	0.12	0.12		
$m_b(m_b)$	4.4 GeV	4.4 GeV	4.4 GeV	4.4 GeV	4.0 GeV	4.0 GeV	4.2 GeV	4.2 GeV		
$m_{\tilde{q}}$	$m_{\tilde{g}}$	$m_{\tilde{h}}$								
90	90	90	157	164	174	179	172	177	180	184
90	90	3000	166	172	183	188	181	185	188	( <sup>a</sup> )
90	3000	90	161	168	181	186	177	182	186	( <sup>a</sup> )
90	3000	3000	169	177	189	195	185	190	( <sup>a</sup> )	( <sup>a</sup> )
3000	90	90	173	181	194	( <sup>a</sup> )	189	( <sup>a</sup> )	( <sup>a</sup> )	( <sup>a</sup> )
3000	90	3000	177	184	197	( <sup>a</sup> )	193	( <sup>a</sup> )	( <sup>a</sup> )	( <sup>a</sup> )
3000	3000	90	158	167	181	187	178	183	188	193
3000	3000	3000	162	171	184	190	181	186	191	( <sup>a</sup> )

<sup>a</sup>For these parameter values,  $\lambda_G > 2$ .

TABLE II. Coefficients of the approximate formula for the top mass, Eq. (32), which is accurate to  $\sim \pm 5$  GeV when the various masses are varied between  $m_Z$  and 3 TeV.

$m_A$	90 GeV	1 TeV	90 GeV	1 TeV	90 GeV	1 TeV	90 GeV	1 TeV
$\alpha_3(m_Z)$	0.11	0.11	0.12	0.12	0.11	0.11	0.12	0.12
$m_b(m_b)$	4.4 GeV	4.4 GeV	4.4 GeV	4.4 GeV	4.0 GeV	4.0 GeV	4.2 <sup>a</sup> GeV	4.2 <sup>a</sup> GeV
$m_t^0$ (GeV)	154	162	173	179	170	177	179	185
$c_3$ (GeV)	4.8	4.3	4.2	3.8	3.8	3.4	3.8	3.3
$c_{\tilde{q},\tilde{g}}$ (GeV)	8.6	8.0	7.9	7.4	7.4	6.9	7.4	6.9
$c_{\tilde{q},\tilde{h}}$ (GeV)	3.9	3.8	3.8	3.8	3.8	3.7	3.8	3.7

<sup>a</sup>For  $m_b(m_b) = 4.0$  GeV and  $\alpha_3(m_Z) = 0.12$ , most parameter choices lead to  $\lambda_G > 2$ .

values of  $\alpha_3(m_Z)$ , we obtain the predictions shown in Fig. 4 as solid lines. The pole mass of the top quark is shown as a function of the running  $m_b$  parameter (discussed in Sec. V) indicated on the upper horizontal axis. The allowed range for  $m_b$  according to Eq. (18), using  $\alpha_3(m_Z) = 0.12$ , is also shown on this axis. We learn that if the SUSY parameters are sufficiently hierarchical that  $\delta m_b$  corrections may be neglected, then the top quark is predicted to be heavier than  $\sim 175$  GeV for  $\alpha_3(m_Z) > 0.115$ . It should be remembered that the prediction for  $\alpha_3(m_Z)$  without GUT-scale threshold corrections is around 0.124, so values much lower than this correspond to large GUT-scale corrections to  $g_3$ . Furthermore, we find that perturbative Yukawa unification demands  $\alpha_3 \lesssim 0.125$ . These last two observations are in rough agreement with previous authors [24,35]. If it turns out that  $\delta m_b$  is significant, our predictions may change considerably and become highly dependent on the SUSY parameters. If  $\delta m_b > 0$ , then  $k'_b > 0$ , and since  $F_b^{(k)}(R) > 0$  the top mass prediction can only *increase*; that is, either the change is small and the top stays near its maximal value of  $\sim 180 - 190$  GeV, or else the change is too big and the corresponding SUSY parameters are excluded (see Sec. VI). On the other hand, if  $\delta m_b < 0$  the top mass prediction can be significantly reduced. We show in Fig. 4 the predictions that result from a light squark spectrum, namely  $m_{\tilde{h}} \sim \mu = 250$  GeV,  $m_{\tilde{g}} = 300$  GeV,  $m_{\tilde{w}} = 100$  GeV,  $m_{\tilde{q}} = m_{\tilde{\ell}} = 400$  GeV and  $m_A = 400$  GeV. The appropriate horizontal axis is now the lower one, which is obtained from the upper axis by  $m_b \rightarrow m_b + \delta m_b$ . (Since  $\delta m_b$  depends on  $\tan \beta$  and thereby on  $m_t$ , we use the central prediction of  $m_t$  in the figure as a rough guide in rescaling the horizontal axis. Also, we have checked that the fit in Eq. (11) is still reasonably valid for these low values of  $m_t$ .) As is evident from the rescaled bounds on  $m_b$  shown in the figure, the top mass prediction now would encompass essentially all the experimentally allowed range. In other words, we obtain a meaningful prediction only if the squarks are much heavier than  $m_{\tilde{g}}\mu$  and  $\mu A_t$  or if we *know* that  $\delta m_b > 0$ . We will discuss whether these requirements are likely to be satisfied elsewhere [20].

