

Supersymmetry, Finite Temperature and Gravitino Production in the Early Universe

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We reconsider post-inflation gravitino production, in the context of hidden sector supergravity models. We discuss the possible role of supersymmetry breaking from finite temperature effects in enhancing the rate for gravitino production and argue that there is no such enhancement. Our conclusion is based on a simple decoupling argument, which is independent of temperature. We also characterize the properties of goldstino-like hydrodynamic fluctuations that arise in supersymmetric models at finite temperature. We show that they cannot lead to enhanced gravitino emission via infrared divergences. We comment on an analogy with axion emission from hadronic matter.

1. Introduction

Supersymmetry is an attractive theoretical framework as it provides a solution to the hierarchy problem for fundamental scalar masses. With dynamical mechanisms for supersymmetry breaking, the gauge hierarchy problem ($m_W \ll m_{Pl}$) can also be explained.

It is of great interest then to study the effects of supersymmetry on various experimental and cosmological data. There is of course as yet no direct evidence for the existence of the supersymmetric partners of the particles of the Standard Model. We must then rely on studying indirect effects. There has been much work done in the last two decades in these regards. In this paper, we will focus on certain cosmological issues.

In particular, the standard cosmological models predict that the early Universe was filled with a plasma; the constituents of this plasma have densities determined by equilibrium thermodynamics. Since the Bose-Einstein and Fermi-Dirac distributions are different, the plasma is populated by different amounts of on-shell fermions and bosons.

In a theory in which supersymmetry plays a role,¹ temperature effects may invalidate various cancellations implied by the symmetry between fermions and bosons. It is important then to investigate what role supersymmetry really plays in the early universe, and to study possible effects of temperature on various calculations in supersymmetric models.

In all extant models of supersymmetry breaking, the theory consists of two sectors. In the first sector, supersymmetry is broken by some mechanism which in general we will not specify. The observable particles, *i.e.*, those of the standard model, are in a second sector. The two sectors are separate in the sense that they interact only very weakly with one another. In the standard hidden sector scenarios, this interaction is provided by non-renormalizable gravitational effects. In models of low-energy supersymmetry breaking, such as those recently discussed by Dine and Nelson[1], the communication of the two sectors is mediated by gauge interactions. We note that in the latter models, the gravitino can be extremely light ($m_{3/2} = F/m_{Pl} \gtrsim m_W^2/m_{Pl}$), and cosmology places no bounds on its interactions, as long as it is sufficiently light.[2] We will thus concentrate in this paper on hidden sector models.

We have been particularly motivated by the recent work of W. Fischler[3] on post-inflation gravitino production, in which it is suggested that temperature effects can greatly enhance the Goldstino component of gravitino production cross-sections, leading to very tight constraints. The plan of this paper is as follows. In the next section, we consider

¹ Here we mean that supersymmetry is unbroken at some scale in the zero-temperature theory.

the equivalence theorem for Goldstinos and gravitinos at high energies/temperatures and give a decoupling argument based on finite-temperature Ward identities that suggests that Goldstino amplitudes will in fact not be enhanced. In the third section, we suggest a calculation that can correctly compute the total Goldstino production rate, making proper use of real-time finite-temperature quantities. This calculation avoids any direct discussion of individual Feynman diagrams; we argue that the total rate can be computed in (re-summed) perturbation theory for weak coupling. In the second half of this paper, we will review older literature on the effects of temperature on supersymmetric theories. We clarify and extend results on how supersymmetry is broken, and its physical manifestation as propagating modes in the plasma.

2. Goldstino-Gravitino Equivalence and Decoupling

In this section we consider the possible relevance of gravitino production in the primordial plasma after reheating. Since we are dealing with temperatures $T \gtrsim m_{3/2}$, the transverse and longitudinal modes of the gravitino field may play very different roles. In particular the transverse (spin 3/2) component couples always with gauge strength $1/m_{Pl}$. Thus the rate of change of its density $\dot{n}_{3/2}$ is at most of order T^6/m_{Pl}^2 . In conventional cosmological scenarios, this small rate gives the usual bounds. If the gravitino is stable, it must be lighter than a few keV, in order that it not dominate the energy density.[4] If it does decay, in the absence of dilution by inflation, nucleosynthesis will constrain $m_{3/2} \gtrsim 10$ TeV.[2] In inflationary scenarios, gravitinos will be diluted, with $n_{3/2}/n_\gamma \sim T_{RH}/m_{Pl}$. However if the reheat temperature T_{RH} after inflation is too high ($T_{RH} \gtrsim 10^{9-10}$ GeV), gravitinos will re-attain sufficient density as to have problematic effects on nucleosynthesis, entropy release after nucleosynthesis, light element photo-dissociation and distortions in the microwave background.[5]

