

# A Note on the Effective Soft SUSY-Breaking Lagrangian Below the GUT Scale

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I consider the superfield derivation of the effective theory of softly broken supersymmetry below the GUT scale. I point out the role of supergauge invariance in determining the form of the result, which is rather restricted in interesting classes of models. As an example I discuss sfermion mass splittings for matter embedded in a single GUT multiplet. Interesting differences arise between the cases of  $SU(5)$ ,  $SO(10)$  and  $E_6$ .

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1. In a recent interesting paper Pomarol and Dimopoulos [1] used the superfield formalism to derive the effective lagrangian of softly broken supersymmetry below the GUT scale. That paper shows that the use of superfields allows a great simplification with respect to the same calculations performed in component notation, like for instance Ref. [2][3]. In this note we will elaborate on the method of Ref. [1] and show a parametrization of the heavy superfields that simplifies the calculation even further. In our derivation it is straightforward to “power count” and keep only the relevant effects, and, more importantly, the origin of certain cancellations, which were missed in Ref. [1], is made clear. The result leads to fairly constrained soft mass splittings in the interesting class of models where susy-breaking takes place in a *hidden sector*. The only mass splitting between light sparticles belonging to the same GUT irreps are mediated by the heavy gauge fields. They generally consist of just the so-called D-terms [2], though in particular cases additional interesting effects may arise.

This note is organized as follows. In this section we focus on the most general soft breaking terms from a *hidden sector*, *i.e.*, also allowing for non-flat Kahler metrics, and described in equation (3) below. In section 2 we discuss sparticle splittings respectively in the case of  $SU(5)$ ,  $SO(10)$  and  $E_6$  unification, including the scenario of  $SO(10)$  Yukawa unification [4]. We point out the possibility of important splitting effects, which had not been discussed before, and which arise when the unified group is enlarged to  $E_6$ . We also comment on case in which the MSSM Higgs doublets are pseudo-Goldstones of “accidental” symmetries [5][6][7]. In section 3 we discuss the case of general soft terms. In sect. 4 we conclude.

In what follows we describe our assumptions for the observable sector. We consider a supersymmetric Grand-Unified Theory with gauge group  $G$  and with a set of chiral matter fields  $\Psi$ . It is assumed that, in the absence of soft supersymmetry breaking terms, the v.e.v.  $\Psi_0$  of the chiral superfields breaks  $G$  down to a subgroup  $H$ , while keeping supersymmetry unbroken. We indicate the set of vector superfields by  $V = (V_A, V_a)$  where  $V_A$  ( $A, B, \dots$ ) are the massive ones corresponding to the broken generators  $T_A$ , while  $V_a$  ( $a, b, \dots$ ) correspond to the unbroken generators. The set of chiral superfields can be decomposed as  $\Psi = (\Phi_A, \Phi_k, \varphi_\alpha)$ , where  $\Phi_A$  ( $A, B, \dots$ ) are the Goldstones eaten via the super-Higgs mechanism,  $\Phi_k$  ( $k, l, \dots$ ) are heavy non-Goldstone fields, and finally  $\varphi_\alpha$  ( $\alpha, \beta, \dots$ ) are the massless non-Goldstone fields corresponding to the matter content of the low-energy effective theory. We choose a basis for the broken generators  $T_A$  such that the vectors  $e_A = T_A \Psi_0 / |T_A \Psi_0|$  form an orthonormal basis of the Goldstone subspace, *i.e.*,

$e_A^\dagger e_B = \delta_{AB}$ . The v.e.v.'s of the various fields are indicated by a “0” sub- or superscript. Unbroken supersymmetry implies  $\Psi_0^\dagger T_A \Psi_0 = 0$ , which in component notation reads  $\Phi_A^0 = 0$ . Notice though that in general  $\Phi_k^0 \sim M_G \neq 0$ . We will assume that there are no light fields that are siglets under both  $H$  and any (possibly discrete) low-energy global symmetries; then there are no GUT v.e.v.'s associated with the light fields, *i.e.*,  $\varphi_\alpha^0 = 0$  and also no terms linear in  $\varphi_\alpha$  can appear in the low energy lagrangian. In the chosen  $\{A\}$  basis the mass matrix of the heavy vectors is diagonal and given by  $M_{AB}^2 = \Psi_0^\dagger \{T_A, T_B\} \Psi_0 = M_A^2 \delta_{AB}$  and  $M_A = \sqrt{2}|T_A \Psi_0|$ . (Notice that since we work at zero momentum we can take the gauge coupling  $g = 1$ ). The degrees of freedom represented by  $\Phi_A$  are superfluous and, as done in Ref. [1], can be eliminated by going to the super-unitary gauge [8]

$$\Phi_A \propto \Psi_0^\dagger T_A \Psi = 0 \quad (1)$$

By going to this gauge, the superpotential is written in terms of the light fields  $\varphi_\alpha$  and shifted heavy  $\psi_k = \Phi_k - \Phi_k^0$  as

$$W = \frac{1}{2} \mu_{kl} \psi_k \psi_l + \frac{1}{2} \lambda_{k\alpha\beta} \psi_k \varphi_\alpha \varphi_\beta + \frac{1}{2} \lambda_{kl\alpha} \psi_k \psi_l \varphi_\alpha + \frac{1}{3!} \lambda_{klm} \psi_k \psi_l \psi_m + \tilde{W}(\varphi) \quad (2)$$