It is important to note that our prediction for  $m_t$ , for given experimental inputs and for  $\delta m_b \simeq 0$ , is considerably larger (by 10 to 20 GeV) than has been previously obtained using lowest order analyses [16,17]. Carena, Pokorski and Wagner [24] have briefly considered the condition  $\lambda_t = \lambda_b = \lambda_\tau$  as a particular case of GUT boundary conditions, and employed 2-loop RGEs to numerically obtain top mass values with which we agree in the minimal scenario. They do not, however, describe the complete dependences on  $m_b$  and  $\alpha_3$  nor do they address the question of  $\delta m_b$  or of superheavy corrections. Finally, we reach different conclusions about the dependence on the superpartner spectrum.

We turn now to the threshold corrections which may be present at the GUT scale. These fall into three classes, corresponding to the three terms that make up  $\epsilon_{t,b,\tau}$  in Eqs. (24): the splitting of the  $\underline{16}_3$  and of the  $\underline{10}_H$  which contribute in proportion to

FIG. 4. Our predictions for  $m_t^{\text{pole}}$  without superheavy corrections using two qualitatively-different superpartner spectra, specifically  $m_{\tilde{h}} \sim \mu = 100$  GeV,  $m_{\tilde{g}} = 300$  GeV,  $m_{\tilde{w}} = 100$  GeV,  $m_{\tilde{q}} = m_{\tilde{\ell}} = 1000$  GeV and  $m_A = 1000$  GeV for the “heavy squarks” case, and  $m_{\tilde{h}} \sim \mu = 250$  GeV,  $m_{\tilde{g}} = 300$  GeV,  $m_{\tilde{w}} = 100$  GeV,  $m_{\tilde{q}} = m_{\tilde{\ell}} = 400$  GeV and  $m_A = 400$  GeV for “light squarks.” The “cloud” indicates the region where  $\lambda_G > 2$ . These predictions carry estimated uncertainties of  $\sim \pm 5$  GeV from various approximations and from the GUT-scale thresholds discussed in the text. Also shown are the estimated allowed mass ranges for the running parameter  $m_b$  as extracted in Sec. V.