The longitudinal (spin 1/2) mode, however, can interact more strongly. Indeed the interactions of this mode at $E \gtrsim m_{3/2}$, are well described by those of the Goldstino via the equivalence theorem.[6] At high energies they are proportional to a power of $(E/m_{Pl}m_{3/2}) \sim E/F$, and could perhaps[3] lead to a large $\dot{n}_{3/2} \propto T^8/F^2$. The cosmological consequences could then be rather strong. In what follows, however, we argue that in the elementary particle models of phenomenological interest the rate of goldstino production is never so enhanced. Indeed it is always suppressed by at least one power of $1/M^2$, where M is the scale of the physics communicating SUSY breaking to conventional

matter. In particular, when SUSY breaking is communicated by gravity, we have $M \sim m_{Pl}$ and the rate for production of helicity 1/2 gravitinos is never more important than the rate for helicity 3/2.

The models of interest consist of a hidden and an observable sector. It is assumed that in the absence of interactions between them the hidden sector breaks supersymmetry while the observable one does not. The fundamental Lagrangian of the models of interest may thus always be written in the form:

$$\mathcal{L} = \mathcal{L}_{obs}(\phi) + \mathcal{L}_{hid}(X) + \sum_n \epsilon^n \mathcal{L}_n(\phi, X) \quad (2.1)$$

where ϵ is a small parameter. In models where the interaction between the two sectors is induced by some physics at a scale M we have $\epsilon = 1/M$. For instance in conventional supergravity models $M = m_{Pl}$. The dynamics of \mathcal{L}_{hid} breaks SUSY at a scale $F = M_s^2$. The SUSY breaking effects in the observable sector are then $O(F/M)$, and it must be $F/M \sim M_Z$ for phenomenological reasons. It is manifest from the form of this Lagrangian that, in the limit $\epsilon \rightarrow 0$, the Goldstone fermion of spontaneously broken supersymmetry does not couple to the fields ϕ . The form of eq. (2.1) then shows explicitly that, as long as perturbation theory is valid, also at finite temperature the rate of Goldstino production from observable matter is suppressed by at least ϵ^2 . We note that the Goldstino production cross-section of Ref.[3] does not satisfy this property. We also note that if we were interested in the production of Goldstinos from a thermal bath of hidden sector particles we would indeed get a rate scaling like T^8/F^2 . However this is not a situation of cosmological interest.

By performing a local field redefinition, the Goldstino coupling with matter can be written in terms of the supercurrent. At lowest order in the Goldstino χ field we have

$$\mathcal{L}_\chi = \frac{1}{F} \partial_\mu \chi \left[S_\phi^\mu + S_X^\mu + \sum_n \epsilon^n S_n^\mu \right] \quad (2.2)$$

where in an obvious notation S_α^μ are the contributions to the supercurrent from the terms in eq. (2.1). The decoupling of goldstino and observable sector is not explicit in eq. (2.2). When calculating scattering amplitudes the decoupling shows up via cancellations among diagrams which are related to each other by supersymmetry. The diagrammatic way of reasoning is however very dangerous when trying to draw conclusions on finite

temperature processes. Indeed the presence of a thermal bath breaks² supersymmetry (as well as Lorentz invariance), and one might then naively expect that the “effective” diagrams will be related to each other up to terms proportional to a power of T . That is, since the static limit of propagators are affected by finite temperature, it is tempting to say that the mass splittings should be replaced by temperature dependent factors. As a result of this, the suppression of processes with goldstinos and ordinary matter would be $O(T/F)$ rather than $\epsilon \sim 1/M$, in clear contradiction with our decoupling argument. However, when using eq. (2.2) to parametrize the goldstino couplings, one should note that S_ϕ^μ is a conserved current in the limit $\epsilon \rightarrow 0$. This means that the decoupling, in this language, is the result of a Ward identity. Real time correlation functions will obey Ward identities that are identical to their zero temperature counterparts, except that the correlation functions should be interpreted as finite temperature averages. In particular, the current S_ϕ is indeed conserved in the limit $M \rightarrow \infty$, and so any correlation function involving a Goldstino and observable matter must also vanish in this limit, irrespective of temperature. The introduction of ensemble parameters like temperature (or chemical potential for abelian symmetries), corresponds to the choice of a particular state, which may not be invariant under some symmetries of the system. Ward identities, on the other hand, express properties of the dynamics, *i.e.*, the Hamiltonian, and as such do not depend on the particular state or ensemble average. In other words, the introduction of ensemble parameters can lead to a Goldstone representation of a symmetry but not to its explicit breakdown. Then an effective finite temperature description of real time processes, if any is possible, must preserve the form of the Ward identities. This is reminiscent of what happens with chiral symmetry in QCD, where the low energy pion lagrangian realizes the same Ward identities of the fundamental QCD dynamics.