where  $\mu_{kl}$  are masses of order  $M_G \simeq 10^{16}$  GeV,  $\lambda$ 's are Yukawa couplings and  $\tilde{W}$  is a piece which depends on the light fields only. Let us now introduce the soft susy breaking terms. We will write them in terms of the spurion  $\eta = m\theta^2$  where  $m \sim m_Z$  [9]. We also follow the conventions of Ref. [10], where  $d^2\theta = D^2/2$  and  $\int d^2\theta\theta^2 = -2$ . As discussed in Ref. [2], the most general lagrangian, inclusive of soft breaking terms, can be written in *hidden sector*<sup>2</sup> scenarios as

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} \int d^4\theta \left\{ \Psi^\dagger (1 + \bar{\eta}\Gamma^\dagger) e^{2V} (1 + \eta\Gamma) \Psi + \Psi^\dagger e^{2V} \tilde{Z} \eta \bar{\eta} \Psi + (\Psi^T \Lambda_1 \Psi + \text{h.c.}) \eta \bar{\eta} \right. \\ & \left. + (\Psi^T \Lambda_2 \Psi \bar{\eta} + \text{h.c.}) \right\} - \frac{1}{2} \left\{ \int d^2\theta (1 + a\eta) W + \text{h.c.} \right\} \end{aligned} \quad (3)$$

where  $\Gamma$ ,  $\tilde{Z}$ ,  $\Lambda_1$  and  $\Lambda_2$  are  $G$ -invariant matrices while  $a$  is just a c-number. In the chosen gauge  $\Psi$  is represented by  $\Psi = \Psi_0 + \psi + \varphi$ , in an obvious vector notation. We are following

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<sup>2</sup> Our definition of *hidden sector* models corresponds to the situation in which, in the parametrization of Ref. [11], the superpotential of the original supergravity splits into the sum of two pieces, one depending only on the fields of the susy-breaking sector and the other only on the observable fields. The form (3) is also stable in perturbation theory [2][12].

the notation of Ref. [1], apart from having “factored” the  $\Gamma$ ’s, *i.e.*, our  $\tilde{Z}$  corresponds to  $Z - \Gamma^\dagger \Gamma$  of that paper. We want now to integrate out the heavy fields and take the double limit  $m = \text{const.}$ ,  $m/M_G \rightarrow 0$ . The low energy effective lagrangian is completely determined by its form at  $V_a = 0$  (we remind that the  $V_a$  correspond to the unbroken generators  $T_a$ ). Indeed, when  $H$  contains an abelian factor  $Y$ , consistently with the low energy gauge invariance, one might expect a Fayet-Iliopoulos D-term proportional to  $V_Y$ . It is however clear that, from eq. (3) at  $\varphi = 0$ , this term is bound to be proportional to  $Y\Psi_0$  which is equal to 0. (Notice however that by integrating out the heavy modes at 1-loop we would get such a term, proportional to  $\text{Tr}Y\hat{m}^2$  where  $\hat{m}^2$  is the soft contribution to the heavy masses). Thus we can derive the effective lagrangian for  $V_a = 0$ , and then obtain the complete one by covariantizing in  $H$  the existing interactions, *i.e.*, just by inserting  $\exp(2V_a T_a)$  in the Kahler potential. The relevant effective interactions, *i.e.*, those that survive the above limit, are suitably characterized by making the following classification of infinitesimal quantities:

$$\eta = \mathcal{O}(\epsilon_1) \quad \varphi = \mathcal{O}(\epsilon_1 \epsilon_2) \quad D^2 = \mathcal{O}(\epsilon_1 \epsilon_2 \epsilon_3) \quad (4)$$

In what follows an operator is defined to be  $\mathcal{O}(\epsilon_1^n)$  when, according to (4), it involves n-powers of  $\epsilon_1$ , and analogously for  $\epsilon_2, \epsilon_3$ . Then in the assumption that  $\varphi$  does not contain any singlet of the full (gauge + global) low energy symmetry, we have that the relevant low energy lagrangian is written as  $\int d^4\theta K_{eff} + (\int d^2\theta W_{eff} + \text{h.c.})$  where

$$K_{eff} \geq \mathcal{O}(\epsilon_1^4), \mathcal{O}(\epsilon_2^2), \mathcal{O}(\epsilon_3^0) \quad W_{eff} \geq \mathcal{O}(\epsilon_1^5), \mathcal{O}(\epsilon_2^3), \mathcal{O}(\epsilon_3). \quad (5)$$