$y_G$ , the splitting of the the superheavy members of the gauge 45 which contributes in proportion to  $\alpha_G$ , and any other superheavy multiplets which may couple to the 10<sub>H</sub> or the 16<sub>3</sub>. In addition, there is the shift in the prediction of the overall superheavy mass scale which occurs if the superpartner mass splittings change the scale at which  $g_1$  and  $g_2$  meet to  $M'_G$ . This shift could also result from GUT-scale threshold corrections to  $g_1$  and  $g_2$ . In any case, such a shift induces an effective GUT threshold correction given by  $\Delta\epsilon_\nu = \frac{1}{4\pi} \ln(M'_G/M_G) \sum_\alpha (K'_\nu{}^\alpha y_G + L'_\nu{}^\alpha \alpha_G + \sum_\sigma K'_{\nu\sigma}{}^\alpha y_\sigma)$  for  $\nu = t, b, \tau$ . After summing over all superheavy particles  $\alpha$ , the last term in  $\Delta\epsilon_\nu$  becomes equal for all  $\nu$  since it only involves complete SO(10) multiplets; therefore we may drop it. The first two terms are  $\frac{1}{4\pi} \ln(M'_G/M_G) \left[ (7, 6, 7)y_G - \left(\frac{223}{10}, \frac{227}{10}, \frac{267}{10}\right)\alpha_G \right]$ , which become [36]  $\frac{1}{4\pi} \ln(M'_G/M_G) \left[ (0, -1, 0)y_G - \left(0, \frac{2}{5}, \frac{22}{5}\right)\alpha_G \right]$  after subtracting out an irrelevant constant. Then an  $M'_G$  anywhere between  $1.3 \times 10^{15}$  GeV and  $1.3 \times 10^{17}$  GeV changes the top mass by less than 2%.

The threshold corrections proportional to  $y_G$  arise from the splitting of the 16<sub>3</sub> into the right-handed neutrino and the rest of the standard-model matter fields, and of the 10<sub>H</sub> into the superheavy Higgs triplets and the standard-model Higgs doublets. Since the right-handed neutrino is but a small part of the 16<sub>3</sub>, its contribution is small, as we saw above in  $\Delta\epsilon_\nu$ , and since its mass is expected to be at or below the GUT scale it can only raise the mass of the top. The splitting of the 10<sub>H</sub> is dominated by the large hierarchy between the doublets and the triplets; in the language of the Pati-Salam subgroup  $G_{\text{PS}} \equiv \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)$  of SO(10), we write 10<sub>H</sub>  $\rightarrow (2, 2, 1) + (1, 1, 6)$ . But in fact any complete multiplet of  $G_{\text{PS}}$  will contribute equally to  $\lambda_t$ ,  $\lambda_b$  and  $\lambda_\tau$ , so the large splitting between the  $(2, 2, 1)$  and the  $(1, 1, 6)$  does not generate any threshold corrections. Furthermore, the  $(1, 1, 6)$  decomposes into a 3 + 3 of color SU(3), which as it turns out *each* contribute equally to the three Yukawa couplings (assuming they are heavier than the right-handed neutrino), so no threshold contributions result from their possible splitting.

Threshold corrections proportional to  $\alpha_G$  would be generated by splittings amongst the superheavy fields in the gauge 45 as well as amongst the members of the 10<sub>H</sub> and 16<sub>3</sub>. Only splittings of  $\lambda_b^G$  from  $\lambda_\tau^G$  are not largely suppressed by the fixed-point behavior, as discussed above. The dominant contributions to such splittings come from three multiplets  $\phi_{1,2,3} \in \text{45}$ , having masses  $M_{1,2,3}$  respectively, which transform as  $\phi_1 \sim (2, 3, \frac{1}{3})$ ,  $\phi_2 \sim (2, 3, -\frac{5}{3})$  and  $\phi_3 \sim (1, 3, \frac{4}{3})$  under  $\text{SU}(2) \times \text{SU}(3) \times \text{U}(1)_Y$ . They yield  $\epsilon_b - \epsilon_\tau = (\alpha_G/\pi) [\ln(M_1/M_2) - \ln(M_3/M_G)]$ . Even in the somewhat extreme case of  $M_2 \sim 10M_1 \sim 10M_3 \sim 100M_G$ , the change in  $m_t$  is only 2 to 6 GeV.