Now one should note that such decoupling arguments can in general be violated by the effects of resonant enhancement. This occurs when a relatively large mixing arises amongst nearly degenerate states, because of a small energy denominator. However, at finite temperature, because of damping phenomena, coherence can be maintained only for finite times $\lesssim t_{\text{damp}}$, and the resonant enhancement is limited. Indeed the only modes that are undamped are those corresponding to Goldstone singularities at zero energy and momentum. It is known that finite temperature Ward identities (even in the absence of spontaneous supersymmetry breaking) require the existence of a fermionic excitation with

² See section 4.

some properties similar to a Goldstino. In the next two sections we will show that these Goldstino-like hydrodynamic modes cannot lead to enhanced gravitino production, simply because of the form of their dispersion relation. In the Appendix, we comment on the case of axion emission³ from thermal pions, where a partial enhancement does indeed take place due to resonant mixing with a Goldstone mode.

3. Goldstino production at finite temperature: a calculation

We now present the outline of a calculation of the production rate of gravitinos at finite temperature. It is well known that the interpretation of finite temperature Green's functions as corresponding to scattering amplitudes is tricky. Essentially the problem is one of boundary conditions: we do not have simply asymptotic states to deal with, but a full plasma. The plasma is assumed to consist of thermalized visible sector particles; the hidden sector must be (nearly) unpopulated. Otherwise, the Goldstino couples directly to those particles and will thermalize with that sector, leading to the usual gravitino problem. This is clearly a condition on inflation models: the inflaton, whose decay is responsible for reheating the universe, must not couple to the hidden sector (except through unavoidable gravitational couplings). This is sufficient to ensure that the decay of the inflaton will thermalize the visible sector fields, but not the hidden sector.

In such a situation, we wish to study the approach of the Goldstino to thermal equilibrium. The idea that we will use here is to attempt to compute the inclusive production rate,⁴ where we sum over all final states including a Goldstino⁵ and average over initial states by using a thermal density matrix $\rho_\beta = e^{-\beta H}/Z$. We thereby avoid having to interpret individual scattering amplitudes.

The Goldstino couples to the visible sector through the supercurrent, with

$$\frac{1}{F}\partial_\mu S_{\alpha,vis}^\mu = -\frac{i}{M}\mathcal{O} \equiv -\frac{i}{M}(aW'_A(\phi^*)\psi_\alpha^A + bG_{\mu\nu}(\sigma^{\mu\nu}\lambda)_\alpha + \dots) \quad (3.1)$$

where the coefficients a, b are of order one. The total transition probability per unit 4-volume for production of single gravitinos may be written:

$$\begin{aligned} w_\chi &= \frac{1}{M^2} \int \frac{d^3q}{(2\pi)^3 2E_q} \sum_{spins} u(\vec{q}, s)_\alpha \bar{u}(\vec{q}, s)_\eta \int d^4x e^{iq \cdot x} \text{Tr} [\rho_\beta \mathcal{O}_\alpha(x) \bar{\mathcal{O}}_\eta(0)] \\ &= \frac{1}{M^2} \int \frac{d^3q}{(2\pi)^3 2E_q} \int d^4x e^{iq \cdot x} \text{tr} \not{q} \langle \mathcal{O}(x) \bar{\mathcal{O}}(0) \rangle_\beta. \end{aligned} \quad (3.2)$$

³ We thank S. Thomas and W. Fischler for bringing this example to our attention.

⁴ We thank T. Banks for discussions on this point.

⁵ States with more than one Goldstino will be parametrically suppressed.

The use of the r.h.s. of eq. (3.1) is justified here, since we are bracketing on on-shell states, which satisfy the full equations of motion. In the first line of (3.2), we have written the result in terms of Dirac spinors and q^μ is the Goldstino 4-momentum. Since we are interested in $T \gg m_{3/2}$ we have taken $q^2 = 0$. We note that the thermal matrix element $\langle \mathcal{O}(x)\bar{\mathcal{O}}(0) \rangle_\beta \equiv \text{Tr} [\rho_\beta \mathcal{O}(x)\bar{\mathcal{O}}(0)]$ is a function of real times, but is *not* a time-ordered quantity. Thus we can also write this expression in terms of the discontinuity (or the imaginary part) of the gravitino self energy[7]

$$w_\chi = \frac{1}{M^2} \int \frac{d^3q}{(2\pi)^3 2E_q} \frac{1}{e^{\beta\omega} + 1} \text{tr} \not{q} 2\text{Im} \langle \mathcal{O}\bar{\mathcal{O}} \rangle_{q,\beta} \equiv \frac{\tilde{w}_\chi}{M^2}. \quad (3.3)$$

We are thus lead to an expression for the total production rate which is suppressed by $1/M^2$ as long as the matrix element is non-singular in the limit $M \rightarrow \infty$, *i.e.*, as long as there is no resonant mixing. In what follows we will argue that this is indeed the case. Notice that a behaviour $w_\chi \sim (1/M)^r$ with $0 < r < 2$ would be already physically interesting; the result of Ref. [3] corresponds to $r = 0$.