(For instance,  $K_{eff}$  is a polynomial of order  $\leq 4$  in  $\eta, \varphi, D^2$ , but of order  $\leq 2$  in  $\varphi, D^2$  and in fact of order 0 in  $D^2$ , since there are no singlets.) To satisfy the above relations we just need to solve the equations of motions for  $V_A$  and  $\psi_k$  to a finite order in the  $\epsilon$ ’s. One important remark here is that the solutions  $V$  and  $\psi$  to the equations of motion are at least  $\mathcal{O}(\epsilon_1)$ . To proceed we need to expand eq. (3) in a power series in  $V$  and  $\psi$ . Before doing so it is useful to perform the following field redefinition

$$\psi_k \rightarrow \psi_k - \mu_{kl}^{-1} \lambda_{l\alpha\beta} \varphi_\alpha \varphi_\beta / 2 \quad (6)$$

which essentially takes care of the lowest order equations of motion for  $\psi$ . In this new parametrization the  $V, \psi$  dependent part of eq. (3) has the form

$$\int d^4\theta \{ V^2 + V(\eta + \mathcal{O}(\epsilon_1^2)) + \psi \mathcal{O}(\epsilon_1) + \text{h.c.} \} + V^3 + V^2 \eta + \psi \bar{\eta} + \psi \eta \bar{\eta} + \dots \} + \left\{ \int d^2\theta^2 [\psi^2 + (\psi - \varphi^2)^2 \varphi + (\psi - \varphi^2)^3] (1 + a\eta) + \text{h.c.} \right\} \quad (7)$$

where we took the GUT masses to be  $\mathcal{O}(1)$ . Notice that the simple terms  $V$  and  $V(\psi + \varphi)$  are absent respectively because of zeroth-order unbroken susy and because  $\psi$  and  $\varphi$  are orthogonal to the Goldstone states. By the dots we mean even higher orders in  $V$  and  $\psi$ . The equations of motion then have the form

$$\psi + \varphi^3 + \bar{D}^2 \mathcal{O}(\epsilon_1) + \psi\varphi + (\psi - \varphi^2)^2 = 0. \quad (8)$$

and

$$V + (\eta + \mathcal{O}(\epsilon_1^2) + \psi\mathcal{O}(\epsilon_1) + \text{h.c.}) + V^2 + V\eta + \dots = 0 \quad (9)$$

Where in eq. (8) the  $D^2$  term comes from varying the Kahler potential. The solution is then  $\psi \sim \epsilon_2^3 + \bar{D}^2 \epsilon_1$ , which manifestly does not give any relevant effects both in  $W$  and in  $K$  (cfr. eqs. (4)(5) and (7) above). In the case of  $V$  the situation seems more complicated since the solution is  $V \sim \epsilon_1 + \epsilon_1^2 + \dots$ , and also the higher order terms  $V^3$  and  $V^2\eta$  contribute to the relevant terms. This would not happen if the term  $V\eta$  in (7) were missing, so that  $V \sim \epsilon_1^2$ . In this situation the only relevant pieces would arise from  $V^2 + V\epsilon_1^2$  by a trivial quadratic integration. In fact eq. (3) possesses a reparametrization invariance, related to the gauge symmetry, and by means of which the  $V\eta$  term can be eliminated. Consider the following gauge-type field redefinition

$$e^{2V} \rightarrow (1 - c_A^* T_A \bar{\eta}) e^{2V} (1 - c_A T_A \eta). \quad (10)$$

It is equivalent to

$$\Gamma \rightarrow \Gamma - c_A T_A = \tilde{\Gamma} \quad (11)$$

in the full lagrangian, since eq. (10), being like a gauge transformation, does not affect the gauge kinetic term. Then eq. (11) defines a reparametrization invariance which turns out to be very useful. The  $V\eta$  term in eq. (7) (see eq.(3)) is in fact  $\propto \Phi_0^\dagger T_A \tilde{\Gamma} \Phi_0$ , which is  $= 0$  if we choose

$$c_A = 2\Phi_0^\dagger T_A \Gamma \Phi_0 / M_A^2 \quad (12)$$

in eq. (11). In what follows  $\tilde{\Gamma}$  is defined by eqs. (11)(12). Notice that, when  $c_A \neq 0$ ,  $\tilde{\Gamma}$  no longer commutes with  $G$ , but  $[T_A, \tilde{\Gamma}]$  is a gauge generator. Then from unbroken susy and the gauge condition (1), we have that both  $\Phi_0^\dagger [T_A, \tilde{\Gamma}] \Phi_0$  and  $\Phi_0^\dagger [T_A, \tilde{\Gamma}] \Psi$  vanish. Notice also that, by the Wigner-Eckart theorem,  $c_A = 0$  when  $T_A$  is not a singlet of  $H$ , so that the above field redefinition is important only when  $\text{rank}(H) < \text{rank}(G)$ . Then, by (12),  $c_A T_A$  is an element of the broken Cartan subalgebra. For instance, in  $SO(10)$  we

have only  $c_A T_A \propto X$ , where  $X$  is the broken Cartan generator which is orthogonal to the hypercharge  $Y$ , *i.e.*,  $SO(10) \supset SU(5) \times U(1)_X$ . As we said, by eqs. (11) and (12) the solution is  $V_A = \mathcal{O}(\epsilon_1^2)$  and the  $V$ -dependent part in eq. (3) reads

$$\frac{1}{4} \int d^4\theta \left\{ M_A^2 V_A^2 + 2V_A [M_A^2 d_A \eta \bar{\eta} + \varphi^\dagger T_A \varphi + (\Phi_0^\dagger T_A \tilde{\Gamma} \varphi \eta + \varphi^\dagger T_A \tilde{\Gamma} \Phi_0 \eta + \text{h.c.}) + \mathcal{O}(\epsilon_1^3)] \right\} \quad (13)$$