Finally, there will in general be threshold effects from various other couplings to the 16<sub>3</sub>10<sub>H</sub>16<sub>3</sub> vertex. These obviously depend upon the Higgs content of the theory, and cannot be estimated without a concrete model. In particular, if some large multiplet

such as a  $\underline{126}_H$  has a large coupling to the  $\underline{16}_3$  and is far from degenerate in mass, then large threshold corrections could result; in that case, the Yukawa couplings would not in effect be unified. Since we cannot rule out such a possibility (even after we impose the restriction that no Landau poles be encountered within, say, an order of magnitude of  $M_G$ ), our predictions will only be valid for models in which either the couplings of all multiplets (except the  $\underline{10}_H$ ) to the  $\underline{16}_3$  are small, or such multiplets are practically unsplit, or they make equal contributions to  $\lambda_b$  and  $\lambda_\tau$ . For this last claim we have used the fact that  $|F_b^{(\epsilon)}(R) + F_\tau^{(\epsilon)}(R)| = |F_t^{(\epsilon)}(R)| \ll 1$ : any threshold effects that are equal for  $\lambda_b$  and  $\lambda_\tau$  or that affect only  $\lambda_t$ , are greatly suppressed by the fixed-point behavior. This applies, for example, to any multiplet that only corrects the  $\underline{10}_H$  leg of the vertex, since it could only distinguish the doublet that couples to down-type fields from the doublet that couples to up-type fields but could not distinguish the  $b$  from the  $\tau$ . Even a 30% threshold correction of this sort would affect the top mass prediction by less than 3%.

In summary, while we cannot eliminate the possibility of large GUT-scale threshold corrections to our predictions, we have shown that all those corrections which are generic to SO(10) SUSY GUTs are not expected to change  $m_t$  by more than a few GeV.

## X. EXTENSIONS

The analysis of Secs. II through VIII has actually assumed that the two light Higgs doublets lie entirely within a single irreducible representation of SO(10). To what extent does the analysis, and the resulting top quark mass predictions, remain valid when the Higgs doublets have components in other irreducible representations? In general there will be a set of mixing angles  $\{\theta_{U,i}\}$  and  $\{\theta_{D,i}\}$  describing the components of the two light Higgs doublets,  $H_U$  and  $H_D$ , in various SO(10) multiplets ( $\underline{10}_H$ ,  $\underline{126}_H$ , or some other representation) labeled by  $i$ . Suppose for now that the third generation masses are generated by a set of Yukawa interactions which are all of the form  $\underline{16}_3 \underline{10}_{H,i'} \underline{16}_3$ , where the  $\{i'\}$  are a subset of the  $\{i\}$ . In this case the GUT boundary condition  $\lambda_b = \lambda_\tau$  will occur regardless of the values of  $\{\theta_{U,i}\}$  and  $\{\theta_{D,i}\}$ ; on the other hand,  $\lambda_t \neq \lambda_b$  if  $\theta_{U,i'} \neq \theta_{D,i'}$  for any  $i'$ . The analysis of this paper still applies to this situation provided an additional term  $\Delta\epsilon_t$  is added to  $\epsilon_t$  to reflect this change in boundary condition. As we saw above, the fixed-point behavior implies a considerable insensitivity to  $\epsilon_t$  (see the corresponding sensitivity function  $F_t^\epsilon$ ). Typically  $\Delta\epsilon_t \sim \theta_{U,i} - \theta_{D,i}$  so for  $\theta_{U,i} - \theta_{D,i} \lesssim 0.3$  the shift in the top mass is less than 5 GeV. (This is not necessarily an uncertainty: in a given model the mixings are computable and so is the top mass shift.) This shows that our results apply even when several  $\underline{16}_3 \underline{10}_H \underline{16}_3$  interactions contribute to the third generation masses and  $H_U$  and  $H_D$  have sizeable components in different  $\underline{10}_H$  multiplets.

A particularly interesting subclass of the above models contains just one pair of  $\underline{10}$ 's, of which only one,  $\underline{10}_1$ , couples to the  $\underline{16}_3$ . The  $\underline{10}_2$  is introduced to make the triplets in  $\underline{10}_1$  heavy. This is achieved while keeping the doublets light via the coupling  $\underline{10}_1 \underline{45}_H \underline{10}_2$ , provided the  $\underline{45}_H$  gets a VEV in the  $B - L$  direction [37]. A more detailed discussion of a model of this type will be given elsewhere [20].