In the limit $M \rightarrow \infty$, the (thermalized) visible sector, in which $\langle \mathcal{O}\bar{\mathcal{O}} \rangle_\beta$ is calculated, corresponds to an exactly supersymmetric theory: the MSSM without soft supersymmetry breaking terms. The singularity required to modify the $1/M^2$ behaviour of eq. (3.3), would be physically interpreted as a mode in a fermionic channel \mathcal{O} , which propagates undamped on the light-cone. The only modes which propagate undamped at finite T , according to prime principles, are Goldstones corresponding to spontaneously broken symmetries. In fact these have to be undamped at $(\omega, k) = (0, 0)$, in order to saturate the broken Ward identities. For $(\omega, k) \neq (0, 0)$ we have no physical reason to expect an undamped mode in an interacting theory, and in fact experience shows that these modes are always damped. It is known that finite temperature leads to a spontaneously broken representation of supersymmetry and the appearance of a Goldstino-like mode. Indeed fermionic Green's functions like $\langle \mathcal{O}\bar{\mathcal{O}} \rangle_\beta$ are singular at $(\omega, k) = (0, 0)$. Fortunately it is possible to characterize in a quite general way the properties of this singularity. As shown in the next section, its dispersion relation in a weakly coupled theory at high T is given by $\omega/k = 1/3$, *i.e.*, strictly *off* the light-cone. In other words, in the limit in which the Goldstino is undamped, only the real part of $\langle \mathcal{O}\mathcal{O} \rangle$ (and not the imaginary one) has a $1/k$ singularity when approaching $(\omega, k) = (0, 0)$ from the light-cone. (Indeed, cfr. (3.3), a much stronger singularity would be needed to affect \tilde{w}_χ). We will thus conclude that no resonant mixing can arise with the zero-temperature Goldstino in eq. (3.3), so that

$w_\chi \sim 1/M^2$. In the Appendix we discuss an example where the Goldstone singularity gets arbitrarily close to the light-cone as $k \rightarrow 0$, so that a deviation from the naive behaviour for a total emission rate does indeed take place.

Indeed, since \tilde{w}_χ is well defined as $M \rightarrow \infty$ it will also be calculable in perturbation theory at weak (gauge and Yukawa) coupling. The leading contributions come from two-loop diagrams involving A-terms (*cf.* the first term in eq. (3.1)), as well as contributions from the gaugino mass (second term in eq. (3.1)). One-loop diagrams do not contribute to \tilde{w}_χ in the susy limit: they correspond to two body decay processes with goldstino emission, which are forbidden by phase space. The differential rate d^3w_χ/d^3k , when computed at momenta $< gT$ requires some care to be taken. The matrix element (3.3) will in general contain infrared divergences when calculated in perturbation theory. These divergences signal a breakdown in naive perturbation theory when the energy carried by a given line (or by all lines entering a vertex) in a graph is of order $g_s T$; fortunately, it is known how to resum the perturbation series.[8] The result, for nonzero external momenta (ω, q) is that all modes are screened at order $g_s T$, including gauge particles. This is consistent with our general argument that \tilde{w}_χ is finite. We stress once more that in order to avoid the $1/M^2$ suppression for Goldstino production, \tilde{w}_χ (*even after resummation*) would have to be *infinite* (we have taken $M \rightarrow \infty$). There is one remaining subtlety in the case of non-Abelian gauge theory; namely in the integration region $\omega, q \lesssim g_s^2 T$, we expect further infrared divergences. These are similar to those encountered in the static limit, where for example, the calculation of the free energy of hot QCD irrevocably breaks down at some order.⁶ Thus ultimately, we cannot calculate the partial rate in this regime. If something dramatic were to happen to the rate in this momentum region, it would happen in a perturbatively incalculable way. But again, consistent with the general physical argument discussed above, we feel that this is implausible. For instance, it is known that in hot QCD[10] the quasiparticle spectrum in the quark propagator can lead to enhancements in the partial rate at low energies. However, it seems reasonable that higher order effects lead to a finite width for these modes which would cut off any zero-energy singularity and, if we may use the analogy here, \tilde{w}_χ would then indeed be finite.