where  $d_A = \Phi_0^\dagger (\tilde{\Gamma}^\dagger T_A \tilde{\Gamma} + T_A \tilde{Z}) \Phi_0 / M_A^2$ . Then, the integral in  $V_A$  is trivial and gives

$$-\frac{1}{2} \int d^4\theta \left\{ \varphi^\dagger T_A \varphi d_A + \frac{|\Phi_0^\dagger T_A \tilde{\Gamma} \varphi + \varphi^\dagger T_A \tilde{\Gamma} \Phi_0|^2}{M_A^2} \right\} \eta \bar{\eta} + \mathcal{O}(\epsilon_1^5). \quad (14)$$

Thus the effective lagrangian for the light fields is given by eq. (14) plus the following

$$\begin{aligned} & \frac{1}{4} \int d^4\theta \left\{ \varphi^\dagger (1 + \bar{\eta} \tilde{\Gamma}^\dagger) e^{(2V_a T_a)} (1 + \eta \tilde{\Gamma}) \varphi + \varphi^\dagger e^{2V_a T_a} \tilde{Z} \eta \bar{\eta} \varphi + [\varphi^T (\Lambda_1 \eta + \Lambda_2) \varphi \bar{\eta} + \text{h.c.}] \right. \\ & \quad \left. - \left[ \frac{1}{2} \Phi_0^\dagger (\tilde{\Gamma}^\dagger \bar{\eta} + \tilde{\Gamma} \tilde{\Gamma} \eta \bar{\eta} + \tilde{Z} \eta \bar{\eta}) + \bar{\eta} \Phi_0^T (\Lambda_1 \eta + \Lambda_2) \right] \mu_{kl}^{-1} \lambda_{l\alpha\beta} \varphi_\alpha \varphi_\beta + \text{h.c.} \right\} \\ & \quad - \frac{1}{2} \left\{ \int d^2\theta^2 (1 + a\eta) \tilde{W}(\varphi) + \text{h.c.} \right\} \end{aligned} \quad (15)$$

where the terms in the second line arise from the  $\psi$  redefinition described in eq. (6). We stress that the use of the reparametrization (11) together with eq. (12) leads to a great simplification. For instance, by using Feynman diagrams as in Ref. [1], we would have just two diagrams from the gauge sector, compared to the eleven of Ref. [1].

**2.** Let us focus on the chirality preserving soft masses induced by the above. Defining, to match the notation of Ref. [1],  $\tilde{Z} = Z - \Gamma^\dagger \Gamma$  we can write the result as

$$\begin{aligned} & \frac{1}{4} \int d^4\theta \left\{ [\varphi^\dagger (Z - 2T_A d_A) \varphi + (|\tilde{\Gamma}_{k\alpha} \varphi_\alpha|^2 - |\Gamma_{k\alpha} \varphi_\alpha|^2) - (|\tilde{\Gamma}_{A\alpha}^\dagger \varphi_\alpha|^2 + |\Gamma_{A\alpha} \varphi_\alpha|^2)] \eta \bar{\eta} \right. \\ & \quad \left. - |\Gamma_{\beta\alpha} \varphi_\alpha|^2 \eta \bar{\eta} + [\varphi^\dagger (1 + \bar{\eta} \tilde{\Gamma}^\dagger)]_\alpha [(1 + \eta \tilde{\Gamma}) \varphi]_\alpha \right\} \end{aligned} \quad (16)$$

where we have used  $M_A = \sqrt{2} |T_A \Phi_0|$  in eq. (14), and summation over  $k, \beta, A$  is also understood. Notice that the second term in the second line is in fact giving no contribution to chiral preserving masses, since it is reduced to  $\varphi^\dagger \varphi$ , by the field redefinition  $\varphi_\alpha \rightarrow [(1 - \eta \tilde{\Gamma}) \varphi]_\alpha$ , which leaves only A- and B-type soft terms from the superpotential. The form of eq. (16) is then fairly restricted. It is remarkable that the superpotential couplings do not enter directly in the above equation, though they obviously affect the  $\Gamma$ 's and  $Z$ 's