The restrictions on the values of  $\{\theta_{D,i'}\}$  are stricter when  $i'$  refers to a  $\underline{126}_{H,i'}$  which couples via  $\xi_{i'} \underline{16}_3 \underline{126}_{H,i'} \underline{16}_3$ , because this leads to  $\lambda_b \neq \lambda_\tau$  at the GUT scale. Using our analysis then requires an additional contribution to  $\epsilon_b - \epsilon_\tau$  of order  $(\xi_{i'}/\lambda_G)\theta_{D,i'}$ . Shifts in  $m_t$  of up to 10 GeV occur when  $(\xi_{i'}/\lambda_G)\theta_{D,i'} \sim 0.1$ . We conclude that while  $H_U$  and  $H_D$  must lie predominantly in a single  $\underline{10}_H$ , they may have a certain amount of mixing with doublets in other multiplets, where the allowed mixing may be quite large if the other multiplet is a  $\underline{10}_H$  but must be small if it is a  $\underline{126}_H$  with a significant coupling to the third generation.

Another contribution to  $\epsilon_{t,b,\tau}$  may arise from operators of the form  $\underline{16}_2 \mathcal{O} \underline{16}_3$  which mix the second and third generations. Such operators must be present in order to generate  $V_{cb}$ ; in fact,  $\mathcal{O}$  must have sufficient structure to give different values for the (2,3) entries of the up and down Yukawa matrices. This implies a non-universal contribution to  $\epsilon_{t,b,\tau}$ . Normally the contribution is of order  $V_{cb}^2 \sim 10^{-3}$ , which hardly affects the top quark mass prediction. However, if in some scheme (see, for example, the last reference in [12]) the operator  $\underline{16}_2 \mathcal{O} \underline{16}_3$  is responsible for generating a sizable fraction of  $m_\mu/m_\tau$  and  $m_s/m_b$  [rather than having these generated by the (2,2) entries of the Yukawa matrices] then the contribution to  $\epsilon_{b,\tau}$  could be large. For example, if there is *no* operator of the form  $\underline{16}_2 \mathcal{O} \underline{16}_2$  to give a mass to the muon or the strange quark, then we would expect  $\epsilon_b - \epsilon_\tau \sim m_\mu/m_\tau \sim 0.04$ , which would lead to a shift in  $m_t$  of  $\sim 8$  GeV. This extreme case is however unnatural since such schemes will not lead to an understanding of why  $V_{cb}^2 \sim m_c/m_t \ll m_\mu/m_\tau \sim m_s/m_b$ ; in any case, the shift in such schemes is calculable and does not introduce much uncertainty into the prediction of the top mass.

## XI. CONCLUSIONS

In this paper we have computed the top quark mass in supersymmetric SO(10) grand unified theories including all contributions larger than about 5 GeV. The assumption underlying this computation is that  $m_t$ ,  $m_b$  and  $m_\tau$  all originate from a single Yukawa interaction with an SO(10) multiplet which dominantly contains the two low energy Higgs doublets. While this is a somewhat restrictive assumption, it has certain virtues. It seems to us to be the simplest assumption, within grand unified theories, which can lead to a top mass prediction in terms of low energy quantities. In particular it does not involve any ansatz about the origin of masses for the lighter two generations. Furthermore