⁶ For recent progress on this problem, see Ref. [9].

4. Temperature Goldstinoes

In this final section, we will discuss some aspects of the behaviour of supersymmetric theories at finite temperature and the appearance of thermal Goldstino modes. Supersymmetry is broken at finite temperature because of the non-zero energy density in the ground state; the breaking thus appears spontaneous and there exists a fermionic Goldstone mode. In this section we study the physics of this mode; we begin by considering theories in which supersymmetry is unbroken at zero temperature. Much of what we have to say is a review of portions of the literature, although we believe some of our comments are new.

In Ref. [11], it is argued that temperature treats fermions and bosons differently, so supersymmetry is expected to be broken. They suggest that it is explicitly broken by the boundary conditions (the antiperiodic boundary condition for fermions leads to a problem in imaginary time Ward identities: the Grassmann variational parameter ϵ cannot be taken to be a constant) and that no massless fermion mode need arise.

In Refs. [12],[13], it was realized that in order to study dynamical effects, one must consider the problem in the real-time finite temperature formalism. It was established that the Ward identities at finite temperature are essentially those of the zero temperature theory with matrix elements interpreted as finite temperature correlation functions. In particular, Boyanovsky[12] established a Ward identity involving a supercurrent-fermion matrix element and found a fermionic pole with no mass gap when $\langle F \rangle_\beta$ is non-zero.

Let us consider the problem now in more general terms. The superalgebra relation

$$\{Q_\alpha, Q_{\dot{\alpha}}^\dagger\} = 2\sigma_{\alpha,\dot{\alpha}}^\mu P_\mu \quad (4.1)$$

implies that a state is invariant under supersymmetry if and only if its energy is zero. Then, when the ground state itself has non-zero energy, supersymmetry is spontaneously broken and a Goldstone fermion is present in the spectrum. The existence of the Goldstino is established by writing eq. (4.1) in its local form, as a Ward identity

$$\partial_\mu \langle T \left(S_\alpha^\mu(x) S_{\dot{\alpha}}^{\nu\dagger}(0) \right) \rangle = \delta^4(x) \langle \theta_\rho^\nu(0) \rangle \sigma_{\alpha\dot{\alpha}}^\rho \quad (4.2)$$

where S^μ and $\theta^{\mu\nu}$ are respectively the supercurrent and the stress energy tensor. By integrating eq. (4.2), a non-zero righthand side implies the existence of a zero momentum singularity in the current-current correlator. Moreover, Lorentz invariance of the vacuum constrains the singularity to be of the form $1/\not{\partial}$, signaling the presence of a massless fermion. Notice, though, that eq. (4.2), is a local operator identity determined by the

dynamics of the system, and, as such, is valid on any stationary, translation invariant state. In particular in a thermal bath, $\langle\theta\rangle \neq 0$, and we expect long range correlations in the system, and the absence of a mass gap. The absence of Lorentz invariance, though, does not allow one to establish the nature of the singularity in a model independent way [14]. For instance in the free massless Wess-Zumino model, the left hand side in eq. (4.2) is easily evaluated and the singular part has the same form as the Klimov-Weldon [15] self-energy from hard thermal loops. The form is involved, with logarithmic singularities on the light-cone, but the only ‘‘pole-type’’ singularity is at the single point $\omega = 0, k = 0$ (where ω and k are respectively energy and three-momentum). Thus there is long-range order, but without a propagating wave.

In fact, as also noticed in Ref. [16], eq. (4.2) is the supersymmetric analogue of the Ward identity expressing the spontaneous breakdown of Lorentz invariance in a thermal bath

$$\int d^4x \partial_x^\lambda \langle T(M_{\lambda\mu\nu}(x)\theta_{\rho\eta}(y)) \rangle_\beta = \langle g_{\mu\rho}\theta_{\nu\eta}(y) - g_{\nu\rho}\theta_{\mu\eta}(y) + g_{\mu\eta}\theta_{\rho\nu}(y) - g_{\nu\eta}\theta_{\rho\mu}(y) \rangle_\beta \quad (4.3)$$

where $M_{\mu\nu\rho}$ is the Lorentz current. In a thermal bath, $\langle\theta_{\rho\eta}\rangle \not\propto g_{\rho\eta}$ and the right-hand side is non-zero. Then the physical interpretation of Eq. (4.3) is that there are propagating hydrodynamic fluctuations (sound waves) corresponding to variations in the stress-energy density. These modes correspond to local boosts of the thermal bath, which in the limit of infinite wave-length cost zero energy.⁷ In addition we also know from hydrodynamics that these modes correspond to propagating waves only at wave-lengths that are much bigger than the mean free path (see for example ref. [17]). In particular, in a free field theory, the mean free path is infinite, and eq. (4.3) does not give rise to any interesting physical wave at finite (ω, k) . This is consistent with our findings for eq. (4.2). By analogy, it also suggests that in an interacting supersymmetric theory a hydrodynamic fermionic wave will describe the fluctuations of the system above some long, but finite, wavelength. It also suggests, that, by a reasoning that parallels the one in hydrodynamics, one should be able to derive the wave equation for the Temperature Goldstino in a quite general way.