via RG evolution. Notice, indeed, that for the subset of the light fields  $\varphi_\alpha$  upon which the broken Cartan subalgebra acts diagonally (as it happens to ordinary matter in most  $SO(10)$  models), we have that  $\tilde{\Gamma}_{k\alpha} = \Gamma_{k\alpha}$  and  $\tilde{\Gamma}_{A\alpha}^\dagger = \Gamma_{A\alpha}^\dagger$  so that the second term in brackets in (16) vanishes. It is interesting to study the splittings of light sfermions embedded within a single GUT multiplet. In the cases of interest, the broken Cartan subalgebra acts diagonally on these fields. Then we are reduced to just consider D-terms and  $\Gamma_{A\alpha}, \Gamma_{\alpha A}$ . Notice though that only the D-terms are of gauge nature, which is to say universal (not with the meaning of “degenerate”!) and generally expected, while  $\Gamma_{A\alpha}$  and  $\Gamma_{\alpha A}$  correspond to mixings between light fields and Higgs multiplets, or more precisely between light fields and heavy vector superfields. Their potential interest then turns out to depend strongly on the original gauge group  $G$ . In what follows we will consider their effect on matter sfermions in the MSSM respectively for  $G = SU(5), SO(10)$  and  $E_6$ . In  $SU(5)$  the massive vectors transform under  $G_{WS} = SU(3) \times SU(2) \times U(1)_Y$  as  $(\bar{3}, 2, -7/6)$  plus its conjugate. Then there is no mixing to the MSSM fields, and the sparticles from the same  $\underline{10}$  and  $\bar{\underline{5}}$  are unsplit. When  $G = SO(10)$  there is an additional set of heavy vectors transforming under  $G_{WS}$  as  $\underline{10} \oplus \bar{\underline{10}}$  of  $SU(5)$ . These can in principle mix with the matter fields  $Q, U_c$  and  $E_c$ . The mixings are induced by the terms  $\int d^4\theta \underline{16}_H^\dagger (\alpha_i \eta + \beta_i \bar{\eta}) \underline{16}_i$ , where  $\langle \underline{16}_H \rangle \neq 0$  and  $\underline{16}_i$ ,  $i = 1, 2, 3$  contain the three light families. However the corresponding  $\Gamma_{A\alpha}, \Gamma_{\alpha A}$  break  $R$ -parity, (i.e. there is no way to define an unbroken  $R$ -parity under which the heavy vectors associated with  $SO(10) \rightarrow SU(5) \times U(1)$  are odd), so that we expect them to be rather small or absent at all. In fact they lead to the  $R$ -odd terms  $H_u L_i$  (both supersymmetric and soft) from the second line of eq. (16). These are generated by integrating out the right-handed neutrinos. Notice that for particular choices of the soft terms or of the neutrino mass matrix (like when the neutrinos  $N_i$  get Dirac masses by mixing to matter singlets  $S_i$ ) these  $R$ -odd masses could be absent at tree level, but they would still be generated at 1-loop. Thus we conclude that for sfermions within the same  $\underline{16}$  of  $SO(10)$  the only relevant source of mass splitting is given by the D-term associated with the only broken Cartan generator. The situation can be fairly different in  $E_6$ . When  $E_6 \rightarrow G_{WS}$ , with respect to the previous case, there are additional heavy vectors in the  $\underline{16} \oplus \bar{\underline{16}} \oplus \underline{1}$  of  $SO(10)$ . Now, we can endow the theory with  $R$ -parity in such a way that the vectors in  $\underline{16} \oplus \bar{\underline{16}}$  are  $R$ -odd, thereby allowing their soft mixing with matter. These are induced by soft terms involving  $\underline{27}_H^\dagger \underline{27}_M$ , where  $\langle \underline{27}_M \rangle \neq 0$  and matter is contained in  $\underline{16}_M \subset \underline{27}_M$  (with obvious notation). The crucial remark here is that if  $\langle \underline{27}_H \rangle$  breaks  $E_6$  down to  $SO(10) \times U(1)$ , the  $\Gamma_{A\alpha}$  between matter and the vector  $\underline{16}$ 's are non zero, and still we have the  $\mathbb{Z}_4$  center

of  $SO(10)$  unbroken. It is then possible that its  $\mathbb{Z}_2$  subgroup  $Z$ , or a combination of it with a global one, survives and corresponds to  $R$ -parity. A necessary condition for this to happen is that all  $E_6$  Higgs multiplets get vacuum expectation values along directions with definite  $Z$ . In this case a combination of  $Z$  and other global discrete symmetries could be the low energy  $R$ -parity. For instance the rank could be further reduced by the  $Z$ -odd vev of a  $\underline{27}'_H$ . In this case there should be a global parity  $Z'$  under which  $\underline{27}'_H$  is odd and  $R = ZZ'$ . Now, the main point is that when  $E_6 \rightarrow SU(3) \times SU(2) \times U(1)_Y$  directly at  $M_G$ , the masses of the  $R$ -odd vectors in the  $\underline{16}$  are in general all split by  $\mathcal{O}(1)$ . In fact this fields will also get mass from  $SU(5)$  breaking vevs like, for instance, a  $\langle \underline{78}_H \rangle$ . As a result the sfermion masses induced by eq. (14) will be clearly respecting only the low-energy gauge symmetry. While it may not be easy to obtain a model with these features, it is an amusing fact that by enlarging the gauge group the symmetry properties of the soft masses are in principle reduced

We are thus lead to the interesting conclusion that, unless  $R$ -odd gauge bosons appear, for matter belonging to well definite GUT irreps, the only sources of soft masses that we expect to be important are the  $G$  invariant ones plus a  $D$ -term for each broken Cartan subalgebra generator. However when  $G \supset E_6$ , there can be  $R$ -odd vectors and a new class of contributions is allowed. When the MSSM Higgs doublets  $H_{u,d}$  sit in specific GUT irreps also this sector is fairly constrained. This is indeed what happens for the interesting  $SO(10)$  Yukawa unified situation [4] in which both light Higgses lie in the same  $\underline{10}$ . In this case the  $\Gamma_{A\alpha}$  terms in (16) is also vanishing for the Higgs doublets, since  $\underline{10}$ 's have zero v.e.v.. Then, neglecting  $R$ -parity violating terms, the soft masses of the third sfermion family + Higgses are completely specified by the three parameters  $m_{\underline{10}}^2$ ,  $m_{\underline{16}}^2$  and the D-term  $d_X$ . It has been shown in Ref. [13] that this constrained form of the soft terms leads to difficulties in radiative electroweak breaking. A more plausible picture of radiative electroweak symmetry breaking, requires additional splittings. As shown above these can arise when  $SO(10)$  is enlarged to  $E_6$ . In sect. 3, we show that they can also arise in  $SO(10)$ , but by allowing very general soft terms.