this assumption leads to an almost unique group-theoretic structure which underlies the top mass prediction: except for a narrow range of (measurable) MSSM parameters, the Yukawa interaction must be of the form  $\underline{16}_3 \underline{10}_H \underline{16}_3$ . The top mass prediction which results from the unification of the three third-generation Yukawa interactions recalls the prediction of the weak mixing angle which follows from the unification of the three gauge couplings. There is however an important difference between these two cases, having to do with threshold effects at the SUSY scale. In the gauge case these effects consist of a renormalization of *only one* operator (the gauge kinetic term), and are therefore small as long as the theory is perturbative. In contrast, there are two possible Yukawa interaction operators below the SUSY scale, corresponding to the two Higgs doublets, that contribute to the mass of each fermion once these doublets acquire VEVs. Thus a hierarchy of VEVs could result in large threshold corrections to the lighter masses without necessarily invalidating perturbation theory. Such a hierarchy is actually mandated in the SO(10) unification under our assumptions, and results in another contrast with previous unification scenarios, specifically the partial Yukawa unification in SU(5). A crucial outcome of the GUT boundary condition  $\lambda_b = \lambda_\tau$  is that the resulting value of  $m_b/m_\tau$  can only be reconciled with experiment if the top quark Yukawa coupling is large. This is why in SU(5) and in SO(10) models the top quark is predicted to be heavy. In the former, the ratio of  $\lambda_t/\lambda_b$  at the GUT scale is an arbitrary parameter, which thus precludes a prediction of  $m_t$ . The SO(10) GUT boundary condition fixes  $\lambda_t/\lambda_b$  to be unity; however, we are forced to take  $v_U/v_D \gg 1$  to account for  $m_t/m_{b,\tau} \gg 1$ , and hence large SUSY threshold effects introduce a new parameter  $\delta m_b$  into the top mass prediction. This feature of SO(10) is an improvement over SU(5): now the undetermined parameter is a ratio of observable mass parameters, which is measurable in principle by future experiments, rather than a ratio of VEVs which will be more difficult to deduce. Furthermore, if we assume a hierarchical structure in the SUSY spectrum (a discussion of such a structure will be presented elsewhere [20]) in which the sfermions are considerably heavier than gauginos and higgsinos, there are indeed no large threshold effects and the top mass is sharply predicted.

With these caveats in mind, and choosing the hierarchical spectrum, our basic results are shown in Fig. 3 and Table I, and are approximated by Eq. (32) and Table II. These show the dependence of the predicted top quark pole mass on  $\alpha_3$  and  $m_b$  and the dominant superpartner spectrum effects. Unless future experiments find a value for  $\alpha_3$  below the range shown in the figure, the hierarchical spectrum predicts a top quark heavier than  $\sim 170$  GeV to within 5 GeV or so. (Of course, various uncertainties could pile up and result in a lower top mass, but this is unlikely.) A smaller experimental value for  $m_t$  would indicate that  $\delta m_b$  is considerable; measuring the various MSSM mass parameters could then test this SO(10) unification scenario as outlined at the end of

Sec. VI. More generally, our prediction reads  $m_t^{\text{pole}} = \lambda_t (177 \text{ GeV}) [1 + f_t(m_A)]$  where  $\lambda_t = F(R) + \sum_n F_n^{(\epsilon)}(R)\epsilon_n + \sum_\nu F_\nu^{(k)}(R)k_\nu$ ,  $F(R)$  and the sensitivity functions  $F_n^{(\epsilon)}(R)$  and  $F_\nu^{(k)}(R)$  are shown in Fig. 2, the  $\epsilon_n$  and the  $k_\nu$  are given in Eqs. (23a-25c), and  $R = \left[ m_b^{\overline{\text{MS}}}(4.1 \text{ GeV})/m_\tau \right] (\eta_\tau/\eta_b) [1 - \alpha_3(\mu_Z)/3\pi] [1 + f_R(m_A)]$ . In any specific MSSM-based GUT model in which  $\lambda_t^G \simeq \lambda_b^G \simeq \lambda_\tau^G$ , both the possible contributions of the diagrams in Fig. 1 and any further deviations from the equality of GUT-scale Yukawa couplings can be calculated and inserted into the above expressions. These then yield an analytic prediction of the top mass to within a few GeV—which is more than sufficient for comparison with top mass measurements likely to be made in the near future.

**Note added:** After this manuscript was completed, we received a paper by M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner [38], in which some of the same issues as in our work are addressed and studied in a particular context, namely that of universal soft masses at the GUT scale. Many of these issues are discussed in detail in our Ref. [20]. A few have also appeared in Ref. [39].