Let us briefly recall the procedure in hydrodynamics. One considers a fluid that in stationary conditions in its restframe is described by a stress energy tensor $\theta_{\mu\nu} = pg_{\mu\nu} + (p + \rho)g_{0\mu}g_{0\nu}$, where p and ρ are respectively the pressure and energy density.

⁷ Notice the subtlety in counting Goldstone modes for this space-time symmetry; only one of three modes propagates.

The Goldstone mode of broken Lorentz (or Galilean) symmetry corresponds to small local boosts of the quiescent system which are described by a space-time dependent velocity three-vector v_i . To lowest order in v , the energy momentum in the “global” rest frame of the bath will have the boosted form

$$\tilde{\theta}_{\mu\nu} = \theta_{\mu\nu} + \Delta_v [\theta_{\mu\nu}] = pg_{\mu\nu} + (p + \rho)(g_{0\mu}g_{0\nu} - v_i g_{i\mu}g_{0\nu} - v_i g_{0\mu}g_{i\nu}) \quad (4.4)$$

The continuity equation for $\tilde{\theta}$ together with the equation of state of the fluid, then give the equation of propagation of the Goldstone mode,⁸ the sound velocity equaling $v_s = \sqrt{\partial p / \partial \rho}$.⁹

Let us now consider the case of supersymmetry. In a thermal bath the density of supercurrent $\langle S_\alpha^\mu \rangle$ vanishes at equilibrium. In the presence of a space time dependent Goldstino oscillation ξ the supercurrent \bar{S} is given by

$$\bar{S}_{\dot{\alpha}}^\mu = \xi^\alpha(x) \{Q_\alpha, \bar{S}_{\dot{\alpha}}^\mu\} = \theta^{\mu\nu} (\bar{\sigma}_\nu \xi(x))_{\dot{\alpha}} \quad (4.5)$$

and conservation of the current corresponds to the Goldstino wave equation. We thus obtain the dispersion relation

$$\omega = \left(\frac{p}{\rho}\right) k. \quad (4.6)$$

The velocity of the Goldstino $v_G = p/\rho$ should be compared to the velocity of sound $v_s = \sqrt{\partial p / \partial \rho}$. In a weakly coupled theory we have respectively in the non-relativistic and relativistic regimes: $v_G = v_s^2 = T/m$ and $v_G = v_s^2 = 1/3$. In the intermediate regimes, though, the relation $v_G = v_s^2$ does not hold.

Eq. (4.6) agrees with the explicit calculation of Ref. [18], where the fermion self-energy in the Wess-Zumino model was studied at low temperature. However we suspect

⁸ Notice that to derive the equation the derivatives of p, ρ have to be considered non zero and of order v , *i.e.*, the longitudinal part of the Goldstone field v “mixes” with other scalar modes. This is because when Lorentz invariance is broken, there can be mixing terms between the Goldstone field and heavier modes which contain just one and not two time derivatives. For example, in the liquid helium case, the Lagrangian contains the term $\mu \Phi \partial_t \pi$ where μ is the chemical potential, Φ the real part of the condensate and π the Goldstone phase. Integrating out the massive Φ produces a relevant quadratic piece for π . As we will see this phenomenon does not take place for Goldstinos due to the linearity in (∂_t, ∂_x) of the wave equation.

⁹ Here we are considering zero chemical potential, so that sound waves are associated with fluctuations in the energy density.