Notice that in Ref. [1] some important cancellations leading to eq. (16) were overlooked, so that other contributions of genuine gauge type in addition to the D-terms were claimed. Indeed these cancellations, in the computation of Ref. [1], arise from the form of the lowest order solution  $V \sim \eta + \bar{\eta}$ , which is a pure gauge configuration, and are manifest by using eq. (11). Notice that eq. (16) agrees with the result in Ref. [2]. Indeed, we have compared our full effective lagrangian with the complete result given in eqs. (3.47), (3.48)



of Ref. [2]. We found agreement for all terms apart from the quadratic (chiral-breaking) B-type ones. For these terms there is a mismatch proportional to the  $c_A$ 's. However, this is probably due to a typographical error, since the missing terms are included in eq. (3.49) of the same Ref., which displays B-type masses only.

Notice that in most cases of interest, it is  $(c_B T_B)_{A\alpha} = 0$  so that  $\tilde{\Gamma}_{A\alpha} = \Gamma_{A\alpha}$ . However, one has in general  $(c_B T_B)_{A\alpha} \neq 0$  in the interesting class of models where the MSSM Higgs doublets have the interpretation of pseudo-Goldstone bosons of an accidental  $G \times G$  symmetry of the Higgs sector of the superpotential. Models of this type have attracted attention [5][6][7] as they offer an elegant solution to the doublet-triplet splitting problem. We devote the remainder of this section to briefly recall their properties and to discuss the implications of (14)(15) on the pseudo-Goldstone masses. The Higgs superpotential is supposed to have the form  $W_h = W_1(\Psi_1) + W_2(\Psi_2)$ , where  $\Psi_1$  and  $\Psi_2$  are separate sets of fields that transform non-trivially under  $G$ . Thus  $W_h$  has a  $G \times G$  symmetry, and we indicate respectively with  $T^{(1)}$  and  $T^{(2)}$  the  $G$  generators acting on each sector (the gauge generators are then given by  $T_A^{(1)} + T_A^{(2)}$ ). When  $\Psi_1^0$  and  $\Psi_2^0$  independently preserve supersymmetry (*i.e.*, the contribution to the gauge  $D$ -terms is zero in both sectors), and the S.S.B. pattern is  $G_1 \rightarrow H_1$ ,  $G_2 \rightarrow H_2$  with  $H = H_1 \cap H_2$ , there is a doubling of the Goldstones belonging to the subspace  $G/H_1 \cap G/H_2$ .<sup>3</sup> For each generator in this set, there is a ‘‘gauge’’ Goldstone eliminated by the super-Higgs mechanism, but in addition there is a physical massless chiral superfield. We can associate these fields to  $G \times G$  generators  $\tilde{T}_{\tilde{A}} = r_{\tilde{A}}^{(1)} T_{\tilde{A}}^{(1)} + r_{\tilde{A}}^{(2)} T_{\tilde{A}}^{(2)}$  ( $\tilde{A}, \tilde{B}, \dots$ ), where  $r_{\tilde{A}}^{1,2}$  are numerical coefficients defined so that the vectors  $p_{\tilde{A}} = \tilde{T}_{\tilde{A}} \Psi_0$ , have unit norm and are orthogonal to the gauge Goldstones, *i.e.*,  $e^\dagger p = 0$ . For instance in [5][6][7]  $G = SU(6)$  and  $H_1 = SU(4) \times SU(2) \times U(1)$ ,  $H_2 = SU(5)$  with  $H = SU(3) \times SU(2) \times U(1)$ , so that the pseudo-Goldstones are just the two doublets  $H_u \oplus H_d$ . The result is just a consequence of the group algebra and of the split form of the superpotential. As already mentioned, it is clear that  $(c_A T_A)_{B\tilde{C}} \neq 0$ , and the general result (14)(15) has to be used, in order to discuss the soft terms. To do so we construct the pseudo-Goldstone superfields as  $\varphi_g = \varphi_{\tilde{A}} p_{\tilde{A}}$ . Then we notice that the  $G \times G$  symmetry of  $W_h$  implies

$$\mu_{kl}^{-1} \lambda_{l\tilde{A}\tilde{B}} = -(\tilde{T}_{\tilde{A}} \tilde{T}_{\tilde{B}} \Psi_0)_k = -(\tilde{T}_{\tilde{B}} \tilde{T}_{\tilde{A}} \Psi_0)_k. \quad (17)$$

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<sup>3</sup> Notice that, consistently with our general assumptions we are limiting ourselves to the case in which  $\Psi_{1,2}^0$  are determined before the introduction of soft breaking terms. In some realistic attempts, however, like Ref. [6] and model I in Ref. [7], this may not be the case. Model II of [7], however, satisfies our assumptions.