## ACKNOWLEDGMENTS

We would like thank B. Ananthanarayan for useful discussions and G. Anderson for comparisons with his numerical results, which will appear in the last reference in [12]. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098, and in part by NSF grant PHY-90-21139.

## APPENDIX: USEFUL FUNCTIONS

We collect below various useful functions needed for calculating corrections to  $m_b$ ,  $R$  and  $m_t^{\text{pole}}$ . The integral which arises in calculating the finite 1-loop corrections to the  $b$  mass in Fig. 1 is given by

$$\begin{aligned} I(x, y, z) &\equiv \int_0^\infty \frac{u \, du}{(u+x)(u+y)(u+z)} \\ &= -\frac{xy \ln x/y + yz \ln y/z + zx \ln z/x}{[(x-y)(y-z)(z-x)]}. \end{aligned} \quad (\text{A1})$$

In some of the SUSY threshold corrections we need the function

$$\begin{aligned} G_1(x, y) &\equiv \int_0^1 u \ln [(1-u)x + uy] \, du \\ &= \frac{1}{(x-y)^2} \left\{ x^2 \left( \ln x - \frac{1}{2} \right) - y^2 \left( \ln y - \frac{1}{2} \right) \right\} \end{aligned}$$

$$- 2y [x (\ln x - 1) - y (\ln y - 1)] \}. \quad (\text{A2})$$

When matching  $R$  in the 2HSM and the 0HSM, we use both  $I(x, y, z)$  and  $G_1(x, y)$  through the function

$$\begin{aligned} 16\pi^2 f_R(m_A) &= (\lambda_b^2 - \lambda_\tau^2) G_1 \left( \frac{m_A^2}{\mu_Z^2}, 0 \right) + \frac{1}{2} \lambda_t^2 G_1 \left( \frac{m_W^2}{\mu_Z^2}, \frac{m_t^2}{\mu_Z^2} \right) \\ &+ \frac{1}{2} \lambda_b^2 G_1 \left( \frac{m_A^2 + m_W^2}{\mu_Z^2}, \frac{m_t^2}{\mu_Z^2} \right) - \frac{1}{2} \lambda_\tau^2 G_1 \left( \frac{m_A^2 + m_W^2}{\mu_Z^2}, 0 \right) \\ &+ \lambda_t^2 m_A^2 I(m_A^2 + m_W^2, m_t^2, m_W^2). \end{aligned} \quad (\text{A3})$$

Finally, for the top quark pole mass, the relevant function is

$$f_t(m_A) = \frac{\Sigma_W(0)}{2m_W^2} + \frac{\delta m_t}{m_t} + \frac{1}{2} \delta_L^t + \frac{1}{2} \delta_R^t \quad (\text{A4})$$

where

$$16\pi^2 \frac{\Sigma_W(0)}{2m_W^2} = 3\lambda_t^2 \left( \ln \frac{m_t}{\mu_Z} - \frac{1}{4} \right), \quad (\text{A5})$$

$$\frac{\delta m_t}{m_t} = 0, \quad (\text{A6})$$

$$16\pi^2 \delta_L^t = \lambda_t^2 G_2 \left( \frac{m_t^2}{\mu_Z^2}, \frac{m_Z^2}{\mu_Z^2} \right) + \lambda_b^2 G_3 \left( \frac{m_t^2}{\mu_Z^2}, \frac{m_A^2}{\mu_Z^2} \right), \quad (\text{A7})$$

$$16\pi^2 \delta_R^t = \lambda_t^2 G_2 \left( \frac{m_t^2}{\mu_Z^2}, \frac{m_Z^2}{\mu_Z^2} \right) + \lambda_t^2 G_3 \left( \frac{m_t^2}{\mu_Z^2}, \frac{m_Z^2}{\mu_Z^2} \right), \quad (\text{A8})$$

and

$$G_2(x, y) \equiv \int_0^1 u \ln [(1-u)^2 x + u y] du, \quad (\text{A9})$$

$$G_3(x, y) \equiv \int_0^1 u \ln |u(1-u)x - uy| du. \quad (\text{A10})$$

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