that in the treatment in Ref. [18], and also [12],[13], higher order corrections are not completely under control. Though these effects will not change eq. (4.6) (apart from the form of higher order corrections to the vacuum energy and pressure), we suspect that they can significantly affect the residue of the Goldstino pole. In what follows we briefly motivate our doubts. For the sake of the argument we will stick to the low temperature case which was studied in Ref. [18],[12],[13]. Eq. (4.2) suggests that the order parameter for supersymmetry breaking is a composite operator, *i.e.*, the stress-energy tensor. We thus expect the Goldstino to lie predominantly in a composite channel, suggesting that the description of this mode in terms of the elementary fields in the lagrangian is indeed non-perturbative. In Refs. [12],[13], it was noticed that in the massive Wess-Zumino model, $\langle\phi\rangle$ is displaced from its zero temperature value $\phi_0 = m/g$ and we have a vacuum expectation value for the auxiliary field $\langle F\rangle = \partial_\phi W \neq 0$ generated at finite temperature; thus the Goldstino has a non-zero overlap with the elementary fermion field in the theory. The fermion and boson masses, as calculated from the tree lagrangian, are thus split by $\Delta = m_A^2 - m_\psi^2 = m_B^2 - m_B^2 = gF$, where A , B and ψ are respectively the scalar, pseudoscalar and fermion. The 1-loop effective fermion self energy $\Sigma(\omega, k)$ is consistently calculated by using the free propagators with the masses corrected by the background F -field value. At zero momentum in real time one finds $\Sigma \sim g^2/\Delta$. In our case, we must have $m + \Sigma(0, 0) = 0$ as required by the broken-supersymmetry Ward identity.[12] The dominant contributions to $\Sigma(\omega, k)$ for $(\omega, k) \ll (\Delta/T, \Delta/\sqrt{mT})$, are determined by denominators of the form

$$\frac{1}{E_\psi^2 - (E_A - \omega)^2} \sim \frac{1}{\Delta + 2pk - 2E_A\omega} \quad (4.7)$$

where p is the integration 3-momentum. The above denominator is the same as that of hard thermal loops, apart from the “infrared regulator” Δ . It is however clear that any higher order contribution affecting the fermion and boson propagators can affect the result, since it is determined in lowest order by the “small” Δ . For instance, the genuine 1-loop corrections to the scalar masses at low temperature (*i.e.*, not those coming from $\langle F\rangle \neq 0$) are not smaller than order Δ . Of course the fact that individual corrections affect the result does not mean that the full correction will, especially since SUSY cancellations might be expected. Nonetheless it is not clear to us how to implement the “nominally” higher order effects in a manifestly SUSY consistent manner. This is especially clear at high temperature where the 1-loop corrections to propagators dominate the tree level. When one is interested in calculating processes at low momenta $\omega \sim gT$ the resummation of hard

thermal loops [19] is the correct approach. However, for ultralow momenta $\omega \ll g^2 T$ we are not aware of any developed technique.

The damping rate of this mode was also calculated in Ref. [18] and was found to be exponentially small at small momentum¹⁰ $\gamma_D \sim \exp(-\beta\Delta/2k)$, where Δ is the effective fermion-boson mass splitting induced by temperature. This exponential suppression arises presumably only at one loop. At this order the attenuation of the Goldstino is determined by a three body process, in which a boson (fermion) in the bath absorbs a thermal Goldstino and becomes a fermion (boson). Since fermions and boson effective masses have a splitting Δ , energy-momentum conservation requires an energy $O(\Delta/\omega)$ for the absorbing particle, thus the Boltzmann suppression at low ω . At higher orders the thermal Goldstino can be absorbed in a four- (or higher) body process. In this case, we do not expect the above phase space exponential suppression, and presumably the only suppression at low (ω, k) will be due to the fact that the Goldstino is derivatively coupled.

The dispersion relation was obtained in Ref. [18] by expanding around $(\omega, k) = 0$, and is apparently not sensitive to Δ . Fortunately, for the discussion of gravitino production, we need only have control of the dispersion relation (4.6). The fact that the “singularity” lies far off the light cone is clear evidence that it cannot play any relevant role in gravitino production, as the mode will not mix with the gravitino. Given this, the behaviour of the Goldstino damping γ_D cannot play any role in our discussion. Nonetheless it would be physically interesting to find a way to calculate this quantity.

Our analysis should be contrasted with the case studied in the Appendix, where the mode that leads to enhanced axion production is given by the pion. In that case, indeed, the relevant mode is not a purely collective one but a thermally “dressed” massless particle. Thus, not surprisingly, the mass shell is very close to the light-cone and a partial enhancement takes place.

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¹⁰ Of course we know that the damping rate must be exactly zero at $\omega, k = 0$, since there must be a Goldstone singularity.

5. Appendix

In this Appendix we show the details of the pion-axion mixing example which was mentioned in the text. We focus on the case of two light quark flavors u, d .