From whence, remarkably, the mass matrix (14)(15) of the pseudo-Goldstones depends very little on the details of the superpotential, the only parameters entering the definition of the  $\tilde{T}$ 's being the two v.e.v.'s  $\Psi_1^0$  and  $\Psi_2^0$ . It is interesting to consider the case in which  $\Gamma \tilde{Z}$  and  $\Lambda_{1,2}$  are also  $G \times G$  invariant. Then from eq. (17) we immediately get that the terms in eq. (15) proportional to  $\Lambda_{1,2}$  vanish for the  $\varphi_{\tilde{A}}$ . By writing the scalar components as  $\varphi_{\tilde{A}} = \sigma_{\tilde{A}} + i\pi_{\tilde{A}}$ , where  $\sigma$  and  $\pi$  are real scalars, it can be shown through straightforward, though tedious, calculations that the mass contributions from (14)(15) are only of the form  $\sigma^2$  and  $\sigma\pi$ , with no  $\pi^2$  terms.<sup>4</sup> Thus there remain flat directions  $\sigma = 0$ ,  $\pi \neq 0$ , which correspond indeed to genuine pseudo-Goldstones.<sup>5</sup> However, we stress that, in contrast with the case of universal soft terms [14], there are in general mixing terms  $m_{\tilde{A}\tilde{B}}^2 \sigma_{\tilde{A}} \pi_{\tilde{B}}$ , so that the mass matrix is not definite semi-positive. The appearance of these mixing terms is closely related to the appearance of D-term type splittings. For instance, when  $\varphi_g = H_u \oplus H_d$  we have

$$\mathcal{L}_{mass} = (H_u^*, H_d) \begin{pmatrix} m_0^2 + \Delta^2 & m_0^2 \\ m_0^2 & m_0^2 - \Delta^2 \end{pmatrix} \begin{pmatrix} H_u \\ H_d^* \end{pmatrix} \quad (18)$$

where  $m_0^2$  and  $\Delta^2$  are soft mass parameters. Then the general signature of models of this type is  $m_u^2 + m_d^2 - 2|B\mu| = 0$  at the GUT scale. Finally, another possibility given by non-universal soft terms is to allow an explicit breaking of the  $G \times G$  symmetry by the  $\Gamma$ ,  $Z$  and  $\Lambda$ 's themselves: in this case a positive diagonal piece can be added to the above mass matrix, giving the possibility to stabilize the GUT scale tree level potential at  $H_{u,d} = 0$ . In this case however the prediction (18) is lost.

**3.** What we have done so far was limited to the scenario in which, in the parametrization of Ref. [11], the source of supersymmetry breaking interacts with observable matter only in the Kahler potential. In more general scenarios, as discussed for instance in Refs. [11] and [15], there will also be susy-breaking feed-down via superpotential couplings. In this situation we do not expect for the soft terms the restricted form of eq. (3). In what follows we just want to sketch how the integration of the heavy GUT modes would be performed in the most general situation.

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<sup>4</sup> We assume that  $\varphi_{\tilde{A}}$  contains no  $H$  singlets. In this situation  $\Phi_0^\dagger \tilde{\Gamma} T_A \Phi_0 = 0$  implies also  $\Phi_0^\dagger \tilde{\Gamma} T_A^{(1)} \Phi_0 = \Phi_0^\dagger \tilde{\Gamma} T_A^{(2)} \Phi_0 = 0$ , which is also of considerable help in the computations.

<sup>5</sup> This could be deduced by inspection of the full-potential inclusive of soft terms, before integrating out the GUT fields. In this respect, the absence of the  $\pi_{\tilde{A}}^2$  terms constitutes a non trivial check of eqs. (14)(15).

By a field redefinition we can write all the soft terms which are linear in  $\eta$  as a  $d^2\theta$  integral. Then, using the same notation as before, the soft terms take the form

$$\mathcal{L}_{soft} = \int d^2\theta \{ \psi^2 + \psi\varphi^2 + (\psi + \psi\varphi + \psi\varphi^2 + \psi^2)\eta + \dots \} \quad (19)$$

were  $M_G \sim 1$ . Again we can proceed as before, and redefine  $\psi \rightarrow \psi - (\eta + \varphi^2 + \varphi\eta + \varphi^2\eta)$ . After which the integration of  $\psi$  and  $V$  are clearly independent and go through along the same lines as before. In particular we can cast the contribution from the  $V$  integration in the form (14). This shows that even with general soft terms the only universal gauge contributions are represented by D-terms. This result, however, can be important only in particular models. In fact, as we show below, in the most general situation the decoupling of the heavy chiral sector can in principle lead to completely split soft terms. In passing, we remind one well known potential problem of the case of general soft terms, which is that of the hierarchy stability [16][12][2]. In order to maintain the hierarchy after the above field redefinition the term  $\varphi^2\eta$  must be absent from the superpotential. The conditions for this to happen are again rather model dependent, though in particular models a symmetry might be at the basis.

Let us now comment on the wide possibility of mass splittings offered by the general case. Consider just the following terms in the lagrangian

$$\int d^2\theta \left\{ \frac{1}{2}(\mu_1)_{kl}\psi_l\psi_k + (\mu_2)_{k\alpha}\psi_k\varphi_\alpha\eta \right\} \quad (20)$$

where  $\mu_{1,2} = \mathcal{O}(M_G)$  and are in general only  $H$ -symmetric. Upon integrating out  $\psi$  we get the soft mass term

$$\int d^4\theta \varphi^\dagger \mu_2^\dagger (\mu_1 \mu_1^\dagger)^{-1} \mu_2 \varphi \eta \bar{\eta} \quad (21)$$

which is also in general only  $H$ -symmetric! In the aligned case discussed in sect. 1, not only  $(\mu_2)_{k\alpha} = (\mu_1\Gamma)_{k\alpha}$ , which already typically implies a bigger symmetry in (21), but there is also an additional term  $-|\Gamma_{k\alpha}\varphi_\alpha|^2$  which exactly cancels the one above (see the second term in eq. (16)); we are assuming the broken Cartan generators to be diagonal on the light states). The implications of eq. (21) are particularly important in a scenario like  $SO(10)$  Yukawa unification [4][17], where reducing the symmetry of the soft terms helps making electroweak breaking more plausible [13]. Indeed, indicating by  $\underline{10}_1$  the multiplet containing the MSSM Higgs doublets, we might have a mixing term  $\underline{10}_1(M + \underline{45}_X)\underline{10}_2\eta$ , where  $\langle \underline{45}_X \rangle \propto X$  and  $\underline{10}_2$  has a direct GUT-scale mass  $M_2\underline{10}_2^2$ . Eqs. (20)(21) then give

the soft masses  $m_{H_i}^2 = m^2|M + v_{45}X_i|^2/M_2^2$ , which split the light fields. In a similar way we can imagine of coupling the matter representations to some heavy vector  $\underline{16} \oplus \bar{\underline{16}}$ . Then we can get “vertically” split masses of the form  $|a + b(B - L) + cT_{3R}|^2$ , where  $a$ ,  $b$  and  $c$  are complex so that there are 5 free parameters which can completely split the  $\tilde{Q}, \tilde{U}_c, \tilde{D}_c, \tilde{L}, \tilde{E}_c$  within a family. In this way we can get very general soft masses, the predictivity on the Yukawas notwithstanding. Of course, in particular models, the terms we are describing could be absent by the same reason that renders the MSSM fields light. For instance, one might expect that  $\underline{10}_1$  couples to heavy fields only via  $\underline{10}_1(\underline{45}_{B-L})\underline{10}_2\eta$  which reproduces the superpotential term implementing the “Dimopoulos-Wilczek” mechanism of doublet-triplet splitting [18][19] ( $\langle \underline{45}_{B-L} \rangle \propto B - L$ ). But this is not necessary. Even when a global symmetry is responsible for the specific form of the original superpotential (like in ref. [17]), it is well possible that the susy breaking spurion  $\eta$  itself, representing now the v.e.v. of a field, transforms under the same symmetry. A new class of soft terms is then allowed. On the other hand, what is really striking about the aligned soft terms of eq. (3), is that, even by allowing light-heavy mixings that individually look like the ones discussed in this section, the final result eqs. (14)(15) still bears a rather good memory of the original gauge symmetry.

**4.** We have presented a rather compact way of deriving the tree level effective lagrangian below the GUT scale in softly broken supersymmetry. We focused first on soft terms coming from a *hidden sector*. We stressed how, in the superfield formalism, supergauge invariance plays an important role in leading to the final result. This is conveniently written in terms of the matrix  $\tilde{\Gamma} = \Gamma - c_A T_A$  and of the so called D-terms. Its form is rather constrained in cases of interest. We discussed the splittings of sparticles embedded in a single GUT multiplet. In  $SO(10)$  and  $SU(5)$  these respect the  $SU(5)$  symmetry, where in  $SO(10)$  the reduction to  $SU(5)$  is determined just by a universal D-term. In  $SO(10)$  a further class of splitting effects is forbidden by the requirement of  $R$ -parity conservation. However effects in this class can become important in  $E_6$ , due to the possible existence of  $R$ -odd heavy vector superfields. These can lead to complete splitting of the sparticles within one family. This is an interesting fact that had not been noticed before. We also have pointed out that a similar result may hold, independent of the unified gauge group, in a situation where the most general soft terms appear. This would suggest a non-minimal scenario for supersymmetry breaking. These last two remarks on intrafamily splittings could be very important for scenarios, like  $SO(10)$  Yukawa unification, which are otherwise rather

constrained by their low-energy implications. They also further confirm that the study of sparticle spectroscopy will be of crucial help in selecting among various scenarios for physics close to the Planck scale.

It is a pleasure to acknowledge stimulating discussions with Riccardo Barbieri, Glennys Farrar, Dan Kabat, Hitoshi Murayama, Alex Pomarol, Uri Sarid, Scott Thomas and Fabio Zwirner. This work was supported by the National Science Foundation under grant PHY-91-21039.

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