The coupling between the axion field a and hadronic matter is given, at lowest order, by the (a, π_0) mass matrix

$$\mathcal{L}_{\rightarrow\pi} = (a, \pi_0) \begin{pmatrix} m_{aa}^2 & m_{a\pi}^2 \\ m_{a\pi}^2 & m_{\pi\pi}^2 \end{pmatrix} \begin{pmatrix} a \\ \pi_0 \end{pmatrix} = B(a, \pi_0) \begin{pmatrix} \frac{m_u+m_d}{f_a^2} & \frac{m_u-m_d}{f_a f_\pi} \\ \frac{m_u-m_d}{f_a f_\pi} & \frac{m_u+m_d}{f_\pi^2} \end{pmatrix} \begin{pmatrix} a \\ \pi_0 \end{pmatrix} \quad (5.1)$$

where $f_a \gg f_\pi$ are the axion and pion decay constants, while $B = \langle \bar{q}q \rangle \sim \Lambda_{\text{QCD}}^3$. We are interested in the rate for axion production from a thermal bath of hadrons. To lowest order in $m_{a\pi}^2$ and using the formalism of Section 3, the rate of axion production per unit time and volume is

$$\frac{dn_a}{dt} = m_{a\pi}^4 \int \frac{d^3k_a}{(2\pi)^3 2E_a} \frac{1}{\exp(E_a/T) - 1} i\text{Disc}[G_{\pi\pi}(E_a, k_a)] \quad (5.2)$$

Where $G_{\pi\pi}$ is the pion propagator in the thermal bath. This function has been calculated at finite temperature up to $\mathcal{O}(1/f_\pi^4)$ in the chiral expansion.[20] Even in the chiral limit, both the real and imaginary parts of $G_{\pi\pi}^{-1}(\omega, k)$ are non-vanishing on the light-cone. The only surviving singularity is at $(\omega, k) = (0, 0)$, consistent with Goldstone's theorem. The thermal self-energy $\Pi_{\pi\pi}(\omega, k)$ can be written as

$$\Pi_{\pi\pi}(\omega, k) = (R + iI) \frac{T^4}{f_\pi^4} k^2 + \frac{T^2}{f_\pi^2} \mathcal{O}(K^2, m_\pi^2) + i \frac{T^4}{f_\pi^4} \mathcal{O}(K^2, m_\pi^2) \quad (5.3)$$

where $K^2 = \omega^2 - k^2$, while R and I are numerical coefficients and $R > 0$. In order to illustrate the infrared enhancement effect we consider the limit $m_\pi \ll T \ll f_\pi$. In this regime the dominant contribution comes from relativistic pions. Then, as will become clear below, we need only consider the term proportional to $R + iI$ in eq.(5.3). The other terms are subleading with respect to the tree-level chirality breaking terms in $G_{\pi\pi}$. The discontinuity of the pion propagator then reads

$$i\text{Disc}G_{\pi\pi}(E_a, k_a) = \frac{2f_\pi^4}{T^4} \frac{Ik_a^2}{[(m_a^2 - m_\pi^2) \frac{f_\pi^4}{T^4} + Rk_a^2]^2 + I^2 k_a^4} \quad (5.4)$$

The above expression behaves like $1/k_a^2$ in the range $k_a \gg k_{\text{inf}} \sim m_\pi(f_\pi^2/T^2\sqrt{R})$. Then, for k_a in this range, the integral in eq. (5.2) is linearly infrared divergent, related to the

bosonic nature of the pion, and its proximity to the light-cone. This divergence is cut-off at momenta of order k_{inf} . Neglecting numerical factors, the total rate behaves like

$$\frac{dn}{dt} \propto m_{a\pi}^2 \frac{f_\pi^2}{m_\pi T} \propto m^{\frac{3}{2}} \quad (5.5)$$

displaying an enhanced and non-analytic behaviour in the quark mass m . Notice also the growth of eq. (5.5) at low temperature. As temperature is lowered the pion density gets smaller, but the overlap of a pion excitation with a massless particle improves due to the smaller real and imaginary parts in $\Pi_{\pi\pi}$. Then there is coherence on a longer scale and a pion is more likely to oscillate into an axion.

Of course, the above results can be qualitatively interpreted by considering the quantum mechanics of pion-axion oscillations taking into account damping phenomena. For a pion emitted at $t = 0$, the probability of finding an axion at time t is given by $P(t) = \epsilon^2 \sin^2(\Delta m^2 t / 2E)$, where ϵ is the mixing angle and $\epsilon \Delta m^2 = m_{a\pi}^2$. However, in the presence of damping, this will be limited by the finite coherence length of the pions. For a damping rate $\Gamma \gg m^2/E$, we have $P_{\pi \rightarrow a} \sim m_{a\pi}^4 / (E\Gamma)^2$. Then we simply find the rate of axion production

$$\frac{dn_a}{dt d^3k} \sim \frac{m_{a\pi}^4}{E^2 \Gamma^2} \Gamma n_\pi, \quad (5.6)$$

qualitatively consistent with the above more formal analysis.